Penerbit Akademia Baru

Journal of Advanced Research in Social and Behavioural Sciences
ISSN (online): 2462-1951 | Vol. 4, No. 1. Pages 73-80, 2016

A Pre-Evaluation Step towards a Guaranteed Consistent AHP-Based Pairwise Comparison

E. M. Nazri [,a], M. Balhuwaisl [b] and M. M. Kasim[c]*

School of Quantitative Sciences, UUM College of Arts & Sciences, Universiti Utara Malaysia, 06010 UUM Sintok, Kedah, Malaysia.
[*,a]*enazri@uum.edu.my*, [b]*abu@fkm.utm.my*, [c]*maznah@uum.edu.my*

Abstract – *Reports by various researchers show that ensuring pairwise comparison consistency is one of the main issues when applying analytic hierarchy process (AHP) in a decision making activity. Normally, in the case where inconsistencies are found, the evaluators might have to redo the pairwise comparisons, hence, disturbing the original preference. In this paper, we proposed an evaluation step prior to conducting the pairwise comparison in order to eliminate inconsistent comparisons. The evaluation step involves the rating of importance of criteria using a scale of 1- 9 whereby 1 represents least important while 9 represents extremely important. The resulted ratings are then used as a guide to construct the AHP-pairwise comparison matrix by using certain simple algorithm that is also proposed in this paper. Once the pairwise comparison matrix is obtained, the weights of criteria can be calculated using the existing AHP-technique. Seven different experiments involving 1000 simulated pairwise comparisons for cases with three, four, five, six, seven, eight and nine criteria, respectively, were conducted. All results yielded consistent pairwise comparisons with consistency ratio values of less than the benchmarked value which is CR = 0.1. This technique can really be an added value to the standard AHP-technique since the users of AHP can now skip the consistency test and stop worrying about producing inconsistent pairwise comparison matrices as reported by many previous researchers.*

Keywords: Analytic hierarchy process, Pairwise comparison, Consistent judgment, Multi-criteria decision making

1.0 INTRODUCTION

Analytic hierarchy process (AHP) is a mathematical technique developed by Saaty in 1970s for complex multi-criteria decision making [1-3]. It enables people to make decisions involving many kinds of concerns including applications in group decision making [4] in fields such as government [5–6], business [7-8], industry [9-10], healthcare [11-12], and education [13-14]. These are done through relative criticality weighting of indicators and relative criticality weighting of evaluators. After identifying the relevant indicators or criteria involved in the decision making process, AHP is conducted in three steps [15]:

- Step 1: Perform pairwise comparisons among the criteria
- Step 2: Assess consistency of the pairwise judgments in step 1
- Step 3: Compute the relative weights of criteria or attributes using the concept of eigenvector and eigenvalue.

Pairwise comparisons of the level of importance between decision criteria can be done with respect to the scale shown in table 1 [15].

Table 1: Preference scale for pairwise comparisons

*Preference Level	Numeric Value
Equally preferred	1
Equally to moderately preferred	2
Moderately preferred	3
Moderately to strongly preferred	4
Strongly preferred	5
Strongly to very strongly preferred	6
Very strongly preferred	7
Very strongly to extremely preferred	8
Extremely preferred	9

*Preference level can also be replaced by importance level, significance level, or any other appropriate levels.

For example, suppose c_{ij} is the pairwise comparison value between criterion *i* and criterion *j*. If criterion *i* is "strongly to very strongly **more** preferred" than criterion *j* then $c_{ij} = 6$. On the other hand, if criterion *i* is "extremely **less** preferred" than criterion *j*, $c_{ij} = 1/9$ (the reciprocal of 9).

Before computing the weights based on pairwise judgments, the degree of inconsistency for the pairwise judgments must be measured using the Consistency Ratio (CR). Perfect consistency implies a CR-value of zero. However, perfect consistency cannot be demanded since, as human beings, we are often biased and inconsistent in our subjective judgment. Therefore, it is considered acceptable for some inconsistency to occur up to a certain degree. In this case as long as the CR-value is less than 0.1, the pairwise comparison can be accepted as consistent.

Some of the more common approaches to measure consistency are the Consistency Ratio (CR) [1], the Geometric Consistency Index (GCI) [16-17], and the Harmonic Consistency Index (HCI) [24]. However, reports by various researchers show that ensuring consistency is one of the main issues when dealing with AHP [18-23]. In the case where inconsistencies are found, the evaluators might have to redo the pairwise comparisons, hence, disturbing the original preference.

The purpose of this study is therefore to formulate an approach to reduce inconsistent pairwise comparisons. If this objective can be achieved, practitioners of AHP will no longer need to test the consistency level of each comparison matrix, thus making the application of AHP easier and faster. In addition, the initial judgment from the evaluators when making the pairwise comparisons can be preserved.

Journal of Advanced Research in Social and Behavioural Sciences
ISSN (online): 2462-1951 | Vol. 4, No. 1. Pages 73-80, 2016

2.0 CURRENT APPROACHES IN TACKLING INCONSISTENT PAIRWISE COMPARISONS

Realizing the difficulty in ensuring consistency when dealing with pairwise comparisons, several researchers have suggested some alternatives. For example, Saaty [1] himself suggested that for some instances, particularly for cases involving qualitative attributes, CR-value can be lowered from the standard CR-value = 0.1 to CR-value = 0.2. However, by doing this, the credibility of the logic becomes less robust. Furthermore, for the cases involving many variables, CR-value can still easily exceeds 0.2. As an alternative, [25] suggested a method for improving the consistency of judgments by manipulating some of [1]'s arithmetic operations. Unfortunately, although the method managed to reduce inconsistent judgment, the method somehow altered the logic presented by [1].

[16] responded by introducing the Geometric Consistency Index (GCR) to replace CR-value, which was later confirmed by Saaty himself as being a valid replacement. However, the original consistency issue is still not resolved since [16] simply introduced a new formula to calculate consistency; they did not improve the pairwise comparison approach. Later, there was an attempt made by [19] with the introduction of a flexible and less restrictive approach for accepting/rejecting the pairwise comparison by introducing different index for different size of comparison matrix. However, a review by [26] revealed that the utilization of various judgment scales on priority estimates in AHP and the use of different indexes do not solve the inconsistency issue entirely.

Some researchers even avoided utilising pairwise comparison altogether. For example, some researchers opted for simpler approaches such as the simple scoring model [27], rank sum method [28-29], ELECTRE III technique [30-31], and rank order centroid [28-29]. However, these techniques also received comments [32-33] due to lack of good attributes that appear in AHP, particularly with respect to the pairwise comparison.

3.0 OUR PROPOSED PRE-EVALUATION STEP

As mentioned in the earlier section, AHP begins by asking the evaluator to compare the level of importance between all possible pairs of criteria, using a scale with values from 1 to 9 as given in table 1 previously. However, instead of asking the evaluator to do the pairwise comparisons, we would like to suggest the evaluator to simply rate the importance of each criterion using the scale of 1 to 9, whereby 1 represents least important while 9 represents extremely important. Here we proposed a similar scale of 1 to 9 to be utilized so that the scale resembles Saaty's scale. Next, these rating evaluation values will be transformed into Saaty's AHP- pairwise comparison table $C = [c_{ij}]_{nxn}$, using the following algorithm.

Suppose we have N criteria. The evaluator must then rate the level of importance of each criterion in determining the weight of that criterion towards the final goal (the decision making process) using the scale of 1 to 9 as mentioned earlier. Suppose also that the evaluator rates criterion i as r_i and criterion j as r_j. Then c_{ij} which is the pairwise comparison value between criterion i and criterion j will be determined as follows:

Let $b = r_i - r_j$

If $b > 0$ then $c_{ij} = b+1$

Penerbit Akademia Baru
Journal of Advanced Research in Social and Behavioural Sciences
ISSN (online): 2462-1951 | Vol. 4, No. 1. Pages 73-80, 2016

If $b = 0$ then $c_{ij} = 1$ (1)

If $b < 0$ then $c_{ij} = 1/(1-b)$

The rationale for the formula is as follows:

i. Suppose the evaluator give the same rating (let's say 6) for both criteria i and j. Using Saaty's logic in table 1, c_{ij} should be 1 since both criteria are equally preferred. Thus, constructing the formula $c_{ij}= b+1$ will give cij = (6 - 6) + 1 (consistent with Saaty's pairwise comparison table 1), instead of 0 if we use $c_{ij}= b$.
ii. For the most extreme case, suppose that criterion i is given a rating of 9 and criterion j is given a rating of 1. We can interpret this as criterion i is 'extremely more preferred" than criterion j. Thus, applying formula 1, c_{ij} = (9-1) + 1 = 9. This is consistent with Saaty's pairwise comparison value in table 1.

Once the pairwise matrix is obtained, the weight for each criterion will be calculated using the standard AHP-technique. And the process of course includes the consistency test.

By preforming the pre-evaluation rating process, we are in a way gauging the logical thinking of the evaluator in assessing the level of importance of each criterion towards the final decision making process (This is particularly more significant for problem involving many criteria). And later, when we convert the ratings into the pairwise comparison table using formula 1 above, the level of consistent comparison is somewhat preserved. In this case, we believe that our suggested approach would give a CR-value of less than 0.1 which is the AHP-standard CR-value.

To rationalize our belief let us compare Saaty's logic in proving consistency. Saaty says that suppose c_{ij} = 2 and c_{jk} = 3. Then for a perfect consistency, c_{ik} = 2 x 3 = 6. Comparing Saaty's logic with our proposed technique, suppose that criterion i, j, and k are given a rating value of 6, 4 and 1 respectively by the evaluator. Then c_{ij} = 3, c_{jk} = 4, and c_{ik} = 6 by our formula 1. To get a perfect consistency c_{ik} should be 12 (i.e. 3 x 4). However, even by Saaty's approach, a pairwise comparison value of 12 is not allowed since the largest value is set at 9. Therefore, Saaty's approach allows for some inconsistencies. Once again, we strongly believe that the inconsistencies in our approach will be within the acceptable range, i.e. CR-value < 0.1.

To justify the belief, we simulated 1000 judgments (J) each for pairwise comparisons involving 3, 4, 5, 6, 7, 8, and 9 criteria respectively. The simulation process was carried out with the assumption that it would represent the actual pairwise comparison exercises and any rating of criteria of scale 1- 9. We illustrate an example of a converted pairwise comparison for a rating involving 5 criteria.

To begin the evaluation process, the evaluator stated the level of importance/preference from the scale of 1 (least important) to 9 (extremely important) of each criterion in determining the final goal. Table 2 gives the simulated result of the evaluation (for the illustration of how the simulation of the rating process using the scale as shown in Table 2 was conducted, please refer to the Appendix at the end of this paper).

Journal of Advanced Research in Social and Behavioural Sciences
ISSN (online): 2462-1951 | Vol. 4, No. 1. Pages 73-80, 2016

Table 2: Rating table involving 5 criteria

Criteria	Rating 1	2	3	4	5	6	7	8	9
A						X			
B					X				
C		X							
D				X					
E				X					

Next, the evaluation result was converted into Saaty's AHP-pairwise comparison table following the conversion formula (1) stated earlier. The CR-value was also calculated with the help of the AHP-software *ExpertChoice*. Table 3 shows both the resulted-comparison table and the CR-value.

Table 3: Saaty's AHP-pairwise comparison table invoving 5 criteria

		Criteria A	B	C	D	E
Criteria	A	1	2	5	3	3
	B	½	1	4	2	2
	C	1/5	¼	1	1/3	1/3
	D	1/3	½	3	1	1
	E	1/3	½	3	1	1

CR-value = 0.01.

Table 4: Summary of pairwise comparison experiments

Number of criteria, *N*	*J*	Min CR	Max CR	Mean CR	Variance
3	1000	.000	.09	.0270	.00053
4	1000	.000	.07	.0272	.00019
5	1000	.000	.05	.0237	.00006
6	1000	.004	.09	.0257	.00058
7	1000	.005	.04	.0261	.00012
8	1000	.006	.05	.0235	.00031
9	1000	.001	.07	.0244	.00034

Journal of Advanced Research in Social and Behavioural Sciences
ISSN (online): 2462-1951 | Vol. 4, No. 1. Pages 73-80, 2016

3.0 RESULT

Based on our experiments, all simulated judgments that have been converted into pairwise comparison tables yielded CR-values less than the allowed value, which is 0.1. Thus all the simulated pairwise comparisons are considered consistent and acceptable. The summary for the results is given in Table 4. Based on the table, all CR-values are less than 0.1, with a minimum CR-value of 0.000 and the maximum CR-value of only 0.09.

4.0 CONCLUSION

In this paper, we showed how a pre-evaluation process in the form of rating using a scale of 1 to 9 can be used as a guide towards producing a consistent pairwise comparison matrix. A simulation involving 1000 trials for cases with 3, 4, 5, 6, 7, 8, and 9 criteria were conducted and the results produced consistent pairwise comparison matrices for all cases. This technique can really be an added value to the existing AHP-technique since if our claim can be proven theoretically, the users of AHP can skip the consistency test and stop worrying about producing inconsistent pairwise comparison matrices as reported by [18-23]. In fact, they do not have to go through the hassle of doing the pairwise comparisons anymore since the pairwise evaluation can be transformed and evaluated directly from the simple rating table. Furthermore, decision making can include as many criteria as they wish without having to deal with the inconsistency issue. However, we propose that the maximum number of criteria to be set at only 9 criteria. Else, when performing the pre-evaluation rating exercise, some of the criteria will be forced to have the same rating score. Saaty in [1] also suggested that the maximum number of criteria to be 9 although for a different reason. Saaty believes that human can handle pairwise comparison exercise "sanely" only up to 9 criteria.

REFERENCES

[1] Saaty, Thomas L. "What is the analytic hierarchy process?." In Mathematical models for decision support, pp. 109-121. Springer Berlin Heidelberg, 1988.

[2] Saaty, Thomas L. "Multicriteria Decision Making: The Analytic Hierarchy Process: Planning, Priority Setting." Resource Allocation 2 (1990).

[3] Saaty, Thomas L. Fundamentals of decision making and priority theory with the analytic hierarchy process. Vol. 6. Rws Publications, 2000.

[4] Saaty, Thomas L., and Kirti Peniwati. Group decision making: drawing out and reconciling differences. RWS publications, 2013.

[5] Huang, Chi-Cheng, Pin-Yu Chu, and Yu-Hsiu Chiang. "A fuzzy AHP application in government-sponsored R&D project selection." Omega 36, no. 6 (2008): 1038-1052.

[6] Syamsuddin, Irfan, and Junseok Hwang. "The application of AHP to evaluate information security policy decision making." International Journal of Simulation, Systems, Science and Technology 10, no. 4 (2009): 46-50.

[7] Hell, Marko, Mira Krneta, and Petar Krneta. "Application Of AHP Method For The Selection Of Business Plan Software." Croatian Operational Research Review 4, no. 1 (2013): 223-234.

Journal of Advanced Research in Social and Behavioural Sciences
ISSN (online): 2462-1951 | Vol. 4, No. 1. Pages 73-80, 2016

[8] Kumar, Sanjay, Neeraj Parashar, and Abid Haleem. "Analytical hierarchy process applied to vendor selection problem: Small scale, medium scale and large scale industries." Business Intelligence Journal 2, no. 2 (2009): 355-362.

[9] Cebeci, Ufuk. "Fuzzy AHP-based decision support system for selecting ERP systems in textile industry by using balanced scorecard." Expert Systems with Applications 36, no. 5 (2009): 8900-8909.

[10] BURDURLU, Erol, and Engin EJDER. "Location choice for furniture industry firms by using Analytic Hierarchy Process (AHP) method." Gazi University Journal of Science 16, no. 2 (2003): 369-373.

[11] Schmidt, Katharina, Ines Aumann, Ines Hollander, Kathrin Damm, and J-Matthias Graf von der Schulenburg. "Applying the Analytic Hierarchy Process in healthcare research: A systematic literature review and evaluation of reporting." BMC medical informatics and decision making 15, no. 1 (2015): 1.

[12] Liberatore, Matthew J., and Robert L. Nydick. "The analytic hierarchy process in medical and health care decision making: A literature review." European Journal of Operational Research 189, no. 1 (2008): 194-207.

[13] Badri, Masood A., and Mohammed H. Abdulla. "Awards of excellence in institutions of higher education: an AHP approach." International Journal of Educational Management 18, no. 4 (2004): 224-242.

[14] Drake, Paul R. "Using the analytic hierarchy process in engineering education." International Journal of Engineering Education 14, no. 3 (1998): 191-196.

[15] Taylor, Bernard W., C. R. Bector, S. K. Bhatt, and Earl Saul Rosenbloom.Introduction to management science. New Jersey: Prentice Hall, 1996.

[16] Aguaron, Juan, and José María Moreno-Jiménez. "The geometric consistency index: Approximated thresholds." European Journal of Operational Research 147, no. 1 (2003): 137-145.

[17] Aguarón, Juan, and José María Moreno-Jiménez. "Local stability intervals in the analytic hierarchy process." European Journal of Operational Research125, no. 1 (2000): 113-132.

[18] Gastes, Dominic, and Wolfgang Gaul. "The Consistency Adjustment Problem of AHP Pairwise Comparison Matrices." In Quantitative Marketing and Marketing Management, pp. 51-62. Gabler Verlag, 2012.

[19] Alonso, Jose Antonio, and M. Teresa Lamata. "Consistency in the analytic hierarchy process: a new approach." International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems 14, no. 04 (2006): 445-459.

[20] Ishizaka, Alessio, and Markus Lusti. "An expert module to improve the consistency of AHP matrices." International Transactions in Operational Research 11, no. 1 (2004): 97-105.

[21] Forman, Ernest, and Kirti Peniwati. "Aggregating individual judgments and priorities with the analytic hierarchy process." European journal of operational research 108, no. 1 (1998): 165-169.

[22] Dyer, James S. "Remarks on the analytic hierarchy process." Management science 36, no. 3 (1990): 249-258.

Journal of Advanced Research in Social and Behavioural Sciences
ISSN (online): 2462-1951 | Vol. 4, No. 1. Pages 73-80, 2016

[23] Crawford, Gordon, and Cindy Williams. "A note on the analysis of subjective judgment matrices." Journal of mathematical psychology 29, no. 4 (1985): 387-405.

[24] Stein, William E., and Philip J. Mizzi. "The harmonic consistency index for the analytic hierarchy process." European journal of operational research 177, no. 1 (2007): 488-497.

[25] Lamata, Maria Teresa, and José Ignacio Peláez. "A method for improving the consistency of judgements." International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems 10, no. 06 (2002): 677-686.

[26] Franek, J. and A. Kresta. “Review on the effects of utilization of various judgment scales on priority estimates in Analytic Hierarchy Process.” Procedia Economics & Finance 12, (2014): 164-173.

[27] Malcom, Jacob W., Whitney M. Webber, and Ya-Wei Li. A simple, sufficient, and consistent method to score the status of threats and demography of imperiled species. No. e1860v1. PeerJ Preprints, 2016.

[28] Alfares, Hesham K., and Salih O. Duffuaa. "Simulation-Based Evaluation of Criteria Rank-Weighting Methods in Multi-Criteria Decision-Making."International Journal of Information Technology & Decision Making 15, no. 01 (2016): 43-61.

[29] Roberts, Ron, and Paul Goodwin. "Weight approximations in multi-attribute decision models." Journal of Multi-Criteria Decision Analysis 11, no. 6 (2002): 291-303.

[30] Balali, Vahid, Banafsheh Zahraie, and Abbas Roozbahani. "Integration of ELECTRE III and PROMETHEE II decision-making methods with an interval approach: Application in selection of appropriate structural systems." Journal of Computing in Civil Engineering 28, no. 2 (2012): 297-314.

[31] Hashemi, Shide Sadat, Seyed Hossein Razavi Hajiagha, Edmundas Kazimieras Zavadskas, and Hannan Amoozad Mahdiraji. "Multicriteria group decision making with ELECTRE III method based on interval-valued intuitionistic fuzzy information." Applied Mathematical Modelling 40, no. 2 (2016): 1554-1564.

[32] Ishizaka, Alessio, and Ashraf Labib. "Analytic hierarchy process and expert choice: Benefits and limitations." Or Insight 22, no. 4 (2009): 201-220.

[33] Velasquez, Mark, and Patrick T. Hester. "An analysis of multi-criteria decision making methods." International Journal of Operations Research 10, no. 2 (2013): 56-66.