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Stability Analysis of MHD Flow and Heat Transfer Passing A Permeable Exponentially Shrinking Sheet with Partial Slip and Thermal Radiation

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ABSTRACT

This paper has investigated the steady magnetohydrodynamic (MHD) flow and heat transfer induced by an exponentially shrinking sheet with partial slip, thermal radiation and suction. Similarity variables are introduced to transform the governing equations into non-linear ordinary differential equations. Then, the bvp4c solver in Matlab software is utilized to solve the transformed ordinary differential equations. The effects of magnetic parameter and mass suction parameter are analyzed and presented. From the results, we notice that first and second solutions exist in certain range of suction parameter. Hence, we continue further in performing a stability analysis. We found the first solution was more stable and the skin friction coefficient increased when suction increased.

1. Introduction

A few decades ago, several authors have studied about heat transfer and fluid flow passing stretching or shrinking sheet. Wang [1] has investigated the problem of a shrinking sheet with the stagnation flow. Saleh *et al.,* [2] investigated steady mixed convection stagnation flow towards a vertical shrinking sheet. Hafidzuddin *et al.,* [3] analyzed the unsteady problem for the three-dimensional flow of the permeable stretching/shrinking surface. In recent years, the studies of magnetohydrodynamic (MHD) have received a great of attention due to its important in many field. The numerous investigations on MHD have been reviewed in the literature [4-8].

Many researchers [9-13] showed that dual solution exist when the velocity ratio exceed unity, i.e., the sheet moves in opposite direction to the free stream. Recently, there has been growing interest in the stability analysis of dual solutions [14-15]. The analysis helps determine the stability and significance of the solutions to the problem. According to Merkin [11], positive eigenvalues mean the solution is stable and negative eigenvalue imply otherwise. Ishak [16] and Yasin *et al.,* [17] study the case of shrinking sheet and they concluded that second solution is unstable while first solution is

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stable. Nazar *et al.,* [18] investigated the stability analysis of three-dimensional flow induced with shrinking sheet in a Cu-water nanofluids. They also found that second solution is unstable and vice versa.

Based on previous study, the present investigation extends the problem by Sharma *et al.,* [12] to the case of magnetohydrodynamic (MHD) flow over an exponentially shrinking sheet. Stability analysis is conducted to identify the stability of dual solutions obtained.

2. Problem Formulation

A steady two-dimensional case of MHD flow passing an exponentially permeable shrinking sheet is considered. The governing equations are written in usual notation as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma\beta_0^2}{\rho}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} \tag{3}$$

The conditions at the boundary are:

$$\begin{gathered} v=v_w(x),\ u=U+N\nu\frac{\partial u}{\partial y},\ T=T_w+D\frac{\partial T}{\partial y}\ \text{at}\ y=0 \\ u\to 0,\ T\to T_\infty\ \text{as}\ y\to\infty \end{gathered} \tag{4}$$

Where $v_w(x)=V_0\exp(x/2L)$, $U=-U_0\exp(x/L)$ and $T_w=T_0\exp(x/2L)v_0$. V_0 is the mass flux velocity with $V_0<0$ (corresponds to the suction) and $V_0>0$ (corresponds to the injection). U represents the velocity of the shrinking, T_w represents the temperature of variable at the sheet, L represents the length, U_0 represents the velocity and T_0 represents the temperature. Here $N=N_1\exp(-x/2L)$ represents velocity slip factor while $D=D_1\exp(-x/2L)$ is the thermal slip factor. Both N_1 and D_1 are initial value for velocity and thermal slip factor, respectively. $N=D=0$ is the no-slip case. For the radiation flux q_r, we used Rosseland approximation and we can write:

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{5}$$

Where k^* is the mean absorption coefficient and σ^* is Stefan-Boltzman constant. We further assumed that the term T^4 can be expressed as a linear function of temperature itself. Thus, T^4 can be expanded using Taylor series. Next, the terms of higher order are ignored and yields:

$$T^4\cong 4T_\infty^3T-3T_\infty^4 \tag{6}$$

Introducing stream function ψ which always be denoted as $u=\partial\psi/\partial y$ and $v=-\partial\psi/\partial x$. Next, we introduce the following similarity variables:

$$\psi=\sqrt{2\nu LU_0}\,f(\eta)\exp(x/2L),\; T=T_0\exp(x/2L)\theta(\eta),$$
$$\eta=y\sqrt{\frac{U_0}{2\nu L}}\exp(x/2L). \tag{7}$$

Using Eq. (7), we get

$$u=U_0\exp(x/L)\,f'(\eta),\; v=-\sqrt{\frac{\nu U_0}{2L}}\exp(x/2L)\left[f(\eta)+\eta f'(\eta)\right] \tag{8}$$

Primes here denote differentiation with respect to η. We are used Eqs. (5) and (7). Thus, Eqs. (2) and (3) become

$$f'''+ff''-2f'^2-Mf'=0 \tag{9}$$

$$\frac{1}{\mathrm{Pr}}\left(1+\frac{4}{3}R\right)\theta''+f\theta'-f'\theta=0 \tag{10}$$

Where $M=\dfrac{2L\sigma\beta_0^2}{u_0\rho e^{x/L}}$ is parameter for magnetic, $\mathrm{Pr}=\dfrac{\nu}{\alpha}$ is the Prandtl number and $R=\dfrac{4\sigma^*T_\infty^3}{k_m k^*}$ is the parameter for radiation. The transformed boundary conditions are:

$$f(0)=S,\; f'(0)=-1+\lambda f''(0),\; \theta(0)=1+\delta\theta'(0)$$
$$f'(\eta)\to 0,\; \theta(\eta)\to 0 \text{ as } \eta\to\infty \tag{11}$$

where $\lambda=N_1\sqrt{U_0\nu/2L}\,(>0)$, $\delta=D_1\sqrt{U_0/2\nu L}\,(>0)$ and $S=-V_0\sqrt{U_0/2\nu L}$. S is the parameter for the constant mass transfer. $S>0$ corresponds to the suction while $S<0$ corresponds to the injection.

The main physical quantities in this study are the skin friction coefficient, C_f and local Nusselt number, Nu_x. C_f and Nu_x are defined as follows

$$C_f=\frac{\mu}{\rho\left[U_0\exp(x/L)\right]^2}\left(\frac{\partial u}{\partial y}\right)_{y=0},\quad Nu_x=\frac{L}{T_0\exp(x/2L)}\left(-\frac{\partial T}{\partial y}+q_r\right)_{y=0}. \tag{12}$$

Substituting Eqs. (5) and (7) into Eq. (12) and we obtain

$$(2\mathrm{Re}_x)^{1/2}C_f=f''(0),\; (2/\mathrm{Re}_x)^{1/2}Nu_x=-(1+R)\theta'(0) \tag{13}$$

Where $\mathrm{Re}_x=(U_0L/\nu)\exp(x/L)$ is the local Reynolds number.

3. Stability Analysis

In this study, we are using the stability analysis developed by Weidman *et al.,* [19]. According to them, a variable τ has to be introduced. To perform stability analysis, unsteady cases are considered. Hence, Eqs. (2) and (3) will be replaced by

$$\frac{\partial u}{\partial t}u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma\beta_0^2}{\rho}u \tag{14}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} \tag{15}$$

Here, t refers to the time. Next, we introduced the new dimensionless variables for the unsteady problem based on the variables (7) previously

$$\begin{aligned}&\psi=\sqrt{2\nu LU_0}\,f(\eta,\tau)\exp(x/2L),\ T=T_0\exp(x/2L)\theta(\eta,\tau),\\&\eta=y\sqrt{\frac{U_0}{2\nu L}}\exp(x/2L),\ \tau=\frac{U_0 t}{2L}\exp(x/L)\end{aligned} \tag{16}$$

So that, Eqs. (14) and (15) change to the following equations

$$\frac{\partial^3 f}{\partial\eta^3}+f\frac{\partial^2 f}{\partial\eta^2}-2\left(\frac{\partial f}{\partial\eta}\right)^2-\frac{\partial^2 f}{\partial\eta\partial\tau}-M\frac{\partial f}{\partial\eta}=0 \tag{17}$$

$$\frac{1}{\Pr}\left(1+\frac{4}{3}R\right)\frac{\partial^2\theta}{\partial\eta^2}+f\frac{\partial\theta}{\partial}-\frac{\partial f}{\partial\eta}\theta-\frac{\partial\theta}{\partial\tau}=0 \tag{18}$$

With the following boundary conditions

$$\begin{aligned}&f(0,\tau)=s,\ \frac{\partial f}{\partial\eta}(0,\tau)=-1+\lambda\frac{\partial^2 f}{\partial\eta^2},\ \theta(0,\tau)=1+\delta\frac{\partial\theta}{\partial\eta}\\&\frac{\partial f}{\partial\eta}(\eta,\tau)\to 0,\ \theta(\eta,\tau)\to 0\text{ as }\eta\to\infty\end{aligned} \tag{19}$$

We can test the stability of the solution $f(\eta)=f_0(\eta)$ and $\theta(\eta)=\theta_0(\eta)$. It is satisfying the boundary-value problem in Eqs. (9) to (11). We can write

$$f(\eta,\tau)=f_0(\eta)+e^{-\gamma\tau}F(\eta,\tau),\ \theta(\eta,\tau)=\theta_0(\eta)+e^{-\gamma\tau}G(\eta,\tau), \tag{20}$$

Here, γ is an unknown eigenvalue. $F(\eta,\tau)$ and $G(\eta,\tau)$ are small relative to $f_0(\eta)$ and $\theta_0(\eta)$. After that, Eq. (20) is substituted into Eqs. (17) and (18) and obtained as below

$$\frac{\partial^3 F}{\partial\eta^3}+f_0\frac{\partial^2 F}{\partial\eta^2}+f_0''F-(4f_0'+M-\gamma)\frac{\partial F}{\partial\eta}-\frac{\partial^2 F}{\partial\eta\partial\tau}=0, \tag{21}$$

$$\frac{1}{\Pr}\left(1+\frac{4}{3}R\right)\frac{\partial^2 G}{\partial \eta^2}+f_0\frac{\partial G}{\partial \eta}+\theta_0{}'F-f_0{}'G-\theta_0\frac{\partial F}{\partial \eta}+\gamma G-\frac{\partial G}{\partial \tau}=0 \tag{22}$$

Together with the following boundary conditions

$$\begin{aligned}&F(0,\tau)=0,\ \frac{\partial F}{\partial \eta}(0,\tau)=\lambda\frac{\partial^2 F}{\partial \eta^2}(0,\tau),\ G(0,\tau)=\delta\frac{\partial G}{\partial \eta}(0,\tau)\\&\frac{\partial F}{\partial \eta}(\eta,\tau)\to 0,\ G(\eta,\tau)\to 0 \text{ as } \eta\to\infty\end{aligned} \tag{23}$$

After that, we set $\tau=0$, $F=F_0(\eta)$ and $G=G_0(\eta)$ to obtain the following:

$$F_0{}'''+f_0F_0{}''+f_0{}''F_0-(4f_0{}'+M-\gamma)F_0{}'=0, \tag{24}$$

$$\frac{1}{\Pr}\left(1+\frac{4}{3}R\right)G_0{}''+f_0G_0{}'+\theta_0{}'F_0-f_0{}'G_0-\theta_0F_0{}'+\gamma G_0=0 \tag{25}$$

And the boundary conditions:

$$\begin{aligned}&F_0(0)=0,\ F_0{}'(0)=\lambda F_0{}''(0),\ G_0(0)=\delta G_0{}'(0)\\&F_0{}'(\eta)\to 0,\ G_0(\eta)\to 0 \text{ as } \eta\to\infty\end{aligned} \tag{26}$$

By relaxing a boundary condition on $G_0(\eta)$ or $F_0{}'(\eta)$ (see [20]), we can determine the possible values of γ. The aim of this procedure is to determine the stability of the problem and for this present problem, we relax $F_0{}'(\infty)\to 0$. Then, we solved the system with the new condition at the boundary $F_0{}''(0)=1$.

4. Methodology

By apply similarity transformation method, the three partial differential equations were transformed to become two ordinary differential equations. After that, we solved numerically Eqs. (9) and (10) together with boundary conditions in Eq. (11) by using bvp4c function in Matlab. In this study, we fixed the values of $R=\lambda=\delta=0.1$ and $\Pr=0.7$. To ensure that our results are correct, a comparison has been made with the results obtained by Sharma *et al.,* [18]. We found that the comparisons are in excellent agreement for $S=2.20$ and $M=0$. Figure 1 represents the skin friction coefficient at the surface for selected values of M while Figure 2 shows the heat transfer rate at the surface for selected values of M. From both figures, we noticed that there are admit dual solutions when $S>S_c$, where S_c is the critical value of suction. To delay or accelerate the separation of the boundary layer, we can set magnetic parameter M as controller parameter because when the value of M is increases, the range of the solution will be increasing. Thus, we can conclude that the changes of the rate of heat transfer and surface stress are influenced by magnetic parameter M.

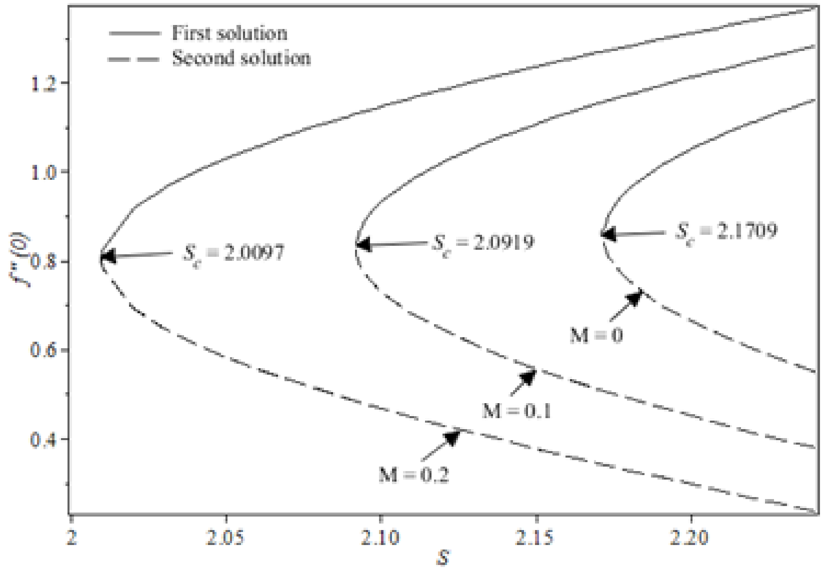

Fig. 1. Variation of $f''(0)$ for several values of M with S

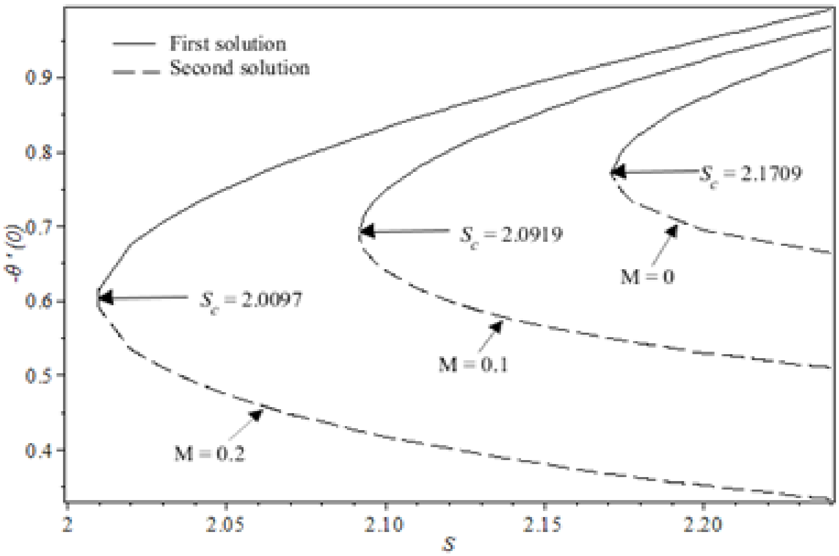

Fig. 2. Variation of $-\theta'(0)$ for several values of M with S

Further, Figures 3 to 6 display profiles to support the dual solutions that exist in Figures 1 and 2. Figures 3 and 5 shows profiles for the velocity while both Figures 4 and 6 show the temperature profiles. All profiles show the thickness of the boundary layer for the second solution always presents greater compared to the first solution. There are display first and second solutions. Hence, a stability analysis is conducted to know which one is unstable and stable. Table 1 presents the smallest eigenvalues γ for selected values of M and S. From Table 1, it is clearly shown that the smallest eigenvalue γ is positive for the first solution and it is in opposite value for the second solution. Thus, the first solution is linearly stable while second solution is linearly unstable.

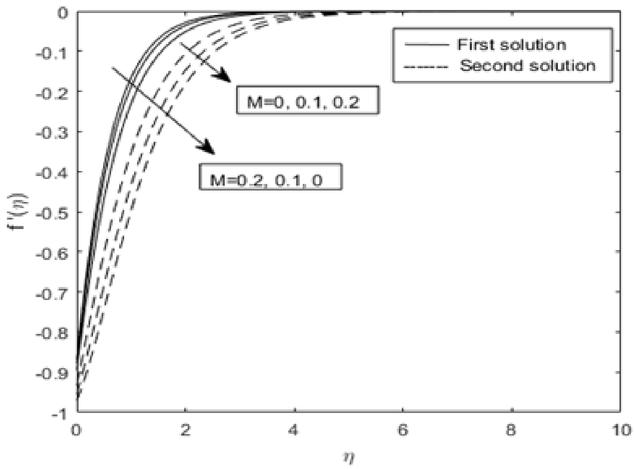

Fig. 3. Velocity profile for different values of M with $S = 2.20$

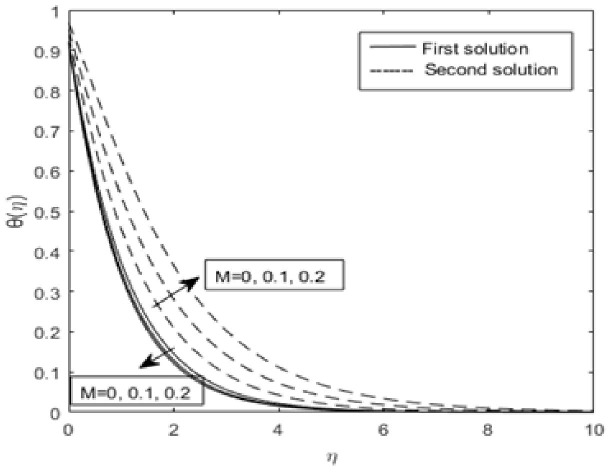

Fig. 4. Temperature profile for different values of M with $S = 2.20$

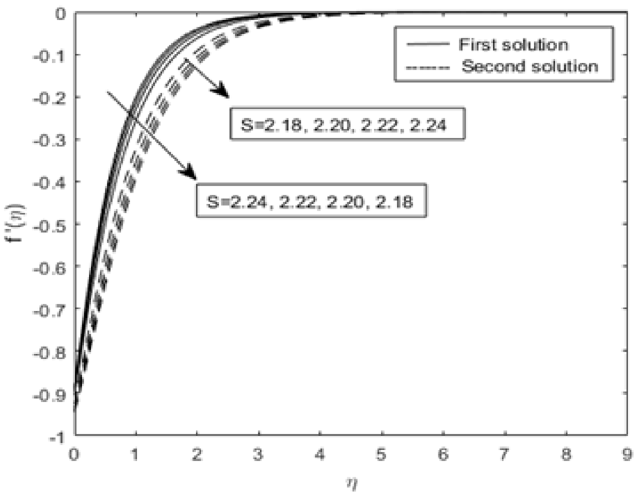

Fig. 5. Velocity profile for different values of S with $M = 0$

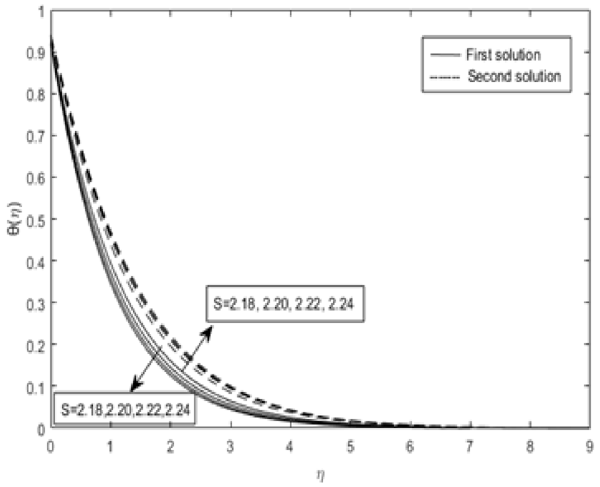

Fig. 6. Temperature profile for different values of S with $M = 0$

Table 1
Smallest eigenvalues γ for some values of M and S

M	S	First Solution	Second Solution
	2.18	0.3015	-0.2985
0	2.20	0.3952	-0.3903
	2.22	0.4758	-0.4671
	2.24	0.5427	-0.5316
	2.18	0.8358	-0.5289
0.1	2.20	0.8774	-0.6394
	2.22	0.8987	-0.6886
	2.24	0.9293	-0.7349
	2.18	1.1906	-0.7827
0.2	2.20	1.2384	-0.8243
	2.22	1.3251	-0.8643
	2.24	1.3749	-0.9028

5. Conclusion

A study on magnetohydrodynamic (MHD) flow induced by shrinking sheet with partial slip, thermal radiation and suction was carried. Similarity transformations were introduce to reduce the partial differential equations into ordinary differential equations. The bvp4c solver was then used to solve the ordinary differential equations and boundary conditions. The results showed that increasing the magnetic parameter, it is contribute to increase the surface stress and heat transfer rate. Then, also we found that there are exists dual solution. The stability analysis was done to determine the stability of dual solutions. Based on the results, the first solution are stable and have physical meaning.

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