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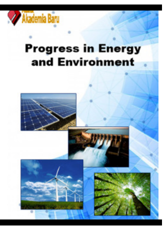

Investigation of Shape Parameter for Exponential Weight Function in Moving Least Squares Method

Research Article

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ABSTRACT

The Moving Least Square (MLS) Method is an approach which is used in meshfree solutions and data approximation. In the formulation of MLS, the exponential weight function is applied to influence the formation of continuous approximated functions. The governing factor in the weight function is the shape parameter, yet the effect of shape parameter to the data approximation is unclear. Therefore the objective of the paper is to investigate the effect of shape parameter to the equation formed via MLS. The three examples are studied and the results show that there is no direct relationship between shape function and prediction accuracy.

1. Introduction

It is perennial in various engineering investigations that data is collected and analysed in order to form an approximated function. Line fitting is the simplest form of statistical data analysis and can also be referred to as linear regression, while curve fitting allows non-linear relationship analysis. Some of the curve fitting techniques are applied in the boundary element techniques [1] to solve large deforming mechanics problems [2-4]. They includes the weighted residual method, collocation method, subdomain method, least squares method, Galerkin Method and Moving Least Squares (MLS) method [5].

The MLS is a highly flexible method, which can be integrated into to a vast variety of fields especially in graphics manipulation and statistical trends analysis. Weight function and its shape parameters may alter the change of prediction curve.

In the study of 3D deformation and image processing, the Euclidean distance between handle and data point would cause unwanted geometrical deformation. Such distance function is changed to the value of the total length of the shortest polyline starting from the handle to the nearest vertex on the mesh and then travels from vertex to vertex through the mesh edges. A damping factor which would slow the transformation down so that the 3D image would not get any unwanted deformation was also added [6]. The accuracy of the MLS was also improved in order to identify sharp edges. The Forward Search on points was analysed in order to identify any outliers which could be ignored, allowing an edge to be produced rather than a smooth curve [7]. Meanwhile in data statistical fitting,

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the smoothness of the curve improves as the basis function polynomial order is increased. The shape parameter also played a large factor in the curve fitting, where the larger the value, the closer the curve will fit to specific points [8]. However this will also sacrifice the smoothness of the curve.

The importance of weight function becomes more apparent when MLS are further developed into a variety of variants. For instance, adaptive MLS approach, where the radius of the influence domain in the weight function varies according to the concentration of the sampled points was studied [9]. Multiscale moving least square where the scale set changes according to the given set point was suggested [10].

Better understanding on effect of shape parameter will bolster the manipulation of MLS in engineering analysis. Therefore in this study, the effect of various shape parameters of exponential weight function on the shape of the weight function plot, unity criterion of weight function and result accuracy are investigated.

2. Moving Least Squared Method

The purpose of the MLS method is to create a smooth function based on random scattered points, usually labelled as nodes. This enables the user to interpolate the data, allowing for better analysis of the field. The array of data can be represented as x,

$$x = [\ x_1\ x_2\ x_3 \ldots\ldots\ldots x_n\ ] \tag{1}$$

$$u^h = P_1 a_1 + P_2 a_2 + P_3 a_3 + \ldots\ldots\ldots + P_m a_m = \sum_{i=1}^{n} P_i a_i \tag{2}$$

where u^h represents the approximated function of summation of P and a. P represents the basis function, a the coefficient of P, and m the number of monomials in the basis function. It can also be expressed in matrix form as:

$$u^h = [\ P_1\ \ P_2\ \ P_3 \ldots\ldots P_m\ ] \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_3 \end{bmatrix} = \mathbf{P}^{\mathrm{T}}\mathbf{a} \tag{3}$$

Minimization of weighted residual, or norm of weighted difference (J) is taken place as in Eq. (4), where R is the residual, W the weight function. J is similar to the standard deviation, σ^2, with the difference that J has an addition of weight, W and the exclusion of number of nodes.

$$\mathrm{J} = \sum_{j=1}^{n} W_j R_j^2 \tag{4}$$

where

$$\sum_{j=1}^{n} R_j = \sum_{j=1}^{n} (u_j^h - u_j) \tag{5}$$

Therefore,

$$\mathrm{J} = \sum_{i=1}^{m} \sum_{j=1}^{n} W_j (P_{i,j} a_i - u_j))^2 \tag{6}$$

The minimum value of constant, a, has to be obtained in order for the results to be accurate. This can be done by considering the differentiation of J with respect to a and let to be equals to 0.

$$\frac{\partial J}{\partial a} = 0 \iff \sum_{i=1}^{m}\sum_{j=1}^{n} W_j P_{i,j}^2 a_i = \sum_{i=1}^{m}\sum_{j=1}^{n} W_j u_j P_{i,j} \quad (7)$$

$$\mathrm{Aa} = \mathrm{Bu} \iff \mathrm{a} = \mathrm{A}^{-1}\mathrm{Bu} \quad (8)$$

where $\mathrm{A} = \sum_{i=1}^{m}\sum_{j=1}^{n} W_j P_{i,j}^2 a_i$ and $\mathrm{B} = \sum_{i=1}^{m}\sum_{j=1}^{n} W_j u_j P_{i,j}$

$$\mathrm{u}^h = \mathrm{P}^T\mathrm{A}^{-1}\mathrm{Bu} = \emptyset \mathrm{u},\ \emptyset = \mathrm{P}^T\mathrm{A}^{-1}\mathrm{B} \quad (9)$$

The weight function (W) is required in calculating the matrices for A and B in Eq. (8). The weight function is used to improve the fit of the line. It allows some of the nodes in the dataset to have more influence over the others on the resulting line, creating a best fit line which caters better to the requirement [1]. The weight function is crucial in creating a function much more accurate as compared to other approaches such as the weighted residual method and collocation method, which do not make use of a weight function [11]. The weight function considered in the study is the exponential weight function and it was compared with the quartic weight function [1].

Quartic Weight Function:

$$W(r) = \begin{cases} 1-6r^2+8r^3-4r^4, & r \leq 1 \\ 0, & r > 1 \end{cases} \quad (10)$$

Exponential Weight Function:

$$\mathrm{W(r)} = \begin{cases} e^{-\left(\frac{r}{\beta}\right)^2}, & r \leq 1 \\ 0, & r > 1 \end{cases} \quad (11)$$

where β is shape parameter. In the study values of the shape parameter are set at the range of $1 \leq \beta \leq 50$ were set with intervals of 10.

3. Numerical Experiments

The effects of shape functions are investigated through some numerical examples. In this section, two type of case studies are considered: (3.1) pure conduction problem; and (3.2) design of experiments studies.

3.1 *Pure Conduction Problem*

Temperature distribution is computed for the case where pure conduction problem was solved with constant boundary temperature 600□, -300□, -400□ and 800□. This heat transfer simulation was conducted using SolidWorks 2016, shown in **Fig. 1**. The data obtained was the resulting temperature for each node on the top face of the model after the heat had been applied. The data was then applied into the code for Moving Least Squares (MLS) with the quartic and the exponential weight function used.

Fig. 2 shows the relationship between the shape parameter and exponential weight function. The general shape of the graph has a downwards trend towards the x-axis. However this trend is not as apparent as the shape parameter value increases, as shown with a value of 50. This trend occurs due to the fact that the shape parameter is the denominator of the fraction value in the exponential weight function. As the value increases, the fraction value decreases, causing the overall weight result to decrease at a less significant rate.

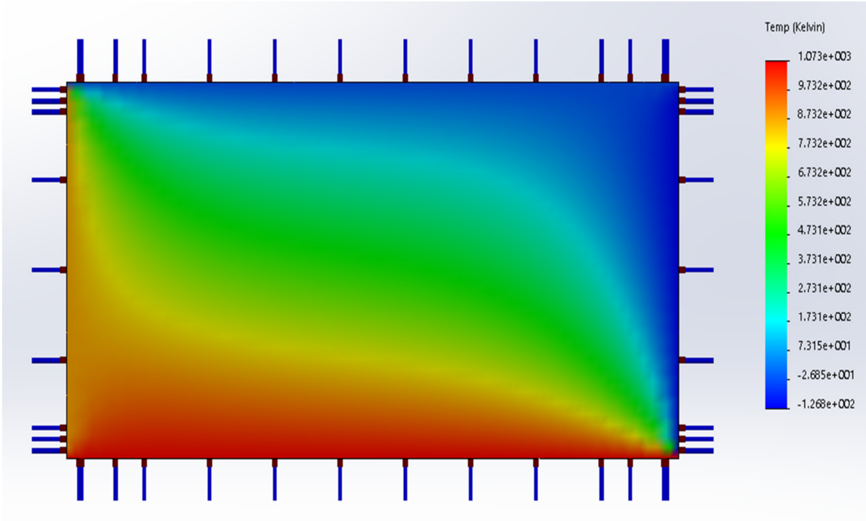

Fig. 1. Solid rectangular block

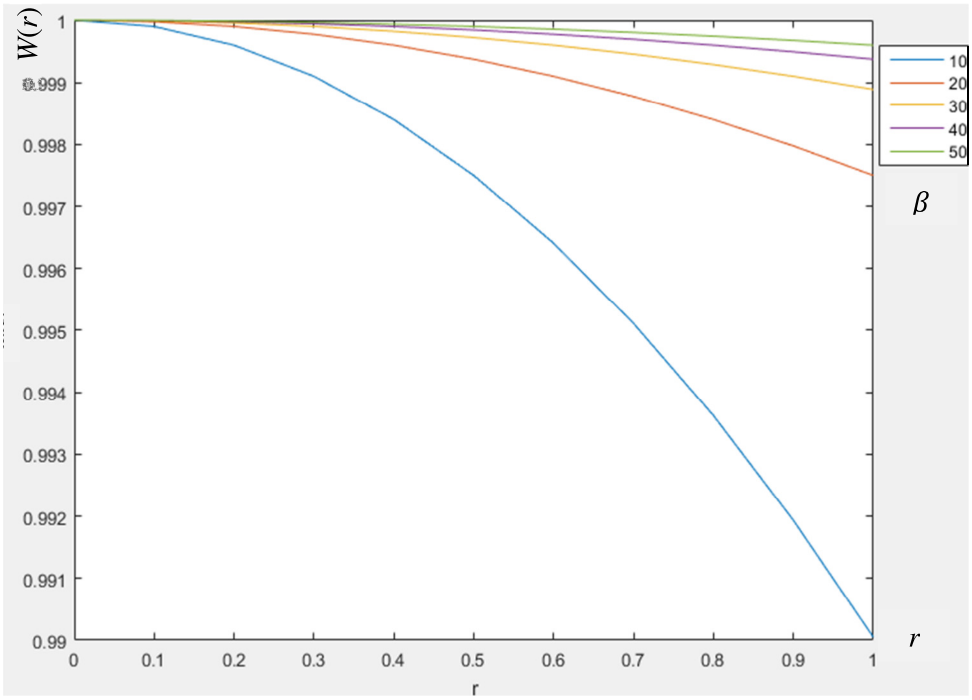

Fig. 2. Effect of shape parameter on exponential weight function

The excerpt of the data obtained for **Fig. 1**, the predicted temperature values obtained using the quartic weight function and exponential weight function is shown in **Table 1**. Significant difference in temperature can be observed between the quartic and exponential weight function. The temperature obtained using the exponential weight function converges much more closely to the original temperature values.

Table 2 displays the average percentage error calculated for quartic and exponential weight. It can be seen that the average error percentage is much lower for the exponential weight function as compared to the quartic one. However the difference in average percentage between the varied β is very small with all variants almost having the same percentage.

Table 1 Calculated temperature values obtained from MLS with Quartic and Exponential weight function

X (mm)	Y (mm)	Temp(K)	Quartic (K)	β (K)				
				10	20	30	40	50
-100	-50	-27	2857.256	245.387	245.459	245.473	245.477	245.479
-100	-42	873	2458.522	363.305	363.381	363.395	363.400	363.402
-73	-47	431	-1394.842	164.312	164.335	164.340	164.341	164.341
-73	47	1050	-501.957	983.340	983.362	983.366	983.367	983.368
-71	-50	-26.9	-1755.579	113.502	113.523	113.526	113.528	113.528

Table 2 Average error calculated

	Quartic	β				
		10	20	30	40	50
Average Error (%)	4926.732	46.257	46.285	46.290	46.292	46.293

3.2 *Design of Experiments Studies*

Rehman [12] investigated the relationship between voltage and time with the deposition yield of bioactive glass composite coatings. The comparison between data of the experiments and the predicted value of MLS can be shown as in **Table 3**. It can be seen that the exponential weight function application has returned a yield data which is much closer to the original one as compared to the quartic weight. As the value of the shape parameter increases, the yield data tends to become more similar in value when rounded off to the third decimal place.

The average error for this data set is significantly higher, as shown in **Table 4**. This is due to the fact that the results obtained in the first row of data on all iterations of the weight functions are very inaccurate. However the average error for exponential weight function at shape parameter 5, has an extremely low average error as compared to the other shape parameter values. This is due to the fact that the parameter is able to generate a function which conforms closely to the original data, where the original data in the first row of the previous table, **Table 3** has a deposition yield of 0.0034% and the new result for shape parameter value of 5, is 0.0104%. This means that in this scenario a lower value for β should be selected when trying to obtain the curve fitting function using the MLS method.

Jahal et al. [14] optimized the conformal cooling channels via design of experiments as well. The difference between experimental and predicted values are shown in **Table 5**. In this scenario, it can be observed that the values obtained both for quartic and exponential weight functions are closely similar to the original cooling time. This also further validates the credibility of the MLS function as it is able to create a best fit function, even though the previous example in **Table 6** had a large difference between the original values and results.

In **Table 6** it can be seen that the exponential weight function is much more accurate than the quartic weight function. A similar trend to the previous example were the larger the shape parameter value, the error tends to decrease at a much more minute rate from 10 to 40. This can be clearly shown in Figure 5 where the accuracy for the shape parameter results are plot against its value. However the error increases slightly after 40. This is different from the scenarios previously addressed. This could be due to the effect of the shape parameter.

Table 3 Yield values obtained from via MLS

Voltage (V)	Time (s)	Deposition yield (%)	Predicted deposition yield (%)						
			Quartic	Exp. with β = 5	Exp. with β = 10	Exp. with β = 20	Exp. with β = 30	Exp. with β = 40	Exp. with β = 50
70	60	0.0034	0.107	0.010	0.0355	0.047	0.050	0.050	0.050
70	120	1.3750	1.332	1.378	1.378	1.374	1.373	1.372	1.372
70	180	1.0101	1.064	1.012	1.023	1.029	1.031	1.031	1.031
80	60	0.4564	0.340	0.408	0.359	0.349	0.347	0.346	0.346
80	120	2.3004	2.278	2.273	2.271	2.274	2.275	2.275	2.275
80	180	1.8437	1.798	1.829	1.808	1.803	1.802	1.802	1.801
90	60	0.4776	0.490	0.584	0.542	0.526	0.523	0.522	0.521
90	120	2.5693	2.684	2.655	2.656	2.661	2.662	2.663	2.663
90	180	2.3179	2.298	2.345	2.327	2.319	2.317	2.317	2.317
100	60	0.5252	0.556	0.539	0.584	0.580	0.579	0.578	0.578
100	120	2.6364	2.550	2.525	2.533	2.534	2.535	2.535	2.535
100	180	2.5321	2.564	2.563	2.580	2.578	2.577	2.577	2.577
110	60	0.5490	0.538	0.274	0.487	0.501	0.513	0.514	0.514
110	120	1.8567	1.877	1.882	1.902	1.894	1.893	1.892	1.892
110	180	2.6050	2.596	2.480	2.568	2.580	2.584	2.582	2.582

Table 4 Average error

	Exponential with Shape Parameter						
	Quartic	5	10	20	30	40	50
Average error	207.1396	20.74456	67.88 7	90.13901	94.844 6	96.52159	97.32105

Table 5 Cooling time obtained with quartic and exponential weight

D(mm)	P(mm)	Cooling time (s)	Predicted cooling time (s)					
			Quartic	Exp. with β = 10	Exp. with β = 20	Exp. with β = 30	Exp. with β = 40	Exp. with β = 50
4	8	14.38	14.645	14.427	14.426	14.427	14.427	14.427
4	12	14.42	14.999	14.800	14.800	14.800	14.800	14.800
4	16	15.03	15.577	15.425	15.425	15.425	15.425	15.425
4	20	16.6	16.379	16.301	16.301	16.301	16.301	16.301
4	24	17.2	17.406	17.429	17.429	17.429	17.429	17.429
5	8	14.03	13.640	13.702	13.702	13.702	13.702	13.702
5	12	14.9	14.211	14.243	14.243	14.243	14.243	14.243
5	16	15.3	14.907	14.918	14.918	14.918	14.918	14.918
5	20	16.1	15.726	15.727	15.727	15.727	15.727	15.727
5	24	16.89	16.667	16.671	16.671	16.671	16.671	16.671
6	8	12.76	12.953	13.082	13.082	13.082	13.082	13.082
6	12	13.9	13.732	13.844	13.844	13.844	13.845	13.844
6	16	14.3	14.511	14.595	14.595	14.595	14.595	14.595
6	20	14.7	15.290	15.334	15.334	15.334	15.334	15.334
6	24	15.89	16.071	16.062	16.061	16.062	16.062	16.062
7	12	12.99	13.557	13.604	13.605	13.605	13.605	13.604
7	16	14.73	14.388	14.457	14.456	14.457	14.457	14.457
7	20	15.13	15.075	15.122	15.122	15.122	15.122	15.122
7	24	15.8	15.619	15.601	15.601	15.601	15.601	15.601
8	12	13.9	13.690	13.523	13.523	13.523	13.523	13.523
8	16	14.5	14.537	14.502	14.502	14.502	14.502	14.502
8	20	15	15.077	15.090	15.091	15.091	15.091	15.091
8	24	15.3	15.311	15.289	15.289	15.289	15.289	15.289

Table 6 Average Error

	Quartic	Exponential with Shape Parameter				
		10	20	30	40	50
Average error (%)	1.9806	1.8809	1.8808	1.8808	1.8808	1.8808

It is noteworthy that the MLS can be a robust alternative tool to analyse the design of experiments, which are normally analysed via Response Surface Methodology.

4. Conclusion

The quartic weight function and exponential weight function were compared extensively in terms of accuracy and it can be found that the exponential weight function is much more accurate as compared to the quartic weight, even when the shape parameter value is varied. When the shape parameter range was set from 1 to 50, it can be found that the trend for the accuracy differs. The heat conduction simulation result showed that the increase in shape parameter value also resulted in a slight decrease in accuracy with the average error increasing. The trend for average error for the composite coating varies with the heat conduction example, where the most accurate result was when the shape parameter value was set to 5. The last example shows a trend where the accuracy improves up to a certain value before decreasing slightly. This variance in results is due to the fact that the data being dealt with in engineering, are usually very random and have no apparent pattern. This causes the shape parameter to have a varied effect on the accuracy of the MLS approach.

The shape of the weight function is heavily affected by the shape parameter, due to the argument of the function, where it is the denominator of the fraction in the exponential weight function. Therefore it can be deduced that the effects of the shape parameter differs from a case to case basis. But the exponential weight function is far superior in obtaining an accurate plot as compared to the quartic.

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