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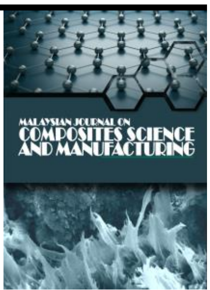

Numerical Investigation on Free Vibration Analysis of Kevlar/Glass/Epoxy Resin Hybrid Composite Laminates

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ABSTRACT

This paper investigates the free vibration analysis of kevlar/glass/epoxy resin hybrid composite laminates. The ABAQUS software is studied to obtain the natural frequency with different fiber hybridization effects. The 3D shell element models evaluate natural frequency and mode shape in the quasi-isotropic laminated composite. The numerical analysis results are studied and compared between ABAQUS and ANSYS software. It was shown that the natural frequency value between ABAQUS and ANSYS software provided almost the same value. The error value between ABAQUS and ANSYS ranged from 1.03 % to 2.30 %. It was found that the error between the experimental and numerical results was from 1.15 % to 13.51 %. It was concluded that the percentage error value was significantly dependent on the mesh size. Furthermore, the reasonable agreement between experimental and ANSYS results can better analyze the free vibration of composite laminates.

Keywords:
Free Vibration Analysis, Natural Frequency, Hybrid Composite Laminate, ABAQUS, ANSYS, Finite Element Modeling

1. Introduction

Fiber-reinforced composite materials have various applications in many engineering fields [1-2]. The fiber-reinforced composite is also being increasingly used as an alternative to conventional materials primarily due to its excellent engineering properties, such as high specific strength and the specific stiffness-to-weight ratio [3]. In addition, the viscoelastic character of composites renders them suitable for high-performance structural applications, such as aerospace, marine, automobile,

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robotics, and sporting appliances [4, 5]. As a result, there is a great deal of interest in researching and comprehending the dynamic behaviour of composite structures. The study of modal parameters based on modal loss factors (damping), modal shapes, and resonance frequencies has played an important role in dynamic structural characterization, dynamic design, damage detection, and condition monitoring [6-12].

The damping of fiber-reinforced composite materials is relatively high and affected by the material's composition. At the constituent level, energy dissipation in fiber-reinforced composites is induced by various processes such as damping at the fiber-matrix interface, viscoelastic matrix behaviour, damping due to damage, and so on. Damping at the laminate level depends on layer orientations, constituent layer properties, stacking sequence, interlaminar effects, etc. The finite element method is now widely used by engineers to conduct structural dynamic response analysis. A numerical method with a convenient solution, perfect function, and good versatility must be established based on the available finite element software. Kyriazoglou et al. [13] proposed a hybrid method that combines the resonance test with equivalent Rayleigh damping to achieve finite element analysis of the vibrational properties of laminated composites. The Rayleigh coefficients were calculated using the resonance test's first-order damping ratio results. Furthermore, Berthelot et al. [14] demonstrated that a strain energy approach was simple to incorporate into finite element schemes. Zhang et al. [15] used the strain energy method in ANSYS to analyze the damping characteristics of two composite sandwich panels $[0_8/d]_S$ and $[(45/-45)_4/d]_S$ and quantified the contribution of composite damping in the two types of sandwich panels.

Furthermore, He et al. [16] used ABAQUS by UMAT subroutine to determine complex modulus values for the carbon/epoxy laminated composites combined with the results of the cantilever percussion free-decay and analysis of modal damping and frequency response for laminated composites. Liu et al. [17] investigated three-dimensional FEA and the transfer matrix method to solve free vibration problems for thick cantilever laminated plates with or without a step-change in thickness in the chord-wise direction. Chakraborty et al. [18] conducted an experimental and numerical study of the free vibration of composite GFRP plates. Modal testing was performed using impact excitation to determine the frequency response functions. Tarobi et al. [19] used ABAQUS software to simulate the delaminated beam and extract the natural frequencies and mode shape in their experimental and theoretical investigation of the transverse vibration of the delaminated cross-ply composite beam. N. Ziane et al. [20], in the study about free vibration analysis of thin and thick-walled FGM box beams, the thin and thick-walled FGM box beams are modelled using ABAQUS (C3D8 element type). Michelle et al. [21] used implicit analysis in ABAQUS to study about dynamic pulse buckling of composite shells subjected to external blast. Radial shell deformations were found to be in good agreement within 7 %. Although much work has been investigated on hybrid composite laminates on free vibration analysis, there is little work on comparison between experimental works and different commercial software modeling results.

This paper investigates the free vibration analysis of kevlar/glass/epoxy hybrid composites, which considers the hybridization effect. Experimental works are validated and compared between two commercial ABAQUS and ANSYS software. The natural frequency of the plate with various laminate schemes is studied with two other numerical analysis results. Moreover, two-mode shapes of hybrid composite laminates are further discussed.

2. Theory Study of Free Vibration Analysis

Free vibration means the motion of a structure without any dynamic equation of external forces or support motion. The motion of the single linear degree of freedom (SDF) system is

$$m\frac{d^2u}{dt^2}+ku=0 \tag{1}$$

where m is the mass, u is the displacement, and t is the time.

Free vibration is initiated by giving a disturbance to the system from its static equilibrium position. Some displacement $u(0)$ and velocity $\dot{u}(0)$ at zero time is defined as the instant the motion

$$u=u(0),\dot{u}=\dot{u}(0) \tag{2}$$

Therefore, the solution to the equation is obtained by standard methods.

$$u(t)=u(0)\,cos\,\omega_n\,t+\frac{\dot{u}(0)}{\omega_n}sin\,\omega_n\,t \tag{3}$$

For the definitions of the circular frequency ω_n, $\omega_n=\sqrt{\frac{k}{m}}$

where k is the spring constant

Time is required for the undamped system to complete one cycle of free vibration on the natural period of vibration of the system.

$$T_n=\frac{2\pi}{\omega_n} \tag{4}$$

$$f_n=\frac{1}{T_n} \tag{5}$$

where f_n is the natural cyclic frequency of vibration

3. Finite Element Modelling of Hybrid Composite Laminates

To validate experimental results, numerical analysis is used, which provides a theoretical study for rapid and accurate analysis of complex structural problems in engineering. In this paper, a finite element analysis was utilized to model accurately for composite laminates. The composite laminates were constructed using ABAQUS software.

3.1 Shell Selection

ABAQUS's shell element library is divided into general-purpose, thin-only, and thick-only shell elements. General-purpose shell elements can solve problems with both thick and thin shells. Thin-only and thick-only shell elements are appropriate for thin and thick shell problems. Sandwich composite constructions typically have very low transverse shear stiffness because their core is softer than their face [22]. As a result, general-purpose shell elements were chosen for this study. This study employs conventional shell elements, one of two types of general-purpose shell elements available in ABAQUS. In blade models, shell thicknesses are much smaller than other global in-plane dimensions.

Conventional shell elements discretize a reference surface by defining the planar dimensions, normal surface, and original curvature of the element. Although the shell element's nodes do not exist throughout the shell thickness, this thickness is defined by section properties. As a result, the composite laminates are represented by a three-dimensional linear shell element (S4R) of ABAQUS, which are general-purpose shell elements suitable for nonlinear geometrical analyses. S4R is an acronym for a four-node, quadrilateral stress/displacement shell element. These elements have three displacements and three rotational degrees of freedom (DOF), allowing for finite strain, large rotations, and transverse shear deformation. It employs reduced integration to precisely integrate their element stiffness, mass, and forces matrices.

3.2 Materials Behaviours

Elastic properties of materials must be accounted for in computational analyses. Because shell elements can define each ply in a laminate, overall laminate stiffness can be calculated prior to or during the analyses. In this paper, the hybrid kevlar/glass fiber with epoxy resin is modeled using an orthotropic elasticity by specifying the engineering constants. The total stress is defined from the total elastic strain's equation.

$$\sigma = D^{el}\varepsilon^{el} \tag{6}$$

where σ is the total stress, $D^{el}$is a fourth-order elasticity tensor, and ε^{el} is the total elastic strain. The equation of engineering constants is as in Equation 7.

$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{12} \\ \gamma_{13} \\ \gamma_{23} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E_1} & \frac{-\nu_{21}}{E_2} & \frac{-\nu_{31}}{E_3} & 0 & 0 & 0 \\ \frac{-\nu_{12}}{E_1} & \frac{1}{E_2} & \frac{-\nu_{32}}{E_3} & 0 & 0 & 0 \\ \frac{-\nu_{13}}{E_1} & \frac{-\nu_{23}}{E_2} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{12}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{13}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{23}} \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{Bmatrix} \tag{7}$$

3.3 Free Frequency Analysis

The frequency analysis of hybrid composite laminates is modeled with single-side constrained, as shown in Figure 1. Vibration analyses are performed with dimensions 200 × 12.7 mm from hybrid composite laminates. The mesh type is used as the shell element S4R. Table 1 summarizes the mechanical properties of GFRP and KFRP composite materials for finite element modeling. The specimen is labeled according to the material type, the number of laminates, and fiber orientation. For example, GFRP represents the glass fiber/epoxy resin composite laminates with 10 layers. KFRP represents the Kevlar fiber/epoxy resin composite laminates with 10 layers. Moreover, H1, H2, H3, and H4 represent 8 CFRP layers /2 KFRP layers, 6 CFRP layers /4 KFRP layers, 4 CFRP layers /6 KFRP layers KFRP, and 2 CFRP layers /8 KFRP layers, respectively.

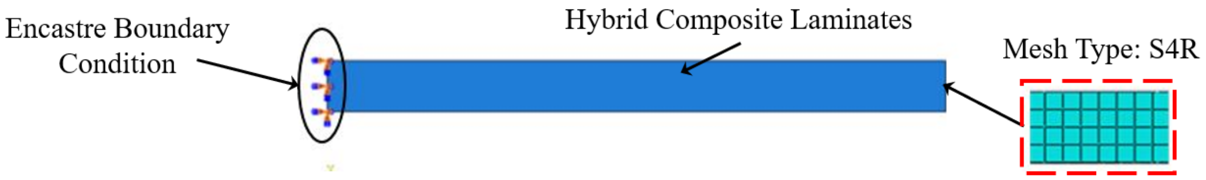

Fig. 1. Boundary conditions for natural frequency analysis of the hybrid composite laminates in finite element modeling

Table 1
Mechanical properties of GFRP and KFRP composites used in finite element modeling

Properties	GFRP	KFRP
E_1/E_2 (GPa)	19.5	26.5
E_3(GPa)	11.7	15.9
v_{12}	0.15	0.09
v_{13}/v_{23}	0.09	0.05
G_{12}(GPa)	3.7	2.5
G_{21}(GPa)	3.7	2.5
ρ (kg/m^3)	1710	1250

Note: GFRP: Glass fiber-reinforced plastic; KFRP: Kevlar fiber-reinforced plastic

3.4 Mode Shape and Natural Frequency Analysis

The corresponding mode shape and natural frequencies of Kevlar/glass/epoxy resin composite laminates are obtained for undamped and non-rotational conditions. The eigenvalues are extended using the linear perturbation scheme and the Lanczos method (Equation 8). In this study, it is assumed that the stiffness matrix is symmetric and positive-semidefinite. Equation 9 expresses the generalized eigenvalue problem of a finite element model.

$$[M]\{\ddot{u}\} + [K]\{u\} = \{0\} \tag{8}$$

where $[M]$ the symmetric positive-definite mass matrix, $[K]$ the stiffness matrix, and $\{u\}$ the displacement vector.

$$(-\omega^2[M] + [K])\{\phi\} = \{0\} \tag{9}$$

where $[\omega]$ is the eigenvalue and $\{\phi\}$ is the eigenvector. Lanczos' method reduces the generalized problem to the standard form of a tridiagonal coefficient matrix, as shown in Equations (10-14). For extracting natural frequencies in interesting ranges, the frequency shift θ_L is introduced to the generalized eigenvalue problem.

$$\omega^2 = \frac{1}{\theta_L} + S_L \tag{10}$$

where S_L is the eigenvalue. By substitution Equation 10 into Equation 8 and will get

$$[M]([K]-S_L[M])^{-1}[M]\{\phi\}=\theta_L[M]\{\phi\} \tag{11}$$

$$\{\phi\}=[U]\{\tilde{\phi}\} \tag{12}$$

$$[U]^T[M][U]=[I] \tag{13}$$

The eigenvector is transformed by using $[U]$, which is defined as the orthonormal matrix of$[M]$. Due to the orthonormality, we have Equation 13. It is noted that $[I]$ is the identity matrix. Thus, the substitution of Equation 12 into Equation 11 and premultiplying it by $[U]^T$ lead to

$$[U]^T[M]([K]-S_l[M])[M][U]\{\tilde{\phi}\}=\theta_L[U]^T[M][U]\{\tilde{\phi}\} \tag{14}$$

It can be rewritten as

$$[T]\{\tilde{\phi}\}=\theta_L\{\tilde{\phi}\} \tag{15}$$

$$[T]=[U]^T[M]([K]-S_L[M])[M][U] \tag{16}$$

Because $[U]$ satisfies the orthonormality of the mass matrix, $[T]$ becomes a tridiagonal matrix. As a result, solutions to the standard eigenvalue problem provide the undamped system's eigenvalues and eigenvectors, and Equation 16 can be solved using the Householder and Q-R algorithms [22-24].

3. Results and Discussion

Experimental and numerical results of ABAQUS and ANSYS software are summarized in Table 3, which is compared with other available numerical analyses (ANSYS) [25]. Table 3 presents the two natural frequencies of hybrid composite laminates with different fiber orientations, which are $[0/90]_{10}$, $[15/-75]_{10}$, $[30/-60]_{10}$, and $[45/-45]_{10}$. Table 3 provided reasonable agreement in two modes, which suggested that ABAQUS and ANSYS can sufficiently model the hybrid composite laminates in vibration analysis. It was found that the numerical results of ABAQUS and ANSYS on GFRP and H1 specimens were smaller than the experimental results on mode 1. Moreover, ANSYS results were obtained with a similar numerical result compared to the experimental results.

Figure 2 illustrates the two mode shapes of hybrid composite laminates with fiber orientation $[30/-60]_{10}$ on ABAQUS software. It was shown that GFRP, H2, and KFRP specimens provided a similar natural frequency on mode 1. For mode 2, the GFRP specimen had a larger natural frequency than the other two material types. Mesh sensitivity analysis was carried out by varying the mesh density within the plane and thickness. Figure 3 presents the sensitivity analysis of the FE model on natural frequency versus mesh size. The 1 mm mesh size was found to provide accurate prediction results.

Table 3
Experimental and numerical results of the natural frequency of hybrid composite laminates on ABAQUS and ANSYS software

Labels	[0/90]$_{10}$				
	Mode 1 (Hz)			Error %	
	Experiment [25]	ABAQUS	ANSYS [25]	Experiment & ABAQUS	Experiment & ANSYS
GFRP	48.57	41.97	45.89	13.58	5.51
H1	55.21	53.19	51.94	3.65	5.92
H2	59.35	61.75	60.32	4.04	1.63
H3	70.18	71.02	69.38	1.19	1.14
H4	73.28	82.07	78.29	12.00	6.83
KFRP	84.33	95.24	93.30	12.94	10.63

Labels	[15/-75]$_{10}$					
	Mode 1 (Hz)		Error %	Mode 2 (Hz)		Error %
	ABAQUS	ANSYS [25]		ABAQUS	ANSYS [25]	
GFRP	38.76	37.90	2.24	242.75	237.16	2.30
H1	46.88	45.94	2.01	278.51	281.38	1.03
H2	54.15	53.08	1.98	290.86	294.34	1.19
H3	61.96	60.77	1.93	301.22	305.16	1.30
H4	70.31	69.03	1.83	310.01	314.29	1.38
KFRP	78.47	77.13	1.71	317.09	321.64	1.43

Labels	[30/-60]$_{10}$					
	Mode 1 (Hz)		Error %	Mode 2 (Hz)		Error %
	ABAQUS	ANSYS [25]		ABAQUS	ANSYS [25]	
GFRP	34.078	33.460	1.813	213.52	209.51	1.878
H1	38.965	38.337	1.612	234.87	238.31	1.464
H2	44.737	44.040	1.558	238.21	242.01	1.595
H3	50.913	50.140	1.518	241.16	245.19	1.671
H4	56.668	55.835	1.470	243.59	247.77	1.716
KFRP	61.011	60.138	1.431	245.13	249.39	1.738

Labels	[45/-45]$_{10}$					
	Mode 1 (Hz)		Error %	Mode 2 (Hz)		Error %
	ABAQUS	ANSYS [25]		ABAQUS	ANSYS [25]	
GFRP	32.29	31.750	1.67	202.43	198.91	1.74
H1	36.25	35.710	1.51	219.18	222.71	1.61
H2	41.54	40.95	1.43	220.32	224.02	1.68
H3	47.19	46.536	1.38	221.57	225.42	1.73
H4	52.22	51.511	1.36	222.57	226.52	1.77
KFRP	55.66	54.91	1.35	222.95	226.94	1.79

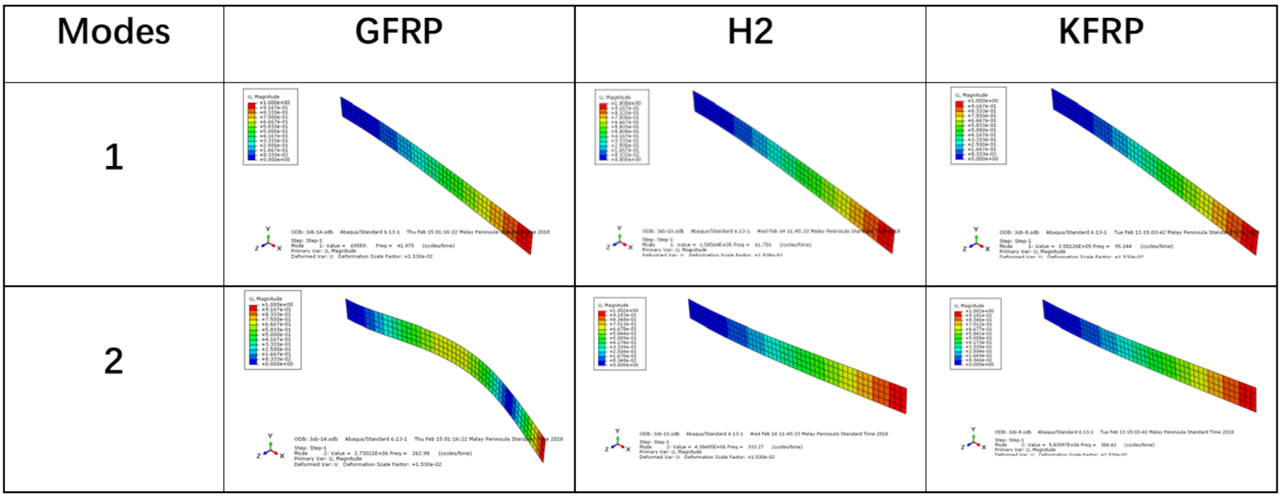

Fig. 2. Mode shape of hybrid composite laminates with $[30/-60]_{10}$ on ABAQUS software

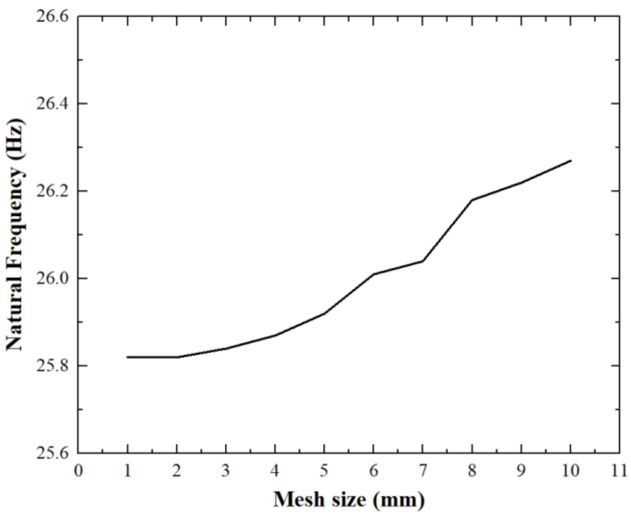

Fig. 3. Sensitivity analysis of the FE model on natural frequency versus mesh size

Figure 4 shows the numerical results on natural frequency for mode 1 between ABAQUS and ANSYS. It was found that KFRP specimens provided the highest natural frequency values on fiber orientation [0/90], which was due to the effect of fiber orientation. Moreover, GFRP specimens had the lowest natural frequency values, which might explain the hybridization effect on composite laminates.

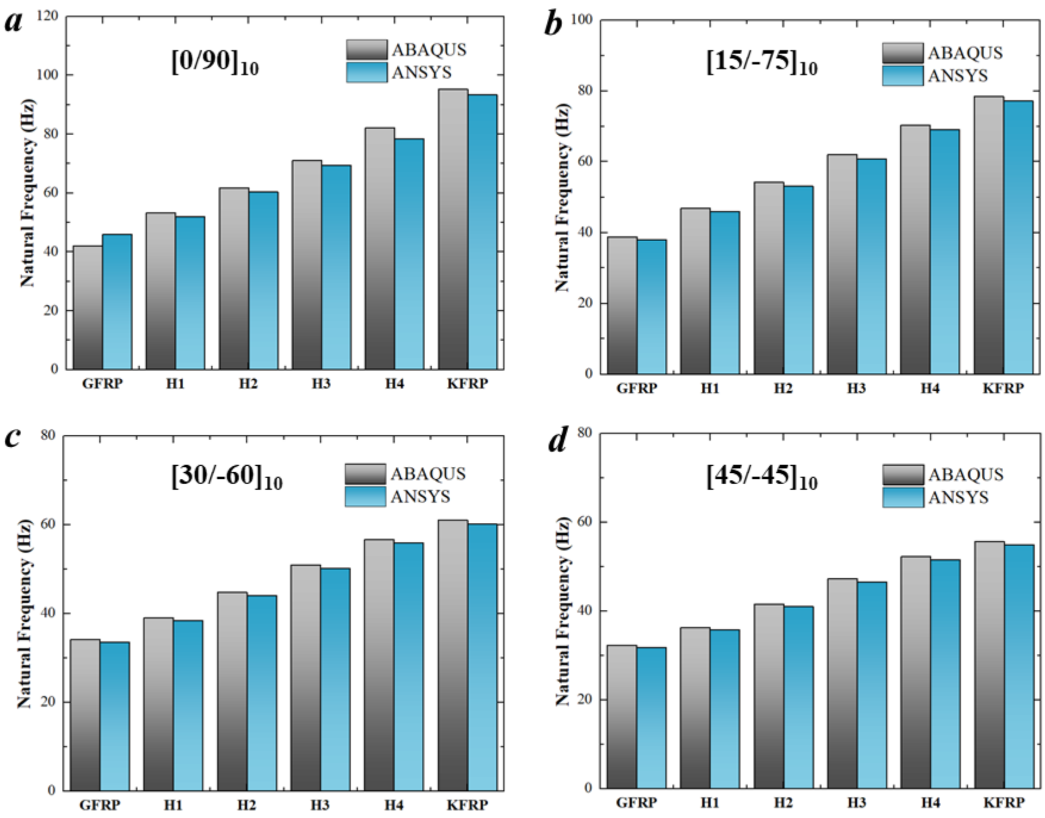

Fig. 4. Numerical results on natural frequency for mode 1 between ABAQUS and ANSYS: (a) $[0/90]_{10}$, (b) $[15/-75]_{10}$, (c) $[30/-60]_{10}$, and (d) $[45/-45]_{10}$

4. Conclusions

The free vibration analysis investigates the numerical analysis for hybrid composite laminate. The ABAQUS software is employed to obtain natural frequencies according to four fiber orientations, and the results are compared with available data in the literature using ANSYS. Numerical analysis shows that frequency values are effective in quasi-isotropic plates using the 3-D shell element method. The average error between ABAQUS and ANSYS is between 1.03 % and 8.53 %. Furthermore, the error between the experimental and numerical results ranged from 1.15 % to 13.57 %, which the mesh size might explain. The excellent agreement with other available data demonstrates the capability of the ABAQUS and ANSYS software, which analyzes the free vibration of the hybrid composite laminates.

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