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Average Friction Factor for Laminar Gas Flow in Microtubes

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ABSTRACT

The average friction factor in micro tubes will help the design engineers to estimate the pressure loss in micro flow devices. The aim of the present study is to obtain numerically average Darcy and Fanning friction factors and Mach numbers between the inlet and outlet of gas flows through adiabatic microtubes. This paper presents the average Poiseuille numbers, $(f_d{\cdot}Re)_{ave}$ & $(f_f{\cdot}Re)_{ave}$, between the inlet and outlet, those are obtained from numerical results for laminar gas flow in microtubes with diameters of 50, 100 and 150 μm and aspect ratios (i.e. length/diameter) of 100, 200 and 400, respectively. Axis-symmetric compressible momentum and energy equations were solved with the Arbitrary-Lagrangian-Eulerian (ALE) method. The stagnation pressure was chosen in such a way that the outlet Mach number ranged from 0.1 to 1.0. The outlet pressure was fixed at atmospheric condition. As a result, the average Darcy and Fanning friction factors between the inlet and outlet were obtained and compared with Moody's chart. The $(f_d{\cdot}Re)_{ave}$ and $(f_f{\cdot}Re)_{ave}$ were also obtained and presented as a function of average Mach number and were compared with the local $f{\cdot}Re$ correlations proposed in the previous study.

1. Introduction

Development of the MEMS technology has increased the need for understanding of fluid flow in micro flow devices such as micro-valves, micro-pumps, micro-reactor and for many other micro fluid systems [1]. A microchannel passage is the basic and important element in the design of micro flow devices and for this reason during the last decade a series of investigations have been conducted with the aim of clarifying the role of scale effects in microchannels [2]. From the early work by Wu and Little [3], numerous experimental and numerical investigations on microchannel gas flow have been undertaken as shown in the comprehensive review by Kawashima and Asako [4].

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The pressure loss determined using an appropriate friction factor between two points is important design information. There are two definitions of friction factors based on pressure drop, namely, Darcy and Fanning friction factors. Both friction factors for gases with large variations in physical properties were obtained solely under the assumption of isothermal flow by most investigators [3, 5-9]. This is because of the measurement limitations in gas temperature flowing through micro-channels. Fortunately, for gas flow through a micro-channel with thermally insulated wall (adiabatic wall), the estimation of gas static temperature at the micro-channel outlet can be obtained using a quadratic equation proposed by Kawashima and Asako [10]. A new data reduction methodology for the average Fanning friction factor calculation between the inlet and outlet, considering the effect of the decrease in gas temperature, has been developed by Hong *et al.,* [11]. Rehman *et al.,* [12] and Hong *et al.,* [13] experimentally and numerically investigated the average friction factor for adiabatic microchannels in compressible gas flows including choked flows.

Attention will now be focused on $f{\cdot}\mathrm{Re}$, the product of a friction factor and the Reynolds number for microchannel gas flows. Asako *et al.,* [14, 15] and Hong *et al.,* [16–19] conducted numerical investigations on gas flow characteristics in microchannels for both no-slip and slip boundary conditions. They proposed local $f{\cdot}\mathrm{Re}$ correlations as a function of Mach number and Knudsen number corresponding to compressibility and rarefaction effects for quasi-fully developed laminar flow. Their local $f{\cdot}\mathrm{Re}$ correlations were in excellent agreement with the experimental data of Hong *et al.,* [9,20]. They also reported pressure variations and mass flow rate for microchannel gas flow using the $f{\cdot}\mathrm{Re}$ correlations [21,22]. However, there seems to be no investigation on the average $f{\cdot}\mathrm{Re}$, the product of average friction factor and Reynolds number, for laminar gas flows in microtubes. This motivated the present study in obtaining numerically the average Darcy and Fanning friction factor and Mach number between the inlet and outlet of gas flows through microtubes.

2. Formulation

2.1 Description of The Problem

The schematic diagram of the calculation domain in a microtube for numerical analysis is shown in Figure 1. The numerical methodology to solve the governing equations is based on the Arbitrary-Lagrangian-Eulerian (ALE) method. The detailed description of the numerical computation was well documented in the previous work [14] and will not be repeated here. Only a brief description is reported here. The flow is assumed to be steady, axisymmetric, and laminar and the fluid is assumed to obey ideal gas law. Compressible gas in a reservoir at the stagnation pressure, p_{stg}, and stagnation temperature, T_{stg}, passes through an adiabatic microtube and is discharged to atmosphere ($p_{\mathrm{out}}=10^5$ Pa) by varying the stagnation pressure. The physical properties assumed to be constant. Inlet values of the velocity, density and temperature are obtained under an isentropic process between the inlet and the stagnation area. It is also assumed that the velocity, pressure, temperature and density profiles at the inlet are uniform and the thermal boundary condition on the wall is adiabatic.

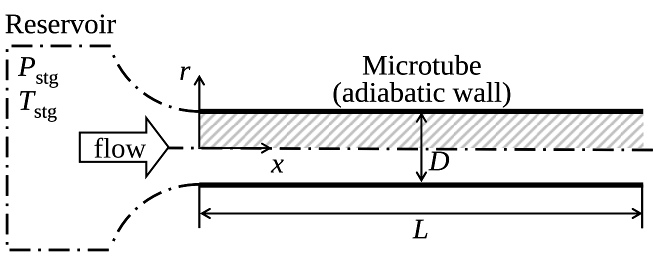

Fig. 1. Schematic diagram of the problem

2.2 Dimensionless Valuables

The Reynolds number and Mach number are defined as

$$Re = \frac{\bar{u} \quad D}{\mu / \bar{\rho}} \tag{1}$$

$$Ma = \frac{\bar{u}}{\sqrt{\gamma(\gamma-1)\bar{\imath}}} = \frac{\bar{u}}{\sqrt{\gamma R \bar{T}}} \tag{2}$$

where D is a tube diameter. $\bar{u}$, $\bar{\rho}$ and $\bar{\imath}$ are the average velocity, density and specific internal energy at the cross-section as follows

$$\bar{u} = \frac{1}{A}\int u \ \mathrm{dA}, \bar{\rho} = \int \rho u \mathrm{dA} \,/ \int u \mathrm{dA},$$

$$\bar{p} = \frac{1}{A}\int p \ \mathrm{dA}, \bar{\imath} = \frac{1}{\gamma-1}\frac{\bar{p}}{\bar{\rho}}, \bar{T} = \frac{\bar{p}}{R\bar{\rho}} \tag{3}$$

The average Mach number between the inlet and the outlet can be defined as

$$Ma_{\mathrm{ave}} = \frac{\bar{u}_{\mathrm{ave}}}{\sqrt{\gamma R \bar{T}_{\mathrm{ave}}}} \tag{4}$$

where the average velocity, $\bar{u}_{ave}$ and the average temperature, $\bar{T}_{\mathrm{ave}}$ can be obtained by integrating between the inlet (x_1) and outlet (x_2) as

$$\bar{u}_{\mathrm{ave}} = \frac{1}{x_2 - x_1}\int_{x1}^{x2} \bar{u} \ dx, \bar{T}_{\mathrm{ave}} = \frac{1}{x_2 - x_1}\int_{x1}^{x2} \bar{T} \ dx \tag{5}$$

In the previous study [11], a new data reduction methodology for the calculation of the average Fanning friction factor between the inlet and outlet through adiabatic microtubes has been developed. The developed average friction factor was employed in the present study. The average Fanning and Darcy friction factors between the inlet (x_1) and outlet (x_2) are

$$f_{\mathrm{f,ave}} = \frac{1}{x_2 - x_1}\int_{x_1}^{x_2} f_f \, dx = \frac{D}{x_2 - x_1}\left\{\int_{x_1}^{x_2}\left(\frac{2}{\bar{p}}d\bar{p}\right) - \int_{x_1}^{x_2}\left(\frac{2\bar{p}}{\bar{\rho}^2\bar{u}^2 R\bar{T}}d\bar{p}\right) - \int_{x_1}^{x_2}\left(\frac{2}{\bar{T}}d\bar{T}\right)\right\}$$

$$= \frac{D}{x_2 - x_1}\left[\begin{array}{l} -2\ln\frac{\bar{p}_{\mathrm{in}}}{\bar{p}_{\mathrm{out}}} + 2\ln\frac{\bar{T}_{\mathrm{in}}}{\bar{T}_{\mathrm{out}}} - \frac{1}{\left(\bar{p}_{\mathrm{in}}^2\bar{u}_{\mathrm{in}}^2 R \times \left(\bar{T}_{\mathrm{in}} + \frac{\bar{u}_{\mathrm{in}}^2}{2c_p}\right)\right)} \\ \times\left\{\frac{\bar{p}_{\mathrm{out}}^2 - \bar{p}_{\mathrm{in}}^2}{2} + \frac{B^2}{2} ln \frac{\bar{p}_{\mathrm{out}} + \sqrt{\bar{p}_{\mathrm{out}}^2 + B^2}}{\bar{p}_{\mathrm{in}} + \sqrt{\bar{p}_{\mathrm{in}}^2 + B^2}} + \frac{1}{2}\left(\bar{p}_{\mathrm{out}}\sqrt{\bar{p}_{\mathrm{out}}^2 + B^2} - \bar{p}_{\mathrm{in}}\sqrt{\bar{p}_{\mathrm{in}}^2 + B^2}\right)\right\} \end{array}\right] \tag{6}$$

$$f_{\mathrm{d,ave}} = \frac{1}{x_2 - x_1}\int_{x_1}^{x_2} f_d \, dx = \frac{D}{x_2 - x_1}\left\{-\int_{x_1}^{x_2}\left(\frac{2\bar{p}}{\bar{\rho}^2\bar{u}^2 R\bar{T}}d\bar{p}\right)\right\}$$

$$= \frac{D}{x_2 - x_1}\left[-\frac{1}{\bar{\rho}_{\text{in}}^2 \bar{u}_{\text{in}}^2 R \times \left(\bar{T}_{\text{in}} + \frac{\bar{u}_{\text{in}}^2}{2c_p} \right)} \times \left\{ \frac{\bar{p}_{\text{out}}^2 - \bar{p}_{\text{in}}^2}{2} + \frac{B^2}{2} ln \frac{\bar{p}_{\text{out}} + \sqrt{\bar{p}_{\text{out}}^2 + B^2}}{\bar{p}_{\text{in}} + \sqrt{\bar{p}_{\text{in}}^2 + B^2}} + \frac{1}{2}\left(\bar{p}_{\text{out}} \sqrt{\bar{p}_{\text{out}}^2 + B^2} - \bar{p}_{\text{in}} \sqrt{\bar{p}_{\text{in}}^2 + B^2} \right) \right\} \right] \tag{7}$$

where

$$B^2 = 4 \times \alpha \frac{\bar{\rho}_{\text{in}}^2 \bar{u}_{\text{in}}^2 R^2}{2c_p} \times \left(\bar{T}_{\text{in}} + \frac{\bar{u}_{\text{in}}^2}{2c_p} \right) \tag{8}$$

3. Result and Discussion

Numerical computations were conducted for microtubes with diameters of D = 50, 100 and 150 μm and aspect ratios of 100, 200 and 400, respectively. The tube diameter and its length, the stagnation pressure and the corresponding Reynolds number and Mach number are listed in Table 1.

Table 1
Tube diameter, length, p_{stg} and corresponding *Re* and *Ma*

D(μm)	L(mm)	p_{stg} (kPa)	*Re*	Ma_{in}	Ma_{out}
50	5	150 - 500	159 - 1514	0.096 - 0.287	0.141 - 1.105
	10	150 - 600	84 - 1454	0.051 - 0.225	0.075 - 1.072
	15	200 - 700	101 - 1245	0.046 - 0.163	0.090 - 0.95
100	10	150 - 375	537 - 2595	0.164 - 0.333	0.236 - 0.981
	20	150 - 450	316 - 2609	0.096 - 0.273	0.140 - 0.986
	40	150 - 500	168 - 2195	0.051 - 0.203	0.075 - 0.859
150	15	150 - 250	1019 - 2550	0.21 - 0.327	0.297 - 0.693
	30	150 - 400	657 - 2578	0.133 - 0.27	0.194 - 0.7
	60	150 - 400	368 - 2342	0.074- -0.206	0.109 - 0.644

3.1 Mach Number

The Mach numbers obtained for microtubes with different diameters are plotted in Figure 2 as a function of *x/L* with almost the same Reynolds number. The pressure distributions are also plotted in Figure 3. The Mach number of the smaller tube is higher than that of the larger tube and steeply increases near the outlet because the gas in the smaller tube expands near the outlet as shown in Figure 3. Also, the Mach number and pressure lines are not straight near the outlet. This phenomenon is dominant for smaller diameter microtubes as a result of compressibility. Therefore, the average Mach number between the inlet and outlet was obtained by integration from inlet to outlet. For *D*=100 μm, both the integral and the arithmetic average Mach number between the inlet and outlet are plotted in Figure 4 as a function of Reynolds number. The inlet Mach number, the integral average Mach number and the arithmetic average Mach number is higher for the shorter length. However, the outlet Mach number is independent of micro-tube length and it can be represented as a function of Reynolds number and diameter. The values of integral average Mach numbers are lower than the arithmetic values because of the flow acceleration due to gas expansion near the outlet. Qualitatively similar results for the microtubes of *D*=50 and 150 μm are also obtained.

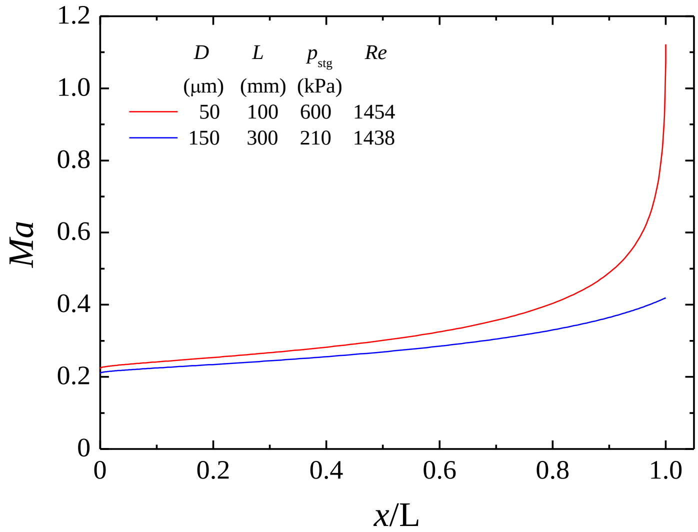

Fig. 2. Mach number as a function of *x*/*L*

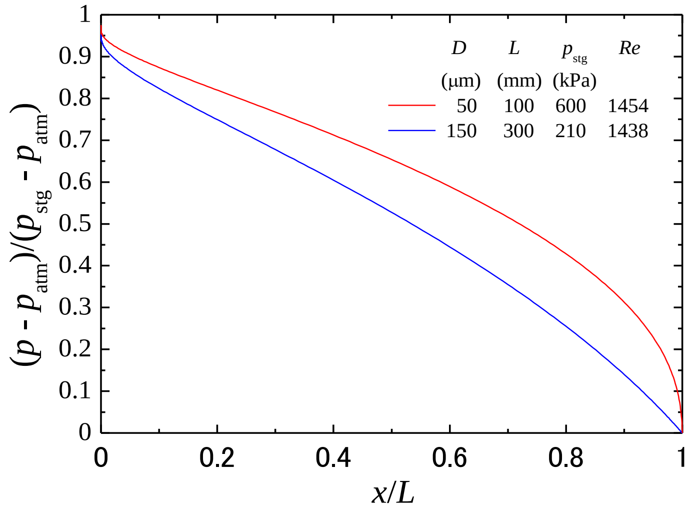

Fig. 3. Pressure distributions as a function of *x*/*L*

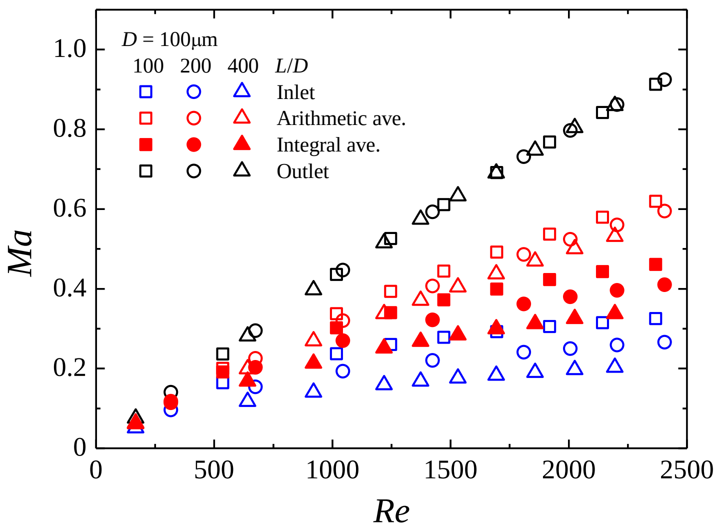

Fig. 4. Average Mach number as a function of Reynolds number

3.2 Average Friction Factors

The average Darcy and Fanning friction factor between the inlet and outlet, $f_{d,ave}$ and $f_{f,ave}$ obtained for all tubes are plotted in Figure 5 as function of Reynolds number with white and solid symbols, respectively. The solid line and the dotted line in the figure were obtained from f=64/Re and f=0.3164/$Re^{0.25}$ for laminar and turbulent incompressible flows, respectively.

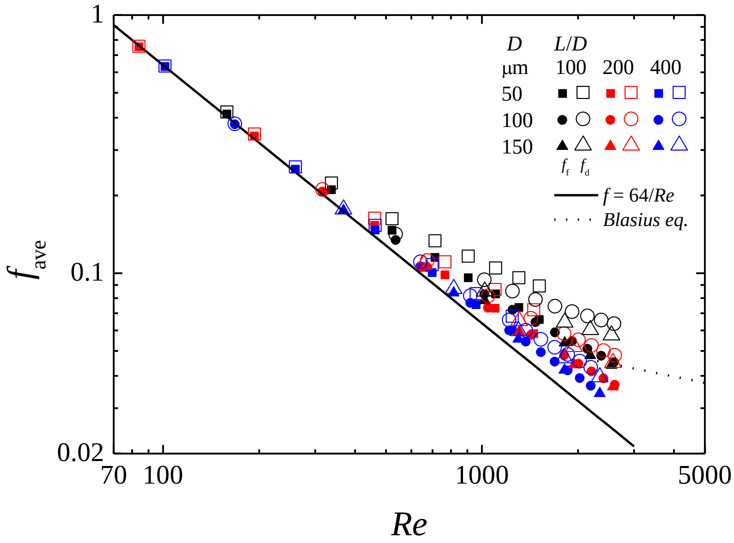

Fig. 5. Friction factors as function of *Re*

For the case of low flow velocities ($Re \leq 300$ and $Ma_{out} \leq 0.2$ as shown in Figure 4), both values of $f_{d,ave}$ and $f_{f,ave}$ are almost identical to the values of the incompressible theory. On the other hand, in the case of high flow velocities ($Re > 300$ and $Ma_{out} > 0.2$ as shown in Figure 4) both values of $f_{d,ave}$ and $f_{f,ave}$ deviate from the incompressible theory with an increase of Re because of the entrance and compressibility effects. The difference between $f_{d,ave}$ and $f_{f,ave}$ is larger for $Re < 300$ since flow accelerates with the increase in Ma_{ave} and Re.

The products of average Darcy & Fanning friction factors and Reynolds number between the inlet and the outlet, $(f_d \cdot Re)_{ave}$ and $(f_f \cdot Re)_{ave}$, for all tubes were obtained and are plotted in Figures 6 and 7 as a function of the integral average Mach number, respectively. Local $f_d \cdot Re$ and $f_f \cdot Re$ correlations proposed by the previous study [15] were employed in order to estimate the entrance losses. In the previous study [15], the local $f_d \cdot Re$ and $f_f \cdot Re$ for quasi-fully developed flows as a quadratic function of Mach number, were numerically obtained and their correlations were proposed as

$$f_d \cdot Re = 64 - 11.99Ma + 263.7Ma^2 \tag{9}$$

$$f_f \cdot Re = 64 + 2.703Ma + 93.89Ma^2 \tag{10}$$

The proposed local $f_d \cdot Re$ and $f_f \cdot Re$ are also plotted in these figures with solid lines, respectively. Both values of $(f_d \cdot Re)_{ave}$ and $(f_f \cdot Re)_{ave}$ increase with an increase in the average Mach number due to compressibility. $(f_d \cdot Re)_{ave}$ is higher than local $f_d \cdot Re$ since $(f_d \cdot Re)_{ave}$ includes losses due to both flow acceleration and entrance. $(f_f \cdot Re)_{ave}$ is also higher than local $f_f \cdot Re$ since $(f_f \cdot Re)_{ave}$ includes losses due to both friction and entrance.

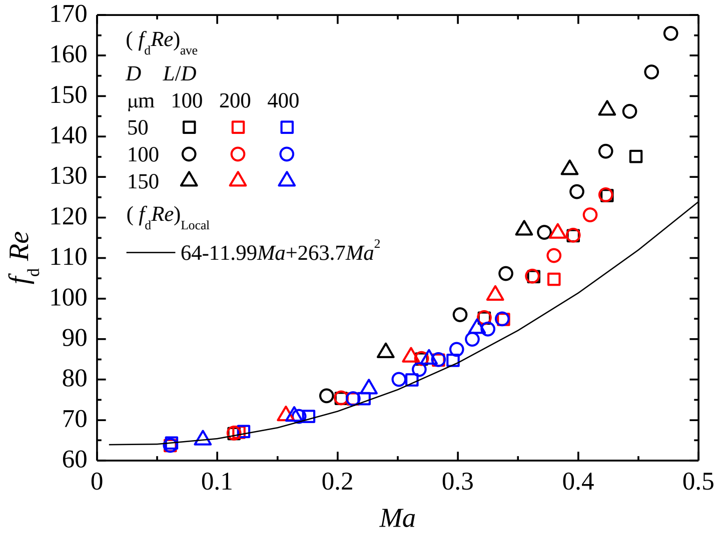

Fig. 6. f_d·Re as function of *Ma*

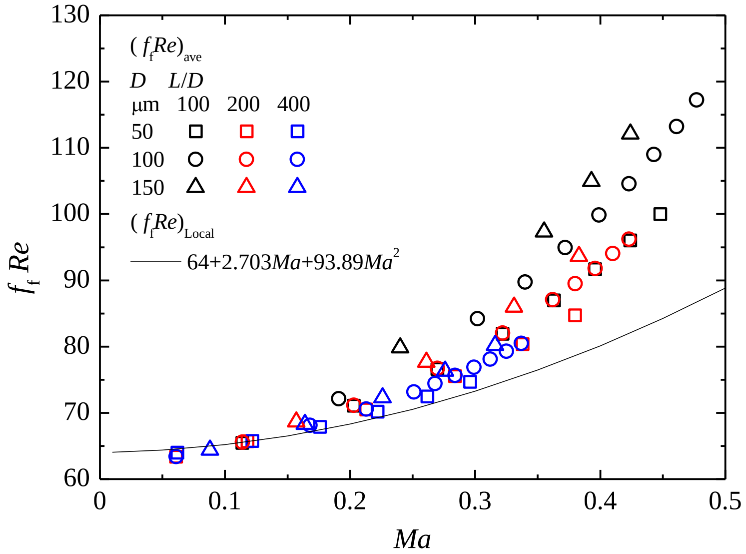

Fig. 7. f_f·Re as function of *Ma*

4. Conclusions

The numerical calculations were performed to obtain $(f_d\cdot Re)_{ave}$ & $(f_f\cdot Re)_{ave}$, the product of average Darcy friction factor and Reynolds number & the product of average Fanning friction factor and Reynolds between the inlet and outlet, for circular microtubes with diameters of 50, 100 and 150 μm and aspect ratios (i.e. length/diameter) of 100, 200 and 400, respectively. The following conclusions were reached.

i. The values of the integral average Mach numbers are lower than those of the arithmetic values because of the flow acceleration due to gas expansion near the outlet.
ii. $(f_d\cdot Re)_{ave}$ & $(f_f\cdot Re)_{ave}$ are higher than local $f_d\cdot Re$ & $f_f\cdot Re$ since $(f_d\cdot Re)_{ave}$ & $(f_f\cdot Re)_{ave}$ have the entrance loss.

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