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Estimation of Radius Ratio in a Fin Using Inverse CFD Model

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Abstract

This article deals with the retrieval of parameters such as the radius-ratio in a rectangular fin using an inverse CFD model involving a mixed boundary condition. At first, the temperature field is obtained from a forward problem using the finite difference method (FDM) in which the inner and outer radii or the radius-ratio is assumed to be known. Next, by an inverse approach using the FDM in conjunction with the genetic algorithm (GA), the inner and outer radii or the radius-ratio is retrieved. To accomplish the task, an objective function represented by the sum of square of the error between the guessed and the exact/measured temperature fields is minimized. Apart from demonstrating the suitability of the FDM-GA, the study provides guidelines for selecting suitable optimization parameters. It is observed that subject to appropriate selection of parameters, very good estimation accuracy could be obtained.

Keywords: fin; parameter retrieval; inverse method; radius-ratio; FDM; GA.

1. Introduction

All processes can be mathematically represented by a set of differential equations. In fluid flow and heat transfer problems, the differential equation involves the Navier-Stokes equation, the continuity equation and the energy equation. From the knowledge of medium properties, initial conditions, and boundary conditions, the solution of such equations yields the required field which may be either the velocity distributions or the temperature distributions. This type of problem is known as the forward/direct problem and is mathematically well-posed [1]. However, there may be instances where either the velocity or the temperature field is known, but some properties, parameters, etc., are unknown and the problem becomes an inverse problem which is mathematically ill-posed [2] and its solution requires regularization using optimization tools.

A good number of studies have been reported to study inverse problems using different CFD models and optimization tools. Using the temperature field in the conducting solid, Chen et *al.* [3] retrieved temperature and heat flux on a surface. FDM in conjunction with the Kalman Filter scheme was used in their inverse analysis. Erturk et *al.* [4] predicted the boundary condition in a furnace using Monte Carlo method in conjunction with the conjugate gradient method (CGM). Huang et *al.* [5] have estimated the conductance in a metal casting process using the FDM and

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CGM. The application of the lattice Boltzmann method, the finite volume method (FVM) and the genetic algorithm (GA) to inverse conduction-radiation heat transfer problems was studied by Das et *al.* [6, 7]. Kim et *al.* [8] used the FVM-GA combination to determine the emissivity in a two-dimensional geometry. Determination of the convective heat transfer coefficient using the FVM in conjunction with the CGM was done by Chen et *al.* [9].The solution of a forward problem is largely done by established CFD methods such as the FDM, the FVM and the finite element methods. But, apart from efficient CFD platform, the solution of inverse problems also depends upon the effectiveness of the optimization algorithm. The optimization algorithms operating on stochastic and evolutionary principles are getting increasing applications for the solution of inverse problems and work better than deterministic based optimization methods [10, 11]. In the past, some studies involving the combination of methodologies such as the FDM and the GA have been reported. Reineix et *al.* [12] have used the FDM-GA to analyse the microstrip antennas. Minimization of absorption/reflection in perfective matched layers terminated by a conduction plane was performed by Michielssen et *al.* [13]. They have also used FDM in conjunction with the GA. Therefore, in the present work we use the GA to estimate the inner and outer radii or the radius-ratio in a rectangular fin problem involving conduction-convection heat transfer using steady-state temperature measurements from a forward model based on the FDM. The paper is organized in five sections. In section 2, the formulation and the solution procedure of the forward problem using the FDM is discussed and in Section 3, the solution procedure of the inverse problem and the optimization algorithm (GA) are provided. Results and discussions are done in Section 4 and conclusions are made in Section 5.

2. Forward Problem and its Validation With Analytical Solution

Consider a cross sectional view of a fin in Cartesian geometry with the details as shown in Fig. 1. The inner and outer radii of the fin are r_1, r_2 and the thickness of the fin is δ. The boundary conditions are of mixed type in which one end of the fin is subjected to a uniform high temperature heating, T_W and the other end is kept insulated ($\nabla.\vec{q} = 0$). The fin is exposed to room temperature, T_0. The heat transfer mode is by conduction and convection. The governing equation representing the energy balance between locations r and $r + \Delta r$ of Fig. 1 can be written as below,

$$q_{c,r} - q_{c,r+\Delta r} - q_{cv} = 0 \qquad (1)$$

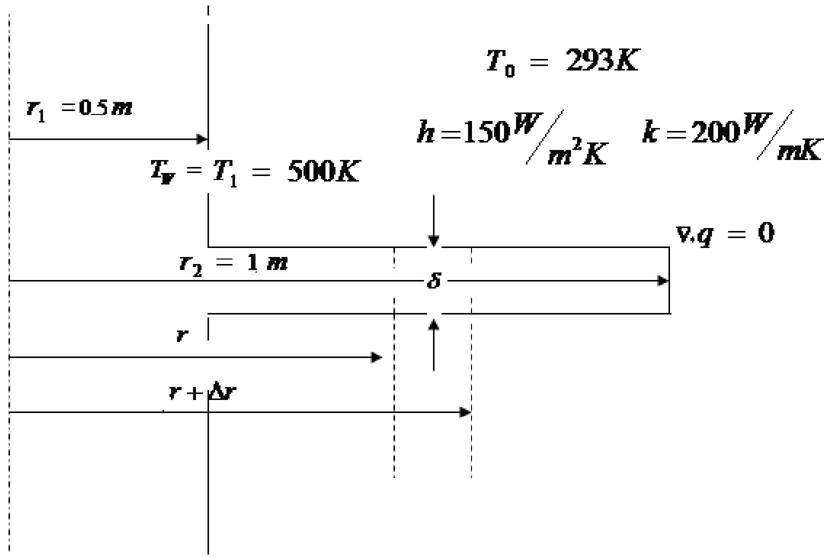

Figure 1: Physical geometry of the problem

Assuming that the conduction and the convection obey Fourier law and Newton's law of cooling, respectively, the Eq. (1) can be rewritten as given by the following equation

$$\left(-kA_{\mathrm{c}}\, {dT}/{dr}\right)_{\mathrm{r}} - \left(-kA_{\mathrm{c}}\, {dT}/{dr}\right)_{\mathrm{r+\Delta r}} - hA_{\mathrm{cv}}\left(T - T_0\right) = 0 \tag{2}$$

where, $A_{\mathrm{c}} = 2\pi r\delta$; and $A_{\mathrm{cv}} = 2\pi\left(2r\Delta r + \Delta r^2\right)$ are the conduction and the convection heat transfer area, respectively. To carryout the solution of the governing equation, we non-dimensionalize the temperature and the distance in the following manner as given below

$$\tau = {T - T_0}/{T_W - T_0} \text{ and } r^* = {r}/{r_2} \tag{3}$$

Substitution of the above non-dimensional terms in Eq. (2) and simplification of the resulting equation yields the non-dimensional form of the governing equation can be represented in the manner as given below

$$r^{*2}\tau'' + r^*\tau' - \alpha^2 r^{*2}\tau = 0 \tag{4}$$

where, $\alpha^2 = {2hr_2^2}/{k\delta}$. The non-dimensional boundary conditions as given in Eq. (3) take the following form

$$\text{at } r^* = {r_1}/{r_2} = 0.5, \tau\left(0.5\right) = 1.0; \text{ and at } r^* = {r_2}/{r_2} = 1.0, \tau'\left(1\right) = 0 \tag{5}$$

The governing equation (Eq. (4)) can either be solved by an analytical method or using any computational scheme. In the present work, the FDM employing central difference scheme for both τ'' and τ' is used to solve the governing equation for the forward problem. For the problem under consideration and discretizing the space using the FDM, the system of equations can be represented in the form of a matrix as mentioned below

$$\left[P\right]\left[Q\right] = \left\{R\right\} \tag{6}$$

where, matrix P consists of the known constant coefficients for the unknown non-dimensional-temperature vector, Q. The vector R represent a set of known values depending upon the boundary conditions and energy balance for the internal nodes. Thus, the non-dimensional steady-state temperature, τ distributions can be obtained at each node using the forward methodology. In order to test the forward CFD model using FDM, a comparison is made with its analytical solution obtained using Bessel functions and equations. The governing energy equation (Eq. (4)), can be represented in form of modified Bessel equations as given below [14-16]

$$r^{*2}\tau'' + r^*\tau' - \left(\alpha^2 r^{*2} - \nu^2\right)\tau = 0 \tag{7}$$

where, ν is the real constant and in the present case equals to zero and I and K represent a set of solutions which is calculated using the procedure described in [14-16]. Applying the boundary conditions and analytically solving the governing equation (Eq. (7)), the final temperature, τ can be calculated and the details can be found in [14-17].

3. The Inverse Method

From the temperature, τ distributions in the medium obtained from the forward problem using the FDM, in the inverse method, the GA is used to retrieve the unknown parameters which in the present work is the inner/outer radii, r_2 and r_1, respectively. This is done by minimizing the square of the errors between a guessed temperature field, τ the exact/measured temperature field, $\tilde{\tau}$. Evidently, the guessed temperature field, τ will represent a guessed value of inner/outer radii, r_2 and r_1. The objective function minimization in the present work is done using the GA. Mathematically, the objective function, J to be minimized by the GA is represented as below

$$J = \sum_{i=1}^{N} \left(\tau_i - \tilde{\tau}_i\right)^2 \qquad (8)$$

where N is the number of measurement points. The exact temperature may also be subjected to a measurement error, $e = \sum_{i=1}^{N} e_i$ in the exact temperature field, τ. Considering the effect of measurement error, the objective function (Eq. (8)) is modified as given below

$$F = \sum_{i=1}^{N} \left\{ \tau_i - \left(\tilde{\tau} + e\right)_i \right\}^2 \qquad (9)$$

The working principle of the GA can be found in literatures [6-8, 10, 11] and the details of the same are not presented here. In the GA, a ranking selection scheme is adopted with a uniform crossover and mutation probability. The stopping criterion is fixed either when a sufficiently small value with $O\left(10^{-4}\right)$ of the objective function is attained or when the GA exceeds a considerably large number of iterations/generations (200, in the present work). In the following section we present the results and discussions for the parameter retrieval using the FDM in conjunction with the GA.

2. Results and discussion

In this section we present the results of the inverse analysis for the retrieval of the unknown inner/outer radii, r_2 and r_1. The temperature, τ distributions in the medium are obtained from the forward problem involving the FDM. In order to validate the the forward problem, we benchmark the results of the FDM with the analytical solution obtained using the approach based on Bessel functions as described earlier in Section 2.

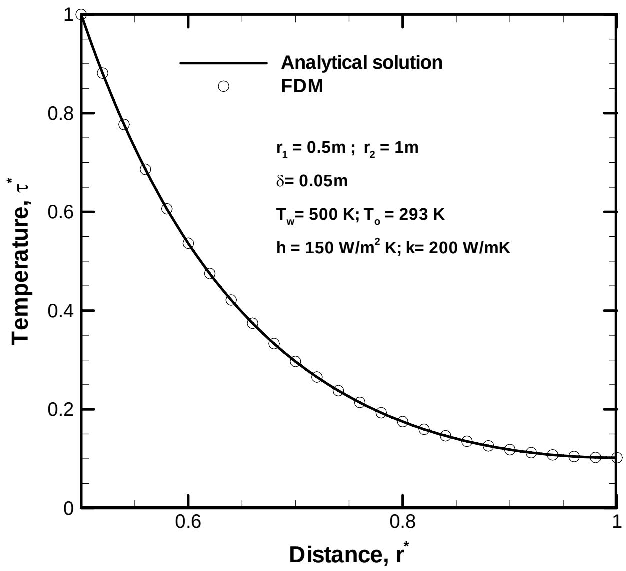

Figure 2: Validation of the forward model (FDM) with analytical solution

From the grid independency tests it is observed that beyond $N = 100$ nodes, there is no significant change in temperature distributions. Therefore, in both the forward and the inverse methods, we carry out our analysis with 100 nodes/measurement points. Figure 2 presents a comparison of the temperature distributions in the medium obtained using the FDM with that of the analytical method. For this comparison the physical parameters such as the thermal conductivity, k, the heat transfer coefficient, h, the inner and the outer radii, r_1, r_2, and the boundary conditions are the same as shown in Fig. 1 and described in Section 2.

TABLE-1 DETAILS OF RUNS CONSIDERED IN THE GA

Runs	Crossover probability	Mutation probability
1	0.70	0.01
2	0.10	0.01
3	0.10	0.10
4	0.70	0.10
5	0.50	0.01

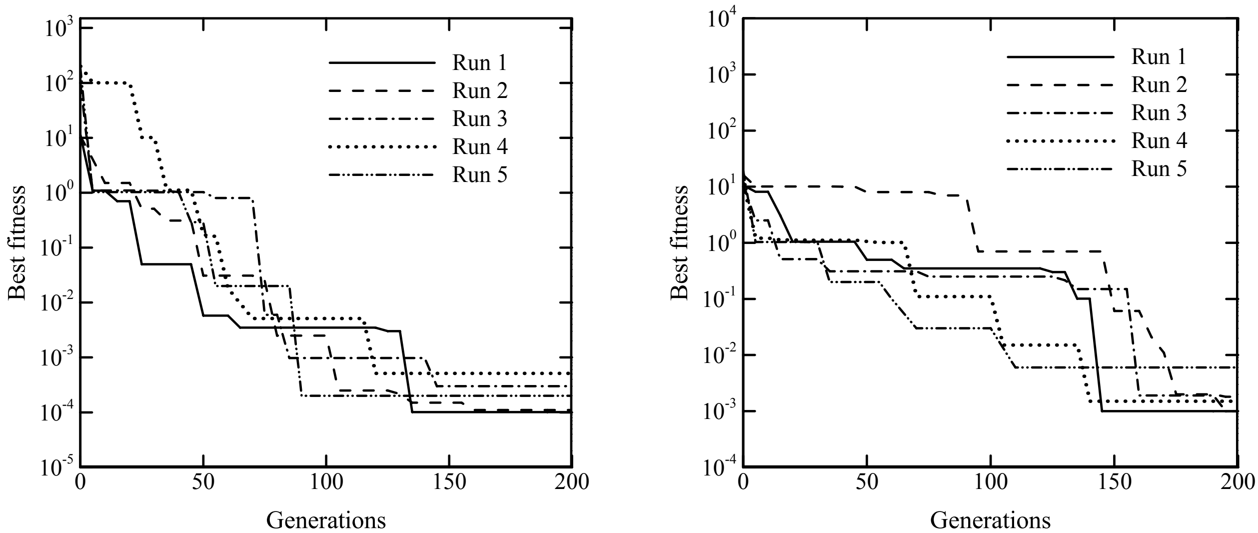

Figure 3: Comparison of objective function with generations for temperature measurements (a) without measurement error (b) with measurement error $e/N = 0.02$; $r_1 = 0.5\,\text{m}, r_2 = 1.0\,\text{m}$, $h = 150\,W/m^2K$, $K = 200\,W/mK$.

In order to demonstrate the suitability of the GA, in Table 1 we present the details of five different cases considered for selecting a proper combination of the crossover and mutation probability. It can be observed from Table 1 that the crossover probability has been varied in the range 0.10-0.70 and the mutation probability has been studied if the range 0.01-0.10. The population size is taken to be 20. For the set of parameters and the boundary conditions as considered in Fig. 1, in Fig. 3 we present the variation of the objective function with generations for the estimation of the inner and the outer radii, r_1, r_2. The study is made by considering the temperature, τ field either to be an exact or consisting of measurement error, $\frac{e}{N} = 0.02$. It is to be noted that for minimizing the objective function, J, for Fig. 3a the relevant equation is given by Eq. (8) and for Fig. 3b, the corresponding equation is given by Eq. (10). It is observed from Fig. 3 that a choice of higher value of crossover probability and a lower value of mutation probability (Run 1) is better and the objective function converges to a lesser value than the other combinations/runs. This is due to the reason that a higher mutation rate tends to deteriorate the solution of the GA. Again a lower crossover probability may also lead to retain larger number of poor solutions in the population. Therefore, for the further study we present the results for the best run (Run 1).

Figure 4 presents a comparison of the measured and the retrieved temperature, τ field with distance, r^*. Figure 4a shows the results for temperature field without error and Fig. 4b represents the case considering a measurement error of $\frac{e}{N} = 0.02$. The retrieved temperature field corresponds to the retrieved values of the inner and the outer radii, r_1, r_2 obtained from Run 1 of the GA. The exact retrieved values are also shown in the figure. It is observed that the measured and the retrieved fields are in excellent agreement with each other. However, the exact and the retrieved values of the radii, r_1, r_2 varies indicating the existence of multiple solutions satisfying a given objective function.

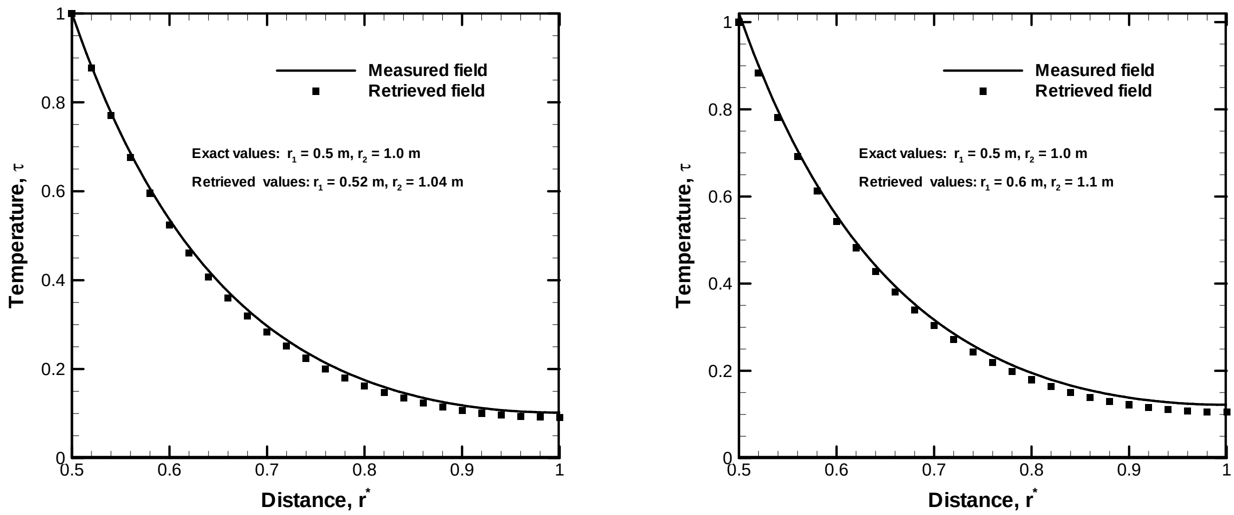

Figure 4: Comparison of local temperature fields at different locations; (a) exact temperature measurements; (b) temperature measurements with measurement error

$$\frac{e}{N} = 0.02. \; r_1 = 0.5\text{m}, r_2 = 1.0\,\text{m}, \; h = 150\, W\!/_{m^2K}, K = 200\, W\!/_{mK}.$$

It is worth to mention here that apart from the FDM, the GA works very well in conjunction with other numerical methods such as the lattice Boltzmann methods [6, 7], the FVM [8], etc. and has been found suitable for estimation by inverse problem. However, it is observed in the present study that the selection of parameters such as the crossover and mutation probabilities needs to be done carefully without which the solution could yield poor results. It is observed that, although the GA is capable of providing good results, the usage of the same is time consuming [6, 7].

TABLE-2 COMPARISON OF EXACT AND THE ESTIMATED PARAMETERS FOR DIFFERENT RADIUS-RATIOS AND EXACT TEMPERATURE MEASUREMENTS

Case	Exact radius, (m) r_1, r_2	Exact radius-ratio, $\frac{r_1}{r_2}$	Estimated radius, (m); r_1, r_2	Estimated radius-ratio, $\frac{r_1}{r_2}$
1	0.25, 1.0	4.0	0.251, 1.020	4.06
2	0.50, 1.0	2.0	0.522, 1.039	1.99
3	0.75, 1.0	1.33	0.733, 0.984	1.34

In order to demonstrate the existence of a multiple combinations of parameters satisfying a given objective function, in Table 2 we compare the exact and the retrieved parameters for three different cases of inner and outer radii, r_1, r_2 or radius-ratios, $\frac{r_1}{r_2}$. The other properties and the boundary conditions are the same as previously considered for other cases. It is observed that from Table 2 that even for same value and with exact temperature measurements, the retrieved values of the inner and outer radii, r_1, r_2 or radius-ratio, $\frac{r_1}{r_2}$ differs. Figure 5 presents the measured and the retrieved temperature fields for the different cases considered in Table 2. It is observed from Fig. 5 that even though the temperature fields are in excellent agreement with each other, the retrieved parameters are different as noted previously in Table 2. It is also observed that with decrease in radius-ratio, $\frac{r_1}{r_2}$, the local temperatures, τ in the medium increases. This is due to the reason that the hot and the cold boundaries moves closer and thus the heat loss is less and heat transfer is fast when the radius-ratio is small.

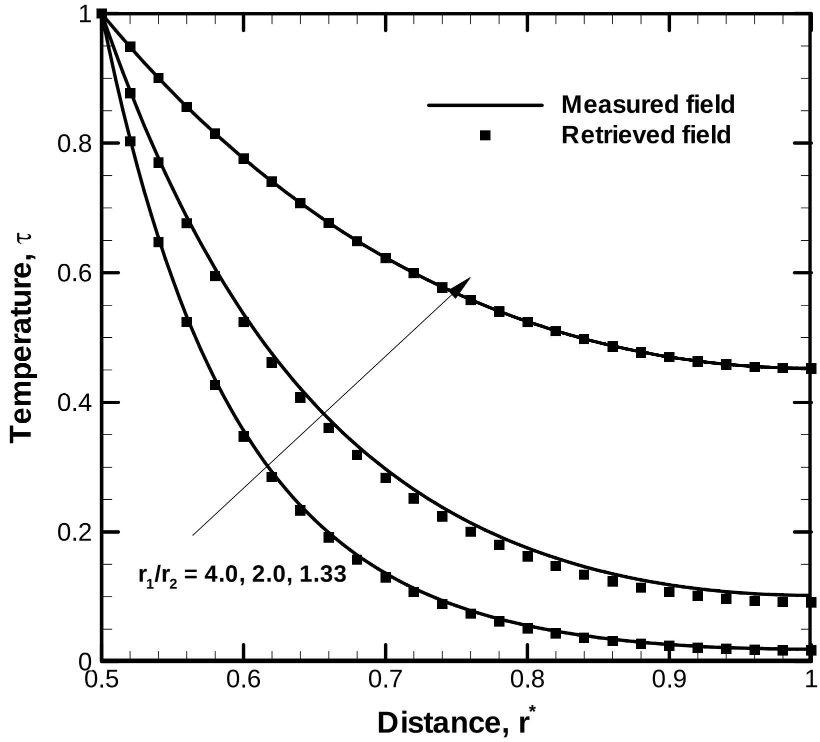

Figure 5: Comparison of the temperature fields at different locations for exact measurements and different radius-ratios; $r_1 = 0.5\text{m}, r_2 = 1.0\,\text{m}$, $h = 150 W/m^2K$, $K = 200 W/mK$.

3. Conclusion

An inverse analysis is carried out to retrieve the inner and outer radii, r_1, r_2 using the temperature field in a fin involving conduction-convection heat transfer and a mixed boundary condition. Problem is first solved by a forward methodology using the FDM and using the obtained temperature field, the GA is used to minimize an objective function for estimating the unknown parameters. The investigation is carried out to identify suitable GA parameters and the effects of retrieved parameters in the temperature field with/without measurement errors. The effect of different sets of the retrieved parameters in temperature filed is also studied. Based on the study, the following conclusions are made:

(1) A higher value of crossover and a lower value of mutation probability are better.
(2) Multiple number of combination of retrieved parameters satisfy a given temperature field.
(3) Lower value of radius ratio provides higher local temperature distribution.

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