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Numerical Study on the Turbulent Mixed Convective Heat Transfer over 2D Microscale Backward-Facing Step

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ABSTRACT

The current numerical study investigated over two-dimensional (2D) flat microscale backward-facing step (MBFS). The boundary conditions and the controls fixed by the finite volume method (FVM) and RNG k-ε model. The upstream and the step of the wall considered adiabatic, while the downstream of the wall heated by constant heat flux. The straightforward wall of the channel fixed at a constant temperature that is higher than the fluid inlet temperature. The Reynolds number (*Re*) range of 5,000 ≤ *Re* ≤ 15,000 was used in the study. The results show that the increase in the *Re* will resulted in increased of Nusselt number *(Nu)*. The study also found that the highest *Nu* was produced from the ethylene glycol case. The current study also found that the recirculating range at steps is more significant than using water under the same *Re* by using ethylene glycol.

1. Introduction

Flow separation and subsequent reattachment occurs when the geometry is pressurized or expands abruptly like backward-facing steps (BFS). This is a normal phenomenon in the design of several engineering applications such as combustion chambers, high-performance heat exchangers, environmental control systems, electronic equipment cooling systems, energy system equipment, cooling passages in turbine blades, and the singularity influences chemical processes. Accordingly, a large amount of low and high energy gets mixed with the region of reattachment. Numerical and experimental studies which have been conducted in early times have proven the existence of the problem of flow on BFS in mixed, natural, and forced geometry [1–9]. In several studies, the use of large-eddy model been employed for the simulation and analysis of turbulent heat transfer over BFS by Labbe *et al.,* [10], Keating *et al*., [11], Mehrez *et al.,* [12], and Avancha *et al*., [13]. The results of these investigations revealed that the rate of heat transfer was significantly enhanced with the region of recirculation, while trends of heat transfer coefficients that correspond to that of past

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experimental data obtained. To investigate the effects which periodic perturbation has on a separated shear layer bordering a recirculation bubble at the back of BFS in a high aspect-ration channel, Dejoan and Leschziner [14] employed the use of eddy simulation.

In the study carried out by Abu-Nada [15], the backward facing steps (BFS) with various expansion ratios were investigated. The results of the study indicated that the entropy generation number (*Ns*) is directly affected by the Reynolds number (*Re*). More so, the *Ns* is also affected by the expansion ratio (*ER*). The maximum value is possessed by the *Ns* with-in the region of recirculation, while the minimum amount observed at the step. Again, a numerical study was carried out by Abu-Nada [16] to investigate the transfer of heat over BFS through the use of the nanoparticle with water serving as the base fluid. In this study, the use of different kinds of nanoparticles with different concentration. It was found that the Nusselt number (*Nu*) improved as a result of increased volume fraction. Besides, the results also indicated that the *Nu* maximum value achieved at the bottom of the wall. Abu-Nada [17] has continued to use suction/blowing to investigate bleeding over a backward-facing step. The study revealed that in the suction case, the increase occurred in the temperature value and velocity gradients. For this reason, as an increase occurred in the *Ns*, while in the blowing, a decrease observed in the *Ns*. While the *Ns* value increased, the enhancement of heat transfer occurred at the bot-tom wall. In a subsequent study by Abu-Nada *et al.,* [18] it was shown that in the case of blowing, *Re* = 800, and the reattachment zone augmented till its determined value was achieved and reduced because of high coefficient of bleed values. As a result of blowing, the flow forcefully detached. With the blowing, the gradients velocity decreased, thereby resulting in the condensation of friction coefficient. Generally, it was found that there was an improvement in the *Nu* by force, while puffing reduced.

Many studies have been conducted by other researchers on the flow over three-dimensional (3D) backward-facing steps (BFS) with different flow regime. Nie and Armaly [19] restrained Laser-Doppler velocity next to the bounding walls of a 3D BFS flow. The study involved Reynolds number *Re* ranging from 100 – 8,000 were presented and therefore encompassing the transitional, laminar and turbulent flow regimes. In the laminar flow regime, an increment observed in the size of reverse flow regions, which also moved further downstream. In the laminar flow regime, there was a decrease in the size of reverse flow regions, while in the transitional flow regime, the proportion remained stable or diminishing as an increase occurred in the *Re*. They also observed that a minimum developed by the width distribution of the boundary line for the reverse flow region adjacent the stepped wall close to the sidewall in the laminar flow regime. However, the disappearance of the minimum in the distribution observed in the turbulent flow regime. In the laminar flow regime as well as the turbulent flow regime, there was a good agreement between the estimates and measurement. In another study by Nie and Armaly [20], the effect of step height on the characteristics of flow and heat transfer were investigated. They also studied a 3D incompressible laminar forced convection flow adjacent the BFS in the rectangular duct and analyzed it numerically. Also, they carried out an experimental examination of the micro scale backward-facing step (MBFS) and enhancement of heat transfer.

The effect of micro scale backward-facing step (MBFS) step height on characteristics of flow and heat transfer studied by Kherbeet *et al.,* [21], the use of EG–SiO_2 nanofluid was employed. Based on the results, there was a development of a complicated 3D flow downstream of the step with whirling and inverse flow areas next to the sidewall. There was a disruption of the flow within the region of separation on the stepped wall, and this disruption led to the development of determined values in the Nusselt number (*Nu*) as well as a least in the length of reattachment. The length of the reattachment, the size of the sidewall reverse flow region, and *Nu* increased resulted to the increase in step height. It also observed that as the height of the step enlarged, the coefficient of skin friction increased, while a reduction occurred in the pressure drop. More so, the results revealed that as the Richardson number (*Ri*) increased, an increment occurred in the local variation of skin friction without dimension along the heated wall.

A study of the 3D flow in micro scale backward-facing step (MBFS) conducted by Hsieh *et al.,* [22], who reported that as the cross-section aspect ratio reduced, a decrease occurred in the velocity magnitude as a result of the extra resistance as of the aspect walls. The study observed that the wall shear stress over the wall at the back of the step was equal to the slip velocity. As a reduction occurred in the cross-section aspect ratio, an increase occurred in the effect of heating, which led to a rise in the local temperature at the side walls. Saldana *et al.,* [23] studied the forced convective airflow on a 3D BFS numerically. The study found that the local *Nu* over the downstream of the wall begins to lie in the surrounding area of the X_u-line and the X_w-line points of the crossing whereabouts the shear stress over at the bottom of the wall is equal to zero.

In the study carried out by Abu-Mulaweh *et al.,* [24], it was found that higher Nusselt number (*Nu*) was produced by the velocity of the lower free stream within the region of recirculation as a result of the change in the area of recirculation. As the velocity of flow or height increased, an increase also occurred in the length of reattachment and size of the recirculation area. With regards to the results for inclined backward-facing step (BFS), a decrease occurred in the local *Nu* as an increase occurred in the vertical side of the inclination angle because of the reduction of the temperature gradient at the heated wall. More so, as the angle of inclination increased, an increase occurred in the place of the determined *Nu* as well as the length of reattachment because of the reduction in the stream-wise buoyancy force. In another study by Abu- Mulaweh *et al.,* [25] the outcome of the BFS heights on turbulent mixed convection flow over a smooth vertical plate was studied. Results of the study showed that as the step height increased, the region of recirculation increased in length, and there was also an increase in the intensity of the passive median transverse velocity ingredient as the height of the step increased.

Nevertheless, the strength of turbulence was enhanced as the step height introduced, thereby causing the flow to become turbulent downstream of the step. The step height increase caused the degree of the strengths of individual velocity, fluctuations in temperature, and measured local Nusselt number (*Nu*) step downstream to increase. In a numerical investigation conducted by Kherbeet *et al.,* [26] the Laminar mixed convection flow over a 2D horizontal micro scale backward-facing step (MBFS) placed in a duct was studied. In their study, they employed the use of the finite volume method to solve the governing equations along with the boundary conditions. They reported that as the Reynolds number (*Re*) and volume fraction increased, an increase observed in the *Nu*. It also noted that the nanofluid of SiO_2 nanoparticles achieved the highest value of *Nu*. More so, the results showed that the reduction in diameter of nanoparticle led to an increase in the *Nu*. Nevertheless, no region of recirculation observed at the step along the duct.

Based on the review of literature, past studies have not focused on the effect of base fluids on turbulent mixed convective heat transfer over a 2D micro scale backward-facing step (MBFS). Therefore, the current research is motivated to fill this research gap. The objective of the present research focused on investigating turbulent mixed convective flow on a 2D MBFS located in a horizontal channel in two kinds of base fluids, which are water and ethylene glycol. Furthermore, the study presented to show the effect of using different base fluids on parameters such as skin friction coefficient (C_f), Nusselt number (*Nu*), distribution velocity in turbulent mixed convection over a 2D MBFS.

2. Mathematical Model

2.1 Physical Model

The contemplate geometry drawing, and the flow conformation used in this investigation presented in Figure 1. Two kinds of base fluids were used to investigate the consequence of the base fluids, which are ethylene glycol and water. To guarantee the fully developed flow in the duct inlet and outlet, the downstream wall length was chosen to be 100 mm, and the upstream wall length was 20 mm. The channel width and height were 30 mm and 900 μm, respectively. The considered step height in this study was 450 μm. The temperature of the straight wall fixed at 300 K and the downstream heat flux of the wall set at 1000 W/m^2. The flow entrance of the channel was hydrodynamically stable and entirely fully developed. The streamwise gradients of all quantities on the exit of the channel were set to be zero. The upstream of the wall and the step of the wall were adiabatic outsides.

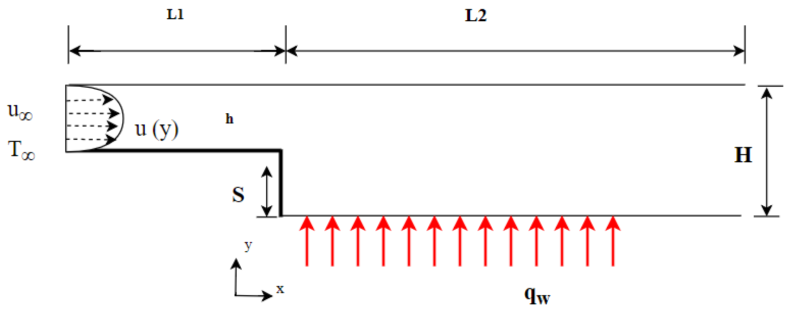

Fig. 1. Horizontal 2D micro scale backward-facing step (MBFS) schematic diagram

The base fluids were considered to have a thermal balance, and there are no slip complaint Newtonian and incompressible fluid flow. The properties of the thermophysical of the base fluids are given in Table 1.

Table 1
Thermophysical properties of the base fluids

Fluid type	ρ (kg/m^3)	μ ($N.s/m^2$)	k (W/m.K)	C_p (J/kg.K)	β (1/K)
E-G	1114.4	1.57E−02	2.52E−01	2415	6.50E−04
Water	998.203	1.01E−03	6.13E−01	4182.2	2.06E−04

2.2 Boundary Conditions

Introducing the next dimensionless measures

$$U=\frac{u}{u_\infty},U=\frac{v}{u_\infty},X=\frac{x}{D_h},\mathrm{Y}\frac{y}{D_h},P=\frac{(p+\rho gx)}{\rho u_\infty^2},\theta=\frac{(T-T_\circ)}{(q_w s/k)}$$

The non-dimensional formula of the momentum, energy governing, and continuity equations, in Cartesian coordinates, are given as

Continuity equation

$$\frac{\partial(\rho\bar{u})}{\partial x}+\frac{\partial(\rho\bar{v})}{\partial y}=0 \tag{1}$$

X-momentum equation

$$\frac{\partial}{\partial x}\rho\bar{u}\bar{u}+\frac{\partial}{\partial y}\rho\bar{u}\bar{v}=-\frac{\partial P}{\partial x}+\frac{\partial}{\partial x}\left[(\mu+\mu_t)\frac{\partial\bar{u}}{\partial x}\right]+\frac{\partial}{\partial y}\left[(\mu+\mu_t)\frac{\partial\bar{u}}{\partial y}\right] \tag{2}$$

Y-momentum equation

$$\frac{\partial}{\partial x}\rho\bar{u}\bar{v}+\frac{\partial}{\partial y}\rho\bar{v}\bar{v}=\ -\frac{\partial P}{\partial x}+\frac{\partial}{\partial x}\left[(\mu+\mu_t)\frac{\partial\bar{v}}{\partial x}\right]+\frac{\partial}{\partial y}\left[(\mu+\mu_t)\frac{\partial\bar{v}}{\partial y}-\frac{2}{3}\rho\frac{\partial k}{\partial y}\right] \tag{3}$$

and the energy equation

$$\frac{\partial}{\partial x}(\rho\bar{u}T)+\frac{\partial}{\partial y}(\rho\bar{v}T)=\frac{\partial}{\partial x}\left[\left(\frac{k}{C_p}+\frac{\mu_t}{Pr_t}\right)\frac{\partial T}{\partial x}\right]\ +\frac{\partial}{\partial y}\left[\left(\frac{k}{C_p}+\frac{\mu_t}{Pr_t}\right)\frac{\partial T}{\partial y}\right] \tag{4}$$

where μ_t is the turbulent dynamic viscosity which is related to k and ε and it is calculated as

$$\mu_t=\rho C_\mu f_\mu\left(\frac{K^2}{\varepsilon}\right) \tag{5}$$

A standardized boundary condition is taken into consideration at the inlet and fully developed outlet flow. The turbulent kinetic energy (k) and the dissipation (ε) are predicted by the RNG k- ε model as follows

$$\frac{\partial(\rho k\bar{u})}{\partial x}+\frac{\partial(\rho k\bar{v})}{\partial y}=\frac{\partial}{\partial x}\left[\left(\mu+\frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x}\right]+\frac{\partial}{\partial y}\left[\left(\mu+\frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial y}\right]+G_k+G_b+\rho\varepsilon-D \tag{6}$$

$$\frac{\partial(\rho\varepsilon\bar{u})}{\partial x}+\frac{\partial(\rho\varepsilon\bar{v})}{\partial y}=\frac{\partial}{\partial x}\left[\left(\mu+\frac{\mu_t}{\sigma_\varepsilon}\right)\frac{\partial\varepsilon}{\partial x}\right]+\frac{\partial}{\partial y}\left[\left(\mu+\frac{\mu_t}{\sigma_\varepsilon}\right)\frac{\partial\varepsilon}{\partial y}\right]+C_{\varepsilon1}f_1\frac{\varepsilon}{k}(G_k+C_{\varepsilon3}G_b)-C_{\varepsilon2}f_2\rho\frac{\varepsilon^2}{k}+E \tag{7}$$

σ_k and σ_ε represent inverse turbulent Prandtl numbers (*Pr*) for k and ε. G_k represents the rate of turbulent kinetic energy generation due to medium speed gradients and is determined by

$$G_k=\mu_t\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)\frac{\partial u_i}{\partial x_j}-\frac{2}{3}\rho k\delta_{ij}\frac{\partial u_i}{\partial x_j} \tag{8}$$

The governing equations boundary conditions are

(i) Conditions of upstream at $X=-\frac{L_1}{D_h}$ and $\frac{S}{D_h}\leq Y\leq\frac{H}{D_h}$

The inlet flow of the channel fully adjusted with the mean velocity u_∞. The distribution of input velocity is, therefore, parabolic

(ii) Conditions of *d* exit at $X=L_2$ and $0\leq Y\ \leq\frac{H}{D_h}$

(iii) The straight wall at $X=-\frac{L_1}{D_h}\leq X\leq\frac{L_2}{D_h}$ and $Y=\frac{H}{D_h}$

(iv) Stepped wall conditions

Upstream of the step at $-\frac{L_1}{D_h} \leq X \leq 0$ and $Y = \frac{S}{D_h}$
At step $X = 0$ and $0 \leq Y \leq \frac{S}{D_h}$
Downstream of the step at $Y = 0$, and $0 \leq X \leq \frac{L_2}{D_h}$

After the above mentioned governing equations and its given boundary conditions, the results of the velocity profiles, Nusselt number (*Nu*), and skin friction coefficient (C_f) over the downstream segment of the micro scale backward-facing step (MBFS) resolved. At this point, it is proper to introduce approximately descriptions

$$Nu = \frac{\mathrm{H}\, D_h}{\mathrm{K}} = \frac{q_w D_h}{K(T_w - T_\infty)}$$

The skin friction coefficient is defined as follows

$$C_f = \frac{2\tau_w}{\rho u_\infty{}^2}$$

where τ_w is the wall shear stress, $Re = \frac{\rho\, u_m\, D_h}{\mu}$, $Pr = \frac{v}{\alpha}$, $Gr_y = \frac{g\, \beta q_w S^4}{K v^2}$

3. Numerical Implementation

The current study solves Eqn. (1) – (4) by using the finite volume approach, with agreeing with the boundary conditions. The SIMPLE algorithm was used to resolve the flow pitch. The second-order upwind differencing scheme takes into consideration the relations of the convective. The second-order central difference gives a decent explanation for the expansion expression in the momentum and energy equations [27]. In the x-direction and nearby area of the step and point of the reattachment, a good grid was used to resolve the steep velocity gradients, while the coarser grid used on the downstream at the same spot. Nevertheless, the fine grid is used in the y-direction near the top walls, the bottom, and straight at the step to make sure the correctness of the numerical study and for saving both computational time and the size of the grid. The non-uniform and quadrilateral elements grid system assumed in this study. The rest of the variables calculated and set at the end of each iteration for the separately conserved variables. The convergence standard wanted the most qualified residual mass built on the entrance mass to be less than $1{\times}10^{-3}$. Table 2 presents the current grid number and the independent test with the Nusselt number (*Nu*).

Table 2
The independence of grid study

Grid No.	$\boldsymbol{Nu}$	e %
65,200	30.3886	-
156,600	30.0619	1.075070
216,800	30.0065	0.184286
331,000	29.9986	0.027667

Figure 2 shows a section of the schematic the grid mesh of the current study.

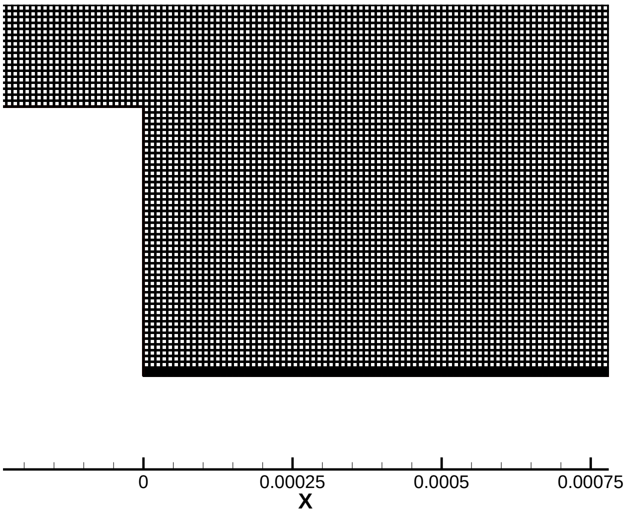

Fig. 2. Schematic mesh grid

4. Validation and Grid Analysis

The grid analysis test was done via numerous grid densities and distributions to confirm an independent grid solution. The grid testing was done using ANSYS Fluent, considered distilled water as an occupied fluid flow over micro scale backward-facing step (MBFS) with Reynolds number (*Re*) of 5,000. Results were performed with various grid numbers. The current numerical solution authenticated with earlier works, as shown in Figure 3 and 4.

Kherbeet *et al.,* [28] offered the work utilized in the justification procedure. In this research, experimental and numerical investigations were showed to reveal that the appearance of nanofluid laminar flow and heat transfer via the micro scale backward-facing step (MBFS). The experiment was shown at the *Re* range from 280 to 470, and Al_2O_3, and SiO_2 particles with water were used as working fluids. Altogether the current outcomes were quite indistinguishable to the results observed by Kherbeet *et al.,* [28].

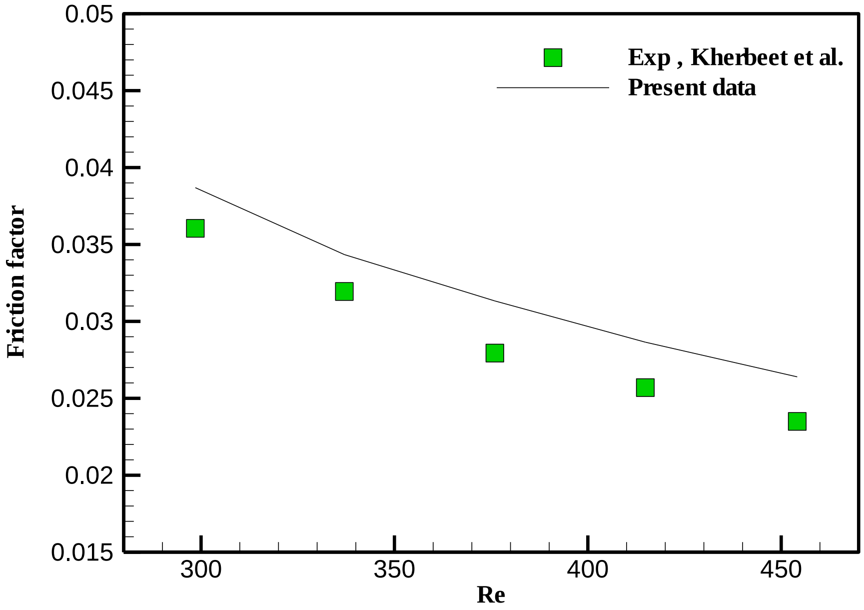

Fig. 3. Validation of present CFD simulation with friction factor

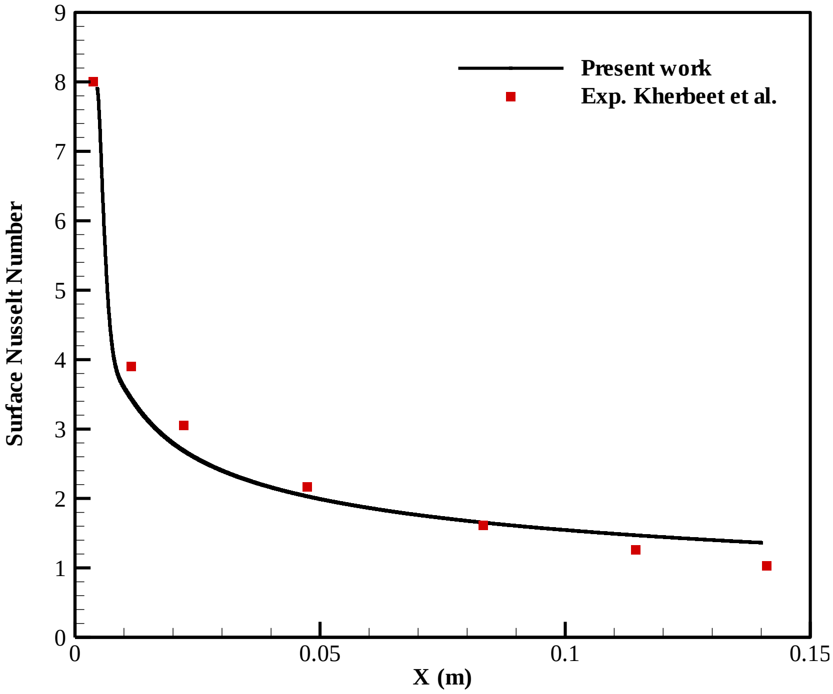

Fig. 4. The validation with the axial distribution of *Nu*

5. Results and Discussion

The current study was conducted in four variations of Reynolds number (*Re*) values of 5,000 ≤ *Re* ≤ 15,000, and two types of base fluids were adopted to investigate the effect of the base fluid; ethylene glycol (EG) and water, while the downstream wall subjected to a persistent heat flux of 1000 W/m^2. The straight-wall temperature steady at 300 K and the height of the step was of 450 μm. The effects of the base fluid on velocity distribution, skin friction coefficient (C_f), heat transfer, and Nusselt number (*Nu*) are discussed, and presented in this section.

5.1 Flow Characteristics

The skin friction coefficient (C_f) in the turbulent flow numerically were calculated. For the micro scale backward-facing step (MBFS), the numerical $C_f$and the deviations between the water and ethylene glycol (EG) results assessed. The effects of using two base fluids with different Reynolds number (*Re*) on the C_f trend presented in Figures 5 - 7. In this section C_f of the base fluid with different *Re* is presented. The numerical study showed that there is a visible change in the C_f of the ethylene glycol and pure water at constant *Re* value as it is showing in Figure 7. The range of *Re* used was 5,000 $\leq$ *Re* $\leq$ 15,000 along with the heated stepped wall results shown in Figure 5 and 6. The significant outcome discovered that the C_f decreases with the rise of *Re* amount and the peak value of *Re* are the lowest value of the C_f peak. It can see from these figures that the C_f rises at the corner that assembly the step of the wall and the wall until it reached the extreme value at the reattachment area. Subsequently, the C_f declines until it reached static amount over the channel, which a sign that the flow passed into the region of the reattachment.

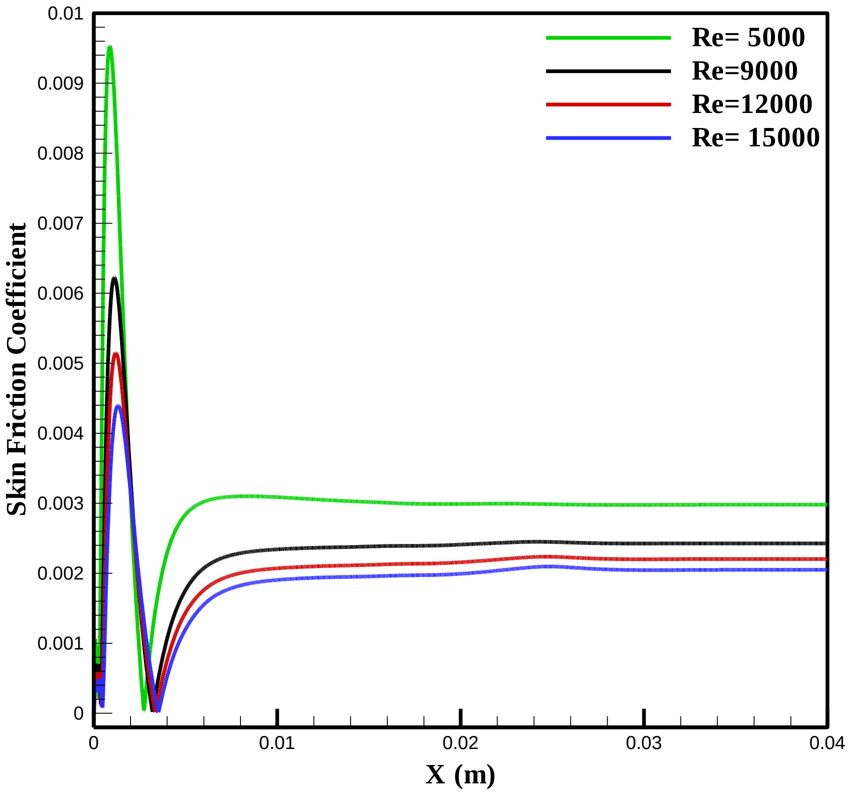

Fig. 5. Water C_f at the bottom wall downstream

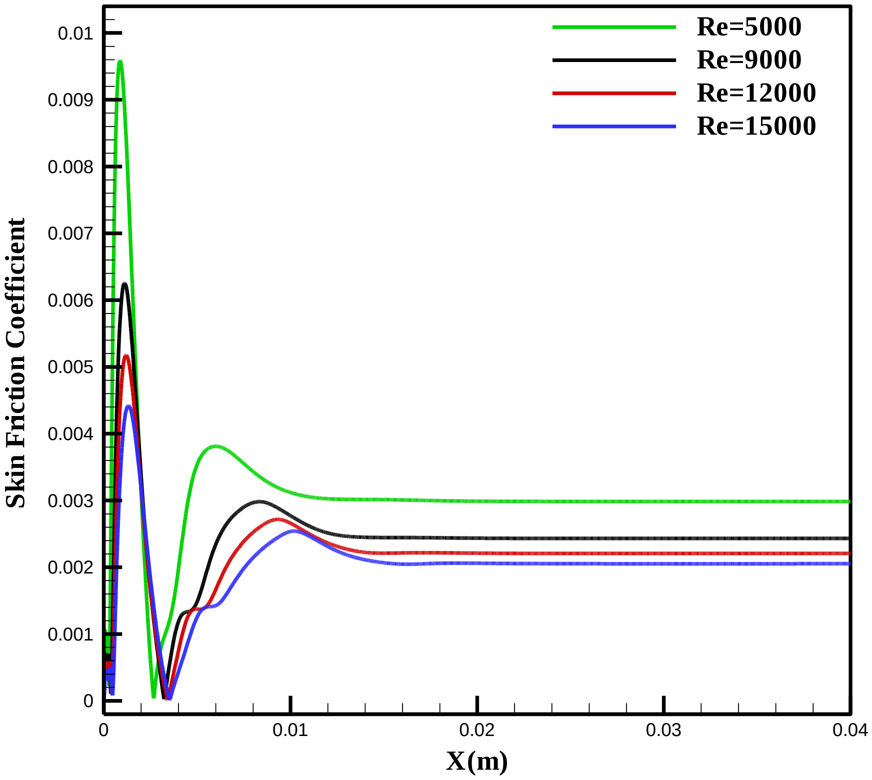

Fig. 6. Ethylene glycol C_f at the bottom wall downstream

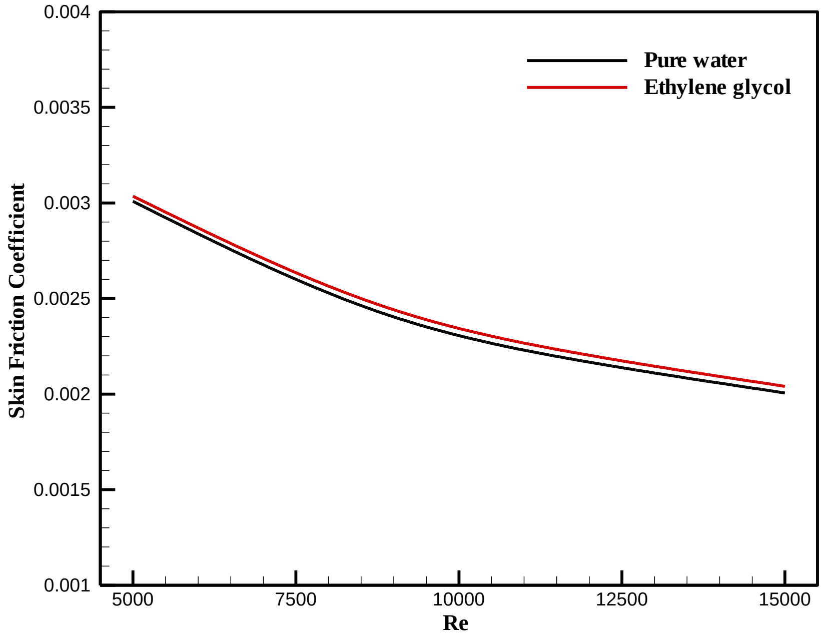

Fig. 7. C_f for water and Ethylene glycol at different *Re*

5.2 Heat Transfer Characteristics

The numerical results of Nusselt number (*Nu*) presented in this section. The results of the study using different Reynolds number (*Re*) and two types of base fluids on the *Nu* showed in the Figures 8 - 10. To investigate the base fluid kind and its effects on the *Nu*, the range of the *Re* was from 5,000 – 15,000. The results demonstrated that *Re* has a considerable impact on the *Nu* results, and *Re* had the highest *Nu* and vice versa, which is due to the highest percentage of convective thermal transfer and velocity.

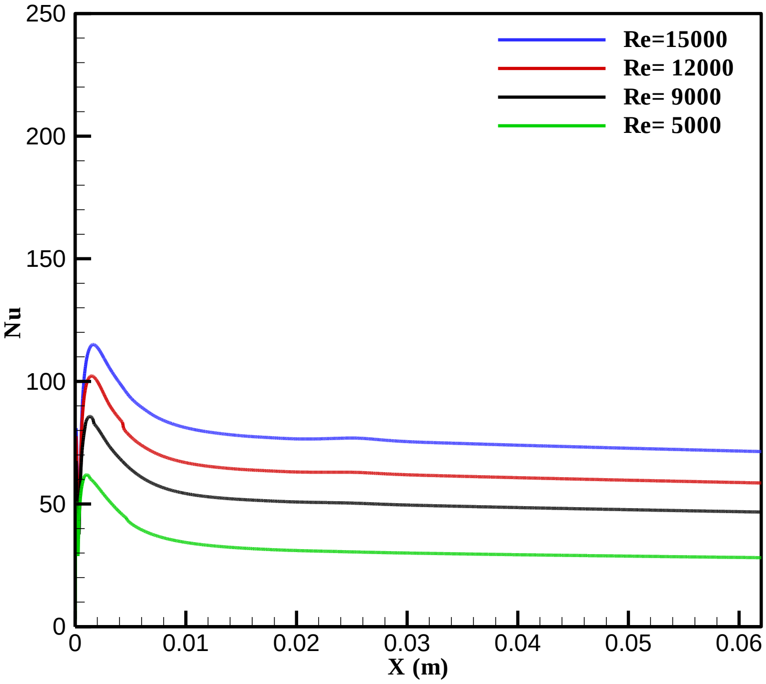

Fig. 8. *Nu* distribution of the water flow at different *Re*

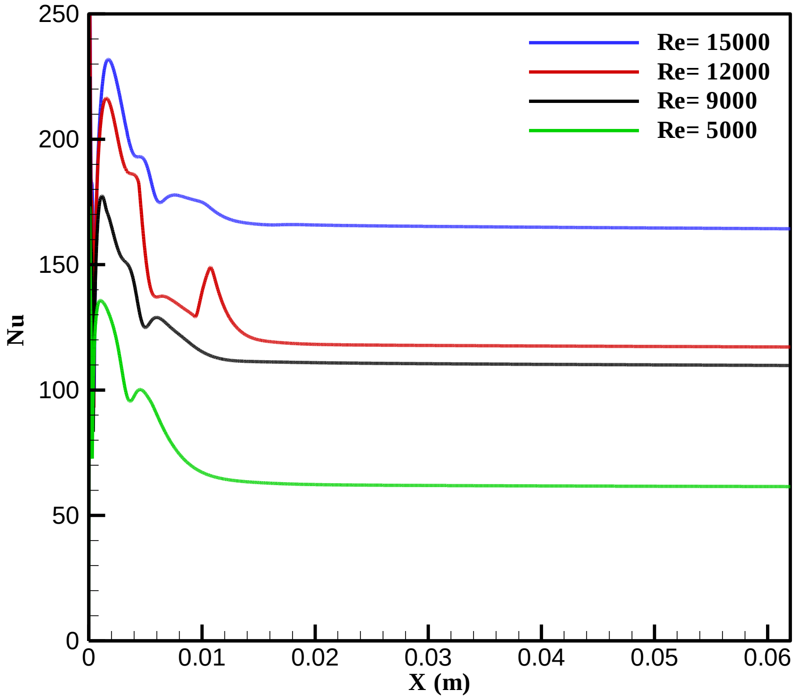

Fig. 9. *Nu* distribution of the ethylene glycol flow at different *Re*

Furthermore, the current study revealed that using ethylene glycol has the highest *Nu*, whereas utilizing the water as the base fluid has the lowest values of the *Nu* as showing undoubtedly in Figure 10. Also, it clearly noticeable from Figure 8 and 9 that the *Nu* increase when the *Re* increased.

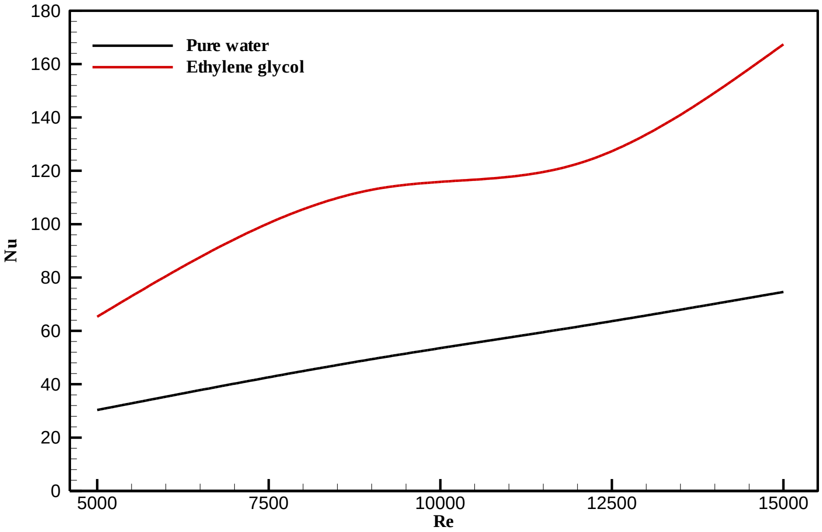

Fig. 10. *Nu* for water and ethylene glycol at different *R*
e

5.3 Velocity Profile

The effects of using different base fluids on the flow field were presented in Figure 11. The velocity profiles showed the streamwise location downstream of the step for stable Reynolds number *Re* for both fluids, which is 5,000, was used. It was found that the base fluid of Ethylene glycol higher parabolic flow in the streamwise path on the step edge as it is showing in Figure 11. The separated flow at the downstream of the edge of the step.

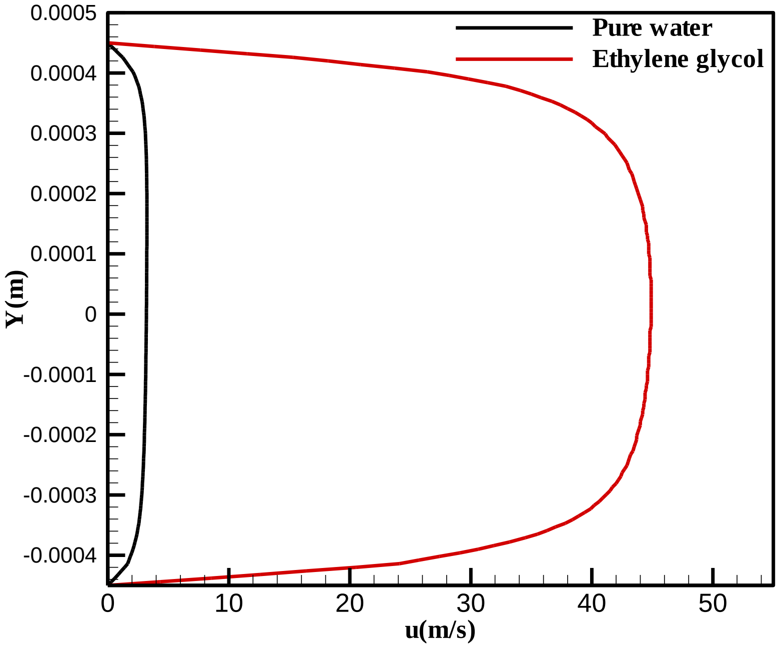

Fig. 11. Ethylene glycol and water velocity distribution at *Re* = 5,000

Figure 12 and 13 display the effect of increasing the *Re* on the velocity profile. The study spotted that the increment of the velocity profile at high *Re* at the same base fluid. Moreover, the increasing of recirculation area observed with increasing of the *Re*. The flow begins to develop at a distance of 0.009 to wholly developed. The stream remains its entirely developed condition until the channel way out.

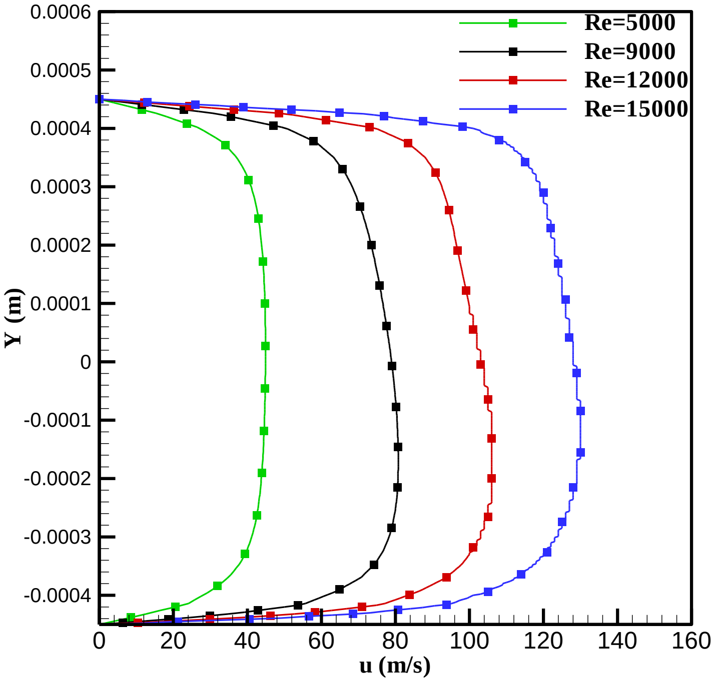

Fig. 12. Ethylene glycol velocity distributions different *Re*

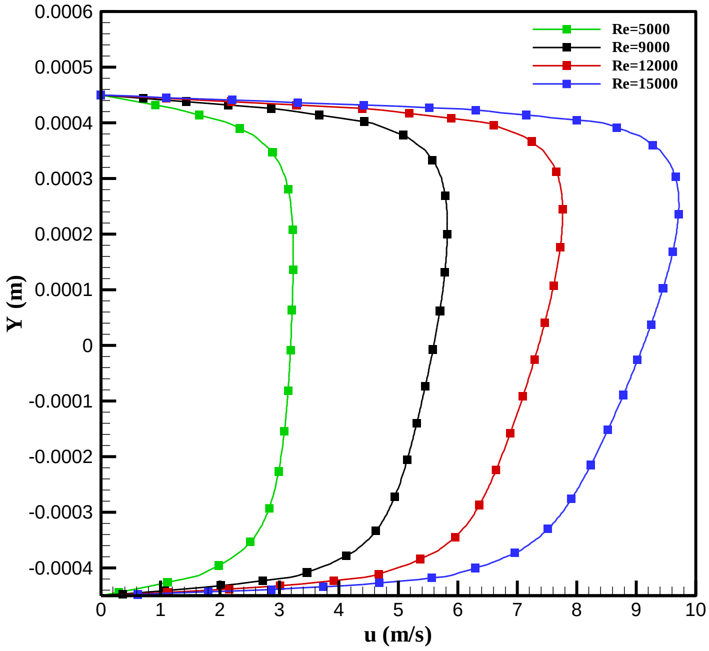

Fig. 13. Water velocity distributions different *Re*

Besides, the recirculation area also expanded downstream from the step wall, and the rest carried along the path of the streamwise. The results found that all the chosen base fluids for this study have a noticeable recirculation area downstream of the step. The study observed that the recirculation area in the case of using ethylene glycol was bigger than the case that uses water at the same *Re*. Figure 14 shows the different of the recirculation area in both ethylene glycol and water respectively at *Re* = 5,000.

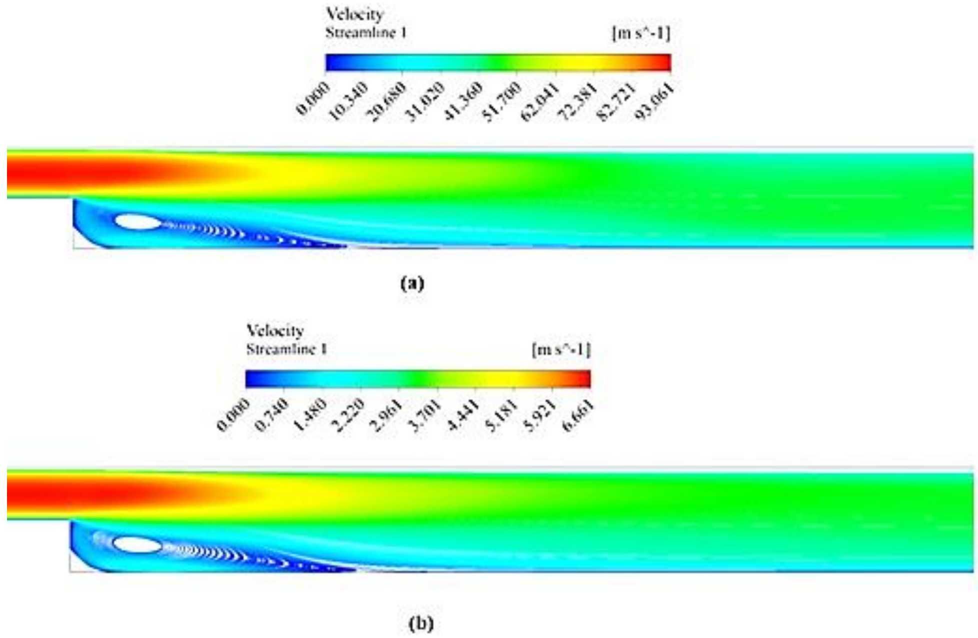

Fig. 14. Velocity streamline contour at *Re* = 5,000 (**a**) Ethylene glycol (**b**) Water

6. Conclusion

Further studies need to be done on the turbulent fluid flow over micro scale backward-facing step (MBFS) by studying the different parameters like different base fluids, Reynolds number (*Re*) ranges, nanofluids, hybrid nanofluids, and various step heights. The current numerical study for 2D turbulent mixed convection over a flat MBFS in a channel using two kinds of base fluids presented and discussed. The effects of various base fluids, the range *Re* on the fluid flow and thermal pitches are studied. The results discovered that the *Re* increment leads to a rise in the Nusselt number (*Nu*) and decrease the skin friction coefficient. Furthermore, the study announced that the ethylene glycol has the highest *Nu* value in comparing to the water. Besides, the recirculation area was observed at the step for the ethylene glycol bigger compare to the water.

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