



Review Article

## Fuel consumption mathematical models for road vehicle – A review



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### Abstract

Since the invention of the automobile, engineers and researchers alike have worked towards improving the automobile in various ways from safety, handling and performance to efficiency and durability. As technology in the IT and computing sector evolves into a very helpful tool for detailed calculations, an advantage and possibility for detailed models is there to assist with very detailed assessment on fuel and energy consumption on today's vehicles. This review is meant to explore in detail what has been achieved by years of joint research through advanced modelling and the following factors such as emissions software and how these models play an important role in sustainable road transport for the masses. The mathematical models also display varying characteristics where models are created striking a balance between complexity, accuracy, and the number of variables to be included.

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## 1 Introduction

Fuel or energy consumption modelling as it has come about today has taken a long journey from where it first began. The factors behind the need to create fuel consumption models come from several factors including on one side, worldwide awareness on the protection of the environment and on the other side from an engineer's point of view is to improve efficiency may it be internal combustion engines or the electric motor. Modern mathematical and simulation software can perform thousands of calculations in a fraction of time which has substantially improved fuel consumption modelling work over a huge leap in the technicality, accessibility, complexity, and accuracy throughout the years to this very day. In this review, a detailed view of some of the key fuel consumption models was conducted which were milestones in the quest for fuel efficiency and fuel consumption prediction and modelling.

The transportation sector in 2019 was responsible for 27% of the world's total energy consumption [1]. As a contributor of greenhouse gases, the transportation sector the second largest source of greenhouse gas emissions with 32% of total CO<sub>2</sub> emissions. Among the fuels consumed were gasoline, diesel, heavy oil, and jet fuel. The statistics are one of the very reasons research into more efficient automobiles and alternative energy usage is crucial to preserve the environment and provide an overview of what engineers need to achieve in predicting fuel consumption rates and how much needs to be improved.

There are various number of different approaches in improving and optimizing fuel consumption. Modelling fuel consumption has been a long-time industry practice which has been researched and improved as there are several different variables to select as a main input variable.

Traffic data is one of the essential inputs to be integrated into tools in specifically modelling and simulating traffic for a given area. Software such as TRANPLAN [2] or MINUTP [3] figures and data

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obtained either through simulation or real-life observation provides output data such as average speed and total kilometers travelled through a given route or arterial network. The output data is then used as a basis to form fuel consumption models.

This paper is arranged in the following chapters to cover various approach of mathematical modelling, consisting of microscopic, mesoscopic, analytical, empirical, and variable-based which are used to model fuel consumption of road vehicles.

## 2 Road Transport Model

### 2.1 Microscopic Model

The main input variables for microscopic models are instantaneous speed and acceleration data to estimate vehicle fuel consumption and emission rates. The output data from the model also results in instantaneous fuel consumption and emissions readings. One can see a general form of a microscopic model in the following Eq. (1) [4].

$$e_i(t) = \sum_j e_i(c_j, x_j(t)) \quad (1)$$

where

- $i$  = pollution particle type or fuel consumption,
- $c_j$  = vehicle type
- $j; x_j(t)$  = instantaneous variables of vehicle  $j$  at time  $t$ ;
- $e_i(c_j, x_j(t))$  = emission of species  $i$  (or the fuel consumption) for vehicle  $j$  at time,  $t$ ;
- $e_i(t)$  = total amount of emissions of species  $i$  (or the fuel consumption) generated at time  $t$  in a given area.

This general form of a microscopic model can then the basic building blocks to form load-based models, regression-based models, neural network models, and emission maps. The resulting output data can then be utilized to form microscopic models capable of estimating instantaneous vehicle fuel consumption and emission output rates. Microscopic models which utilize instantaneous power can also be used as a base variable to be derived instantaneous fuel consumption and in the aggregated form, enables compatibility with the estimation of network-wide effectiveness. Discrete combinations of both vehicle characteristics, and road data stand as essential requirements for the estimation of the output data for these models. As fuel data characteristics display high fluctuations between different vehicles, these models are more suited to individual projects instead of industrial applications. Among the known microscopic models widely used are:

- Comprehensive Modal Emission Model
- VT-Microscopic Model

#### 2.1.1 Comprehensive Modal Emission Model

In 1997, the Comprehensive Modal Emission Model (CMEM) was designed by An et al. [5] was created to produce a wide databank of emissions inventory to be later be compatible with a number of transportation models. Several required parameters are needed to execute computations including calibration to estimate these where at its highest resolution models were representing microscopic transportation information e.g., vehicle trajectories including vehicle location, speed and acceleration. Selected driving cycles (speed versus time) are also computed on a second-by-second basis. Other aggregated data include average travelling speed, acceleration statistics. At a lower resolution, these microscopic models can produce total vehicle volume and average speed about the entire designated regional network. The CMEM model takes acceleration, air-fuel equivalence ratio, fuel rate, speed, road grade, and in more detail, accessory use as inputs [5].

300 vehicles were tested where modelling data were collected from a dynamometer. The model being as comprehensive as it is uses six main modules which are used to produce output data for engine power, engine speed, air-fuel ratio, fuel usage, engine-out emissions, and a catalyst pass fraction. The core module, the fuel usage module correlates with vehicle power demand, engine speed, and air-fuel ratio [5].

CMEM consists of six different modules but the module specifically related to fuel consumption is through the calculation of the fuel rate as in Eq. (2):

Fuel Rate Module

$$FR \approx \phi \left( kNV + \frac{P}{\eta} \right) \frac{1}{44} \quad (2)$$

where

$FR$  = fuel-use rate (g/s)

$P$  = engine power output (kW)

$k$  = the engine friction factor, which can be determined based on a measured EPA urban cycle fuel economy miles per gallon

$N$  = engine speed (rps)

$V$  = engine displacement (liter); and

$\eta$  = a measure of indicated efficiency.

The main unit of measure for CMEM are emissions but fuel consumption, power demand, and combustion efficiency are the main variables needed to calculate fuel consumption in high detail. Although the model has its drawbacks it is capable of executing detailed outputs and considered to be among the best emissions models created [6].

### 2.1.2 VT-Microscopic Model

The VT-Micro model [7] was the work of three researchers from Virginia Tech University and is capable of precisely predicting real-world fuel consumption. Instantaneous speed and acceleration are the main inputs in the VT-Micro model as the outputs are instantaneous fuel consumption and emission rates.

The VT-Micro model covers four different driving characteristics or modes which include accelerating, idling, cruising, and decelerating. Between the transition of driving characteristics, the model takes in account the emissions rate of sensitivity which are also coupled with natural algorithms making sure the models produce non-negative fuel consumption and emission rates. Rakha et al. [7] combined linear, quadratic, and cubic speed and relation terms in the final regression models as it was compatible with the ORNL data having mostly at least 0.70 MOE readings producing the least number of R2. MOE represents derivations of Method of Effectiveness of the VT Micro model as displayed below in Eq. (3).

$$\ln(MOE_e) = \sum_{i=0}^3 \sum_{j=0}^3 (K_{i,j}^e \times u^i \times a^j) \quad (3)$$

where

$MOE_e$  = instantaneous emission rate (mg/s)

$K_{i,j}^e$  = Model regression coefficient for MOE (“e” at speed power “i” and acceleration power “j”)

$u$  = Instantaneous vehicle speed (km/h)

$a$  = Instantaneous vehicle acceleration (m/s<sup>2</sup>)

As a significant difference exist between acceleration and deceleration modes due to the sensitivity of the fuel consumption rate to speed, the models for positive acceleration and deceleration are develop in separate segments to compensate this difference. Another feature behind the methodology of the VT-Micro is to compensate for negative fuel consumption readings. To overcome this, natural algorithms are then integrated as part of the models. These two advantages make the model help buffer out these discrepancies to aim for better accuracy in output. ORNL test vehicles were used in the validation of the model [7].

Park et al. [8] further increased vehicle test speeds, as he claimed that past microscopic models, like CMEM and VT-Micro has a vehicle test limit of 128 km/h and had developed four new drive cycles which are the STEP cycle for cruising conditions with an increment to the limit of 160 km/h. The other three cycles named FAST, NORMAL, SLOW covers specific variations of acceleration and deceleration profiles. The model is validated with a recorded accuracy of as low as +-4% compared to empirical data.

VT-Micro provides further flexibility in usage expansion as it can also be incorporated within the INTEGRATION simulation microscopic traffic package to further assist with traffic engineering [8].

## 2.2 Mesoscopic Model

The elemental model which was proposed by Herman [9] expressed fuel consumption in urban conditions with a function relating linearly the average time per unit distance. These models were only applicable to speeds up to 55 km/h as they did not include coefficient of dynamic drag at higher speeds.

Akcelik and Biggs with the support of the Australian Road Research Board [10] model extended basic formulation and is one of the earliest examples of how models are re-adjusted to adapt to the future coming automobiles by integrating adjustment factors and calibration with reference to empirical data. The base model is shown as Eq. (4) below.

$$F_x = \frac{f_i}{v_s} + 20.7 + 0.0443M \quad (4)$$

where

$f_i$  = idle fuel rate (ml/h)

$v_s$  = average speed in km/h ( $3600 x_s/t_s$ , where  $x_s$  is the total distance travelled in km and  $t_s$  is the total time travelled including any stopped time in seconds)

$M$  = (vehicle mass in kg)

The number 20.7 and 0.0443 are calibration coefficients depending on calibration needed to suit empirical data. The model is however unable to account for fuel consumption in transient changes of speed. There are no explainable mathematical trends to the data as it requires calibration for each group sample of vehicle being modelled.

Huan Yue et al. [11] used the VT-Micro model as a base to create VT-Meso. Huan also claimed that there were discrepancies in the VT-Micro where the model does not cover the effect of additional engine loads being an influence to fuel consumption and emissions. The VT-Meso model requires the average trip speed, the number of vehicles stops, and the stopped delay to create a drive profile which will be further be the building blocks for drive cycles. VT-Meso base its modelling on 4 operating modes, idling, acceleration, cruising, and deceleration. The model has been validated but requires further enhancement as it is only suitable for SI engines. Early findings from the model reveals that the VT-Meso model delivers satisfactory accuracy but can be further enhanced. The validation also unveils that the VT-Meso is a middle point between modelling speed and complexity. The model has been found to have been able to predict fuel consumption and emissions rates matching the accuracy of microscopic models for a number of different scenarios with an average offset of up to 17% between the two models.

In 2017, Chen et al. [12] revived the mesoscopic approach with the Mesoscopic data driven model with a different regression approach. In comparison to macroscopic models, mesoscopic models save computational time due to the absence of look up tables as data is processed with the use of statistical relationships. A major disadvantage of microscopic models integrated into eco-routing approach is unfortunately the heterogeneity in driving nature which reveals as two designated test routes driven produces the same fuel consumption rates. Computational demand is reduced with the application of mesoscopic models as discrete intervals of seconds as per the accuracy of microscopic models are not required in a mesoscopic model. However, accuracy is not heavily compromised as these models do not emit important factors such as real world driving data as usable input and is suitable for eco routing in fuel consumption estimation [13]. However, accuracy is not heavily compromised as these models do not emit important factors such as real world driving data as usable input and is suitable for eco routing in fuel consumption estimation [13]. Eq. (5) below is the fuel consumption model.

$$m_f = \dot{m}_s(T_e, \omega_e) + \sum_{i=0}^3 \sum_{j=0}^3 \alpha_{i,j} \cdot v^i \cdot a^j \quad (5)$$

where  $m_f$  (cc/s) is the transient fuel consumption,  $\dot{m}_s(T_e, \omega_e)$  (cc/s) is the steady-state fuel consumption,  $T_e$  (Nm) and  $\omega_e$  (rpm) are engine torque and engine speed, respectively;  $\alpha_{i,j}$  is the model regression coefficient, and  $v$  (km/h) and  $a$  ( $m/s^2$ ) are the instantaneous vehicle speed and acceleration, respectively.

These optimizations being instilled provided for very promising results. The validation process was conducted similar if not identical to other recognizable models. Mean Average Percentage Error (MAPE) was the metric being chosen to validate Error values in the prediction of fuel consumption. As the model was compared to MOVES (look-up table and drive-link schedule) and EMPFAC results show an MAPE

difference of 17% lesser. The model is new and shows a lot of potential for improvement with further research.

### 2.3 Analytical Model

These models are focused mainly on modelling multi sub models representing various sub systems involving in-cylinder parameters, pump systems, crankshaft dynamics, etc. These models often involve vehicle powertrain parametric specific to the vehicle along with principles of physics and are represented with detailed mathematical formulas. Processing the input within these models are usually very time consuming and meant for a very specific system.

Khayyam et al. [14] in 2008 develop the Analytical Model Fuel Consumption (AMFC) with the modelling approached based on a number of modules starting first with a power balance equation where left-hand-side term calculates the generated power, and the right-hand-side terms give the consuming powers. With the equation values a simulation is ran based on factors such as road-slope, road friction combustion, wind-drag, accessory, and vehicle efficiencies. The iteration of the simulation is then calculated to produce speed and acceleration. Four sets with different values from the factors mentioned above. The model concluded that friction plays an avid role in fuel consumption. The model is very calculation intensive. The model begins with the involvement of combustion energy in the form of an Eq. (6) and (7).

$$\dot{m}_{fuel} = \frac{P_{road-friction} + P_{drag} + P_{slope} + P_{accessory}}{\left( q_{(combustion)} + \eta_{otto} + \eta_{fuel-air} + \eta_{mechanical} + \eta_{heat loss} \right)} \quad (6)$$

using the basic formula above, a simplified model derived from the fuel map obtained is shown in Eq. (7) below.

$$y = 0.002x^2 + 0.0481x + 1.2557 \quad (7)$$

where  $y$  = fuel flow (litres/second) as a function of  $x$  = mechanical power (kW).

To optimize the mass flow rate of fuel combustion, the quantities of the denominator in the right-hand side of equation must increase, or the quantities of the numerator must decrease. To achieve this, it is necessary to analysis the thermal behaviour of the vehicle under different operating conditions. The total fuel consumption in this process is displayed in Eq. (8).

$$m_{fuel} = \int_0^T \dot{m}_{fuel} \times dt \quad (8)$$

Hillion et al. [15], demonstrates a modelling approach which focuses on KIM (Knock Integral Model) which includes crankshaft angle parameters at the start of the injection process and combustion. Within cylinder parameter cylinder pressure, cylinder temperature, the rate of gas burned and fuel-air ratio. In this case the model was adjusted to function with HCCI diesel engines with a variable turbocharger. Livengood and Wu [16], were the authors of the KIM model and is referred to as essential by many analytical model creators which involve injection modelling.

### 2.4 Empirical Model

Empirical modelling, which usually has an approach of model building with heavy reference on actual data collected by experiments conducted in real life conditions has been adopted in many research papers in the field of powertrain modelling and control. Datasets can either be collected manually or be obtained online from several sources. Sources include kms travelled, fuel consumption throughout this specific distance.

Scillieri et al. [17] with the concept of feedforward control of a direct-injection SI engine including dynamic parameters (engine speed, intake manifold pressure, intake manifold burned gas rate, in-cylinder crank angle domain, torque, EGR implementation). The controller design, the baseline controller involved an air fuel loop and fuel loop module. The model was run through a nonlinear based simulation with a feedforward algorithm to improve transient responses of the model. The study is valid only for SI engines. The core equation for this study in relation to fuel consumption is down to an equation which dictates the fuel rate through a PID controller which is represented by a fuel command Eq. (9).



$$W_f = \frac{(1 - \hat{F}_i) \hat{W}_{cyl}}{\rho} \quad (9)$$

where  $W_f$  is fuel command,  $\hat{F}_i$  is intake manifold burned fractions,  $\hat{W}_{cyl}$  is cylinder flow rate, where the internal variables of the equation involve much deeper additional mathematical models.

Pu and Yin [18] made good use of the Dynamic Programming (DP) method in order to reduce computational complexity with followed integration into MATLAB for improve computation efficiency. As already noticed hybrid systems were relatively new at the time as equations to integrate the relationship of both internal combustion engine and the electric motor had to be derived. Assisted braking is however not included in the model. However, a start-stop module and a module to account for frequent gear changes was introduced in the form of two augmented cost functions. The model was then compared to real life results involving vehicles being run on a chassis dynamometer and proven that the proposed (DP) methodology is suitable for HEV within an acceptable period. The models fuel consumption module of the model is calculated by a system cost function,  $J$  which is the accumulated fuel consumption over any driving cycle in its core version represented by Eq. (10).

$$J = \int_0^{t_f} L(x(t), u(t), t) dt \quad (10)$$

where

$u(t)$  = admissible fuel control

$x(t_f)$  = final state fuel control

$L(x(t), u(t), t)$  = the fuel consumption rate of the internal combustion engine at the time

Rakha et al. [19] developed a model with modules involving driver accelerator and brake pedal input engine speed, two transmission modules (automatic and manual), gear shifting algorithms, gear selection module for manual transmission and a torque converter module for automatic transmission. The intent of the model was to also integrate complex subsystem modelling into traffic software simulation as at the time this was widely not implemented with sub system modelling. Tests were run at Virginia Tech's test facility with ideal acceleration conditions and were calibrated to be constant. GPS providing outputs such as latitude, longitude, altitude, speed, and heading was recording at time stamp intervals of one second. The proposed model being compared to another model for fuel consumption modelling figures being developed, the (VT-CPFM-1) model at the time showcases errors of up to 12.8%. The model needs calibration but without the need of field data collection and depends mainly on driver throttle input. There are two fuel consumption mathematical models which were built to be easily calibrated as shown below in Eq. (11) and (12).

$$FC(t) = \alpha_0 + \alpha_1 P(t) + \alpha_2 P(t)^2 \quad (11)$$

$$FC(t) = \beta_0 \omega_e + \beta_1 P(t) + \beta_2 P(t)^2 \quad (12)$$

where  $FC(t)$  is the vehicle fuel consumption rate (1/s) at instant  $t$ ,  $\alpha_0$ ,  $\alpha_1$ ,  $\alpha_2$  and, are model constants that require calibration and  $P(t)$  is the instantaneous total power in kilowatts (kW) at instant  $t$ .

Semi-Empirical Fuel Use Modelling (SEFUM) modelled by Orfila et al. [20] for fast computation of fuel consumption by using a polynomial function with variables including air density, gravity, rolling resistance, fuel energy density, fuel density, mass, frontal cross-sectional area of the vehicle, and fuel use at idle. The model was then compared with the equivalent of 600 km of experimental data with another three models for fuel consumption estimation accuracy with high performance. The model is also feasible for online simulation as the computational times were very low. The model is already being put in service by the ESI-CIVITEC Company [21], where one equation could actually cater to all vehicle types in all states. The has also been tested in France for further assistance in the development of eco routing through the simulation of driver behavior [22]. The fuel consumption used in this method relies on convert in theoretical energy consumed in a time step which is evaluated in the following formula Eq. (13).

$$dE_{theo} = \left( \frac{1}{2} \rho_{air} S C_x v_{\alpha}^2 + C_r r m g + m p + m a_{\alpha} \right) v_{\alpha} dt \quad (13)$$

where

- $\rho_{air}$  = 1.2 kg.m<sup>-3</sup> is the density of the air
- $S$  (m<sup>2</sup>) = end face
- $C_x$  = the longitudinal drag coefficient,
- $C_r$  = 0.015 is the coefficient of rolling resistance
- $m$  = vehicle mass (kg)
- $g$  = 9.81 ms<sup>2</sup> is the standard gravity
- $p$  = road grade (%)
- $a_{\alpha}$  = vehicle acceleration (m/s<sup>2</sup>)
- $dt$  = time step (s)

## 2.5 Main Input Variable-Based Model

### 2.5.1 Average Speed Model

As average speed is usually the used variable for traffic simulation application and is an aggregated value. Many researchers approach modelling with average speed as it also contributes to computational speed.

Ding and Rakha [23] in a research with 1,080 artificial trips to create datasets made within a given set time using average speed, speed changes and number of vehicle stops factors concluded that the use of average speed for estimating fuel consumption and emissions however selection of a suitable model involving average speed needed to be optimized where in the research it does mention the use of a log transformation which does lower the accuracy of the model but results in higher methods of effectiveness (MOE). The proposed statistical models in the paper were found to compute fuel consumptions and emissions within 88% to 99% of microscopic models. These models were validated with promising results. The fuel consumption model being used in this model is based on a VT-Micro system with a model regression coefficient as in Eq. (14).

$$\ln(fuel) = a_0 + \frac{a_1}{u} + a_2 \bar{u}^2 + a_3 \sigma_u^2 + a_4 S + a_5 A + a_6 E_k \quad (14)$$

where

where:

- $a_{profile}$  = Instantaneous vehicle acceleration (km/h/s)
- $\bar{u}$  = Average vehicle speed
- $\sigma_u^2$  = Speed variability
- $E_k$  = Kinetic Energy
- $A$  = Total acceleration noise
- $S$  = Number of vehicle stops

Yan-Tao Zhanga et al. [24] proposed a model that would fulfill the demands of hybrid electric vehicles (HEVs) fuel consumption modelling by proposing a transient corrected model based on steady state mapping where the model accounts for the split of power source between the battery module and engine. The model is validated through simulation based on 4 drive cycles which are the steady state cycle, the Highway cycle, the UDDS cycle, and the US06 cycle. The model consists of first a Kinematic model which takes variables including wheel output torque, vehicle mass, aerodynamic air resistance coefficient, road grade angle, and vehicle speed. Torque values are then converted onto engine power value requirements. The fuel consumption model is interesting as it takes engine power value and converts this into an electric consumption rate equation which accounts for battery power demand. Fuel consumption is then modeled as effects of different factors such as wind and road grade as an extra observation to the corrections needed where the results of the model were compared to those of from the basic VT-Micro model. Ahn et al. [25] created the first VT-Micro which displayed results showcasing, the fuel consumption of hybrid vehicles is suitable for high density traffic, inclusion of calculations involving external factors such as wind and slope grade can be effectively executed in the VT-Micro

model. As traffic is flowing, fuel consumption is not significant as the engine is operating under optimal condition. The fuel consumption model here is represented by the Eq. (15) below.

$$Q_g = \frac{P_e \cdot b}{361.7 \rho_g g} \quad (15)$$

where  $\rho_g$  is the density of gasoline,  $\rho_g g$  is usually set as 7 N/Litre, and variable  $b$  (g/KW h) is the instantaneous consumption rate corresponding to the current engine torque and rotational speed. The value of  $b$  can be obtained through engine test.

Christos Samaras et al. [26] integrated modelling approaches to combine the different characteristics of traffic modelling. Three existing models were used to be able to create a hybrid model which was (AIMSUN) (microscopic model) a microscopic traffic model software, (CRUISE) (microscopic model), is used to refine the average speed model functions in (COPERT) (macroscopic model). The model further focuses on the simulation segments where the roads are free and congested. The model was validated through comparison with HBEFA estimates and the results are satisfactory. The results show the importance of factoring in congestion factors for better fuel consumption and emissions estimation. The model concludes that congested roads increase fuel consumption up to 50% in certain conditions.

### 2.5.2 Instantaneous Speed Model

As instantaneous speed allows for a short time stamp between data inputs, many researchers have opted for the inclusion of this variable for fuel consumption and emissions rates modelling. Instantaneous speed as a variable can be found in many types of models including empirical and microscopic models.

Rakha and Ahn [27] claimed that average speed can produce widely different instantaneous speed and acceleration profiles, and thus provides different fuel consumption and emissions output levels. With this conclusion improvements had to be made as there was back then the need to integrate model into the Intelligent Transportation System (ITS) directive. THE INTEGRATION [27] model was born with includes models with car-following compatibility, vehicle dynamics, lane changing logic, energy, and emission models turn instantaneous speed and acceleration level inputs to model emission and mobile source emissions. The final regression version of the model included linear, quadratic and cubic linear equations with an  $R^2$  value above 0.92 for all MOEs (method of effectiveness) where validation found the INTEGRATION model to be satisfactory and consistent compared to the Oak Ridge National Library (ORNL) field data. The validity of the model was demonstrated using sample test scenarios that include travelling at a constant speed, travelling at variable speeds, stopping at a stop sign, and travelling along a signalized arterial. The fuel consumption model here is instantaneous and is expressed in a single independent variable as in Eq. (16).

$$F = \alpha + \beta P_{tot} \quad \text{for } P_{tot} \geq 0 \quad (16)$$

$$F = \alpha \quad \text{for } P_{tot} < 0 \quad (17)$$

where

$F$  = instantaneous fuel consumption rate (ml/s)

$\alpha$  = vehicle parameter, idle fuel consumption rate (ml/s)

$\beta$  = vehicle parameter

$P_{tot}$  = instantaneous total power (kW)

Ahn et al. [25] further enhanced the VT-Micro model with emphasis on categorizing vehicles into Emitter categories and also accommodate the fuel consumption demands of coming EV vehicles. A dual regime evaluation module is suggested to model positive and negative acceleration as it significantly reduces errors. The high emitter category displays various types of suitable vehicles to be instilled in the validation of the modelling which differs significantly in air-fuel ratio variants with different power outputs and different transient characteristics. The model was validated with satisfactory results overcoming major drawbacks such as sensitivity levels with respect to the environmental impact of operational-level projects in which lane changing character and before and after characteristics of lane changing is critical. Error can be seen for the enhancement to having errors below 17%.

Rakha et al. [28] combines the use of a vehicle dynamics model in conjunction with an energy and emissions model with relation to evaluate fuel consumption and emissions rates to be integrated to the then newly introduced ITS. The model includes lane changing logic as the vehicle reaches a certain



speed. A car following model is also implemented and position of the vehicle is updated upon a decisecond duration in relation to the INTEGRATION model. Datasets for the model were also obtained from the ORNL dataset where regression models were integrated to reduce the data storage requirements of the models and also provide an average estimate of MOE rates. Acceleration constraints were also fitted to account for the relationship between maximum acceleration and vehicle speeds. Constant and variable speed, and stop sign, and traffic signal scenarios were also taken into account. The model concluded that fuel consumption and emissions rate were easier to model at constant speeds and relying on just average speed is not sufficient. The fuel consumption model here was created using actual data with additional regression models fitted as below in Eq. (18).

$$\log Z_k = \sum_{i=0}^3 \sum_{j=0}^3 B_{ij}^k \times u^i \times a^j \quad (18)$$

where

$Z_k$  = measure of Effectiveness  $k$  (e.g.  $Z_1$  = fuel consumption and  $Z_2$  = HC emissions)

$B_{ij}^k$  = constant for speed degree  $i$ , acceleration degree  $j$ , and MOE  $k$  (e.g.  $B_{1,2}^3$  = CO constant for term  $ua^2$ )

$u$  = vehicle speed

$a$  = vehicle acceleration

Froschhammer et al. [29,30] with a component-based modelling approach sub system were modelled such as valve systems and injectors making it cost effective and less reference based as to models which rely on look-up tables. The model named SIMPACK models in real time engine parameters and has been used by BMW before. Although performance of the model provides satisfactory results it is known to be very labor intensive.

### 2.5.3 Specific Power Model

Specific power has been long been widely used as favorable variable in the construction of fuel consumption models as it contributes to computational load being lesser while providing a decent amount of accuracy. Specific power with the right adjustment is also suitable for the building of hybrid models as power can be expressed in an equation which involves electric powered powertrains.

Frey et al. [31] modelled a Vehicle specific power (VSP) - based model used to model fuel consumption and emissions rate of diesel and hydrogen cell fuel buses. Data sets of the buses were obtained for validation of the mode. The model considers aerodynamic drag, rolling resistance and road grade. Fuel consumption rates of both diesel and hydrogen fuel cell buses were compared where accuracy to within 10%. The model is concluded to be appropriate for application to many vehicles with further studies. The fuel consumption model is utilizing instantaneous vehicle power VSP-based model with a trip-based approach as in Eq. (19).

$$E = \sum_{j=1}^J FR_j \times TVSP_j \quad (19)$$

where  $E$  is the total trip fuel consumption (in litres for diesel buses and in grams for hydrogen buses);  $j$  the VSP mode index,  $J$  is the number of VSP modes ( $J = 8$  for diesel buses and 6 for hydrogen buses as explained later);  $FR_j$  is fuel consumption rate for VSP mode  $j$  (l/s for diesel bus and g/s for hydrogen bus); and  $TVSP_j$  is the bustrip time spent in VSP mode  $j$ (s).

Wang et al. [32] utilized VSP for the modelling for fuel consumption with Portable Emissions Measurement System (PEMS) equipment. Four modes were initiated which were idle, low speed moderate speed and high speed. The research found that fuel consumption increases during acceleration and displayed little change during deceleration. Specific times and road were designated as the test route. The model has an accuracy between 15% to 20%.

The equation being formulated for this model is based on VSP with EPA's simplification and being later implemented into a trip-based fuel consumption model shown by Eq. (20) and (21).

$$VSP \left( \frac{kW}{ton} \right) = 2.73 \times \sin(slope) \times v + 0.05 \times v \times a + 0.0593 \times v + 0.000653 \times v^3 \quad (20)$$

here  $v$  and  $a$  are the vehicle speed and acceleration in mile/h and mile/h/s; and slope is the road slope in degrees. This is then converted into a trip-based fuel consumption model.

$$FC = \sum_{i=0}^I FR_i \times T_i \quad (21)$$

where

$FC$  = trip fuel consumption

$i$  = speed-VSP bin index and  $I$  is the number of bins

$FR_i$  = fuel consumption rate for speed-VSP bin  $i$ , (L/s)

$T_i$  = vehicle trip time spent in speed-VSP bin  $i$ , (s)

Duarte et al. [33] created a Vehicle Specific Power (VSP) model with the emphasis of generic application with propulsion systems such as spark-ignition engines, compression-ignition engines and hybrid vehicles. In the model, (SFC) specific fuel consumption place an important role as 3 further fuel consumption profiles were created to suit 3 categorized power ranges. The proposed methodology highlights how in models develop in the generic direction will have dependence on how much certification fuel consumption data is available and requires additional extrapolation in every cycle. Tests were ran comparing the VSP model with more detailed secondly intervals discrete models with a specified driving mode and drive cycle with 14 vehicles put to the test an absolute deviation of 9.2% was observed.

This model uses VSP (Vehicle Specific Power) and is later converted to a Specific Fuel Consumption (SFC) model as displayed below in Eq. (22).

$$SFC = \frac{Fuel}{|VSP_i|} \left[ \frac{g_{fuel} / s}{W / kg_{vehicle}} \right] \quad (22)$$

where  $i$  being the VSP bin.

### 3 Latest Models

Kan et al. [34] proposed an improved fuel consumption model through the GPS big data method by utilizing today's advances in networking and data storage. Although the study concludes the little differences between high- and low-resolution GPS data, more emphasis is focused on the vehicles mobile and stationary activities. The two activity modes are further expanded into hot and cold emissions representing. A route being labeled as Space-Time Segments (STPs) to quantify the path and moving parameters of a vehicle through a given route. Adopting fuel consumption equations from COPERT the researcher then implies Stationary Activity (SAs) factors to improve accuracy. Total emissions are then estimated at hot, cold and during temperature changes. The model was able to achieve a maximum accuracy of 88.6% after all refinements. The equation for fuel consumption for this model is based on a moving average COPERT based model as stated below in Eq. (23).

$$FC_{MA} = \left( 217 + 0.253V + 0.00965V^2 \right) / \left( 1 + 0.096V - 0.000421V^2 \right) \quad (23)$$

where

$FC_{MA}$  = fuel consumption (litres/s)

$V$  = vehicle speed (km/h)

Fiori et al. [35] proposed the Comprehensive Power-based EV Energy consumption Model (CPEM) to cater specially for the ever-moving trends of the automotive world that is trending towards the manufacturing of electric and hybrid vehicles. The model approach starts by evaluating power at the wheels (backward model). Instantaneous speed and EV characteristics are taken as input variables while output variables of the models are energy consumption instantaneous power and state-of-charge. The factors involved in the model are mass, vehicle acceleration, road grade, frontal area of the vehicle, aerodynamic drag, and outside environments (road surface type, road condition and vehicle tire type) represented by coefficient values. Included is also a regenerative braking model as energy is recovered. Available datasets from renowned research were the basis for the validation of this model. The latest drive cycles including the NEDC, WLTP, HWFET, WMTC, UDDS, and US06 were implanted in the validation process of the model. For this reason, the average error on the entire six driving cycles analyzed is 5.86%.

## 4 Conclusion

The models being displayed above displays a wide array of ways on how fuel consumption can be estimated whereby the next future fuel consumption models will be tailored around velocity and acceleration of a vehicle rather than specific analytical modules. The derivation of fuel consumption models such as analytical and microscopic models involves looking at the detailed internal systems of an Internal Combustion Engine(ICE) which is suited more fuel consumption estimation of a very specific system whereas moving on to models such as the empirical and variable based models are suited to a more generic and general mode of application where there are more emphasis on assumptions and constants being integrated into the models to reduce the number of variables with the aim to derive the equation of the fuel consumption model to take on a more linear form for the reduction of computational load while being applicable for a wide range of vehicles. The aim for the creation of a mathematical model is to first identify compatibility and suitability for each application and the environment of which the model is to be used. Considering the wide array of vehicles being produced involving various methods of accelerating a vehicle, simple linear models which are based on a vehicle's power, acceleration and weight could be the flexible direction in the way future fuel consumption models are created.

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## Declaration of Conflict of Interest

The authors declared that there is no conflict of interest with any other party on the publication of the current work.

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