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On Numerical Investigation of Semi-Empirical Relation Representing Development Length for a Fluid Flow in a Closed Conduit

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ABSTRACT

The development of the velocity gradient at the opening of a closed conduit plays an essential role in monitoring the heat transfer behavior. Hence it is indispensable to examine the minimum critical length at which the velocity gradient is fully developed. Ultimately the flow is fully developed. The developing length determination is very useful for many industrial problems. This can be concluded from the literature survey. The role played by the wall shear in creating the profile is of strong consideration against the Reynold number of flowing fluid. Therefore, the development length for laminar and turbulent flow are defined separately. The determination of average Reynold number through the conduit is mandatory to be known in terms of developing length. In the latter part of the article, semi-empirical relations are defined for developing length separately for laminar and turbulent flow. In order to minimize the effort on experimentation, simulations are carried out in the present studies in ANSYS CFX 18. The developing length carries 1/6th power to the Reynold number. For determination of developing length in terms of Reynold number, SST turbulence model was used.

1. Introduction

The development of flow profile changes gradually with the effect of shear stress. This plays a vital role in accurately predicting the phenomenon happing due to the fluid outlet. Heat transfer is one of the significant areas where development of length plays a vital role in the enhancement and degradation of heat transfer. The most straightforward case study is heat exchanger, where the development of length plays a vital role in heat transfer. Actually, the retardation produced by shear stress at the wall to the flow generates a velocity gradient perpendicular to the contact surface. Due

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to this velocity gradient, at some point a low velocity and at other point high velocities are observed. Generally, the developed flow profile is precisely opposite to that of the shear stress profile.

1.1 Literature Overview

Durst *et al.,* [1] investigated pipe as well as channel flow using analytical, numerical and experimental methods. But only the numerical method fetched results. Correlations were given using L/D $= C$. For pipe flow, $L/D = [(0.619)^{1.6} + (0.0567Re)^{1.6}]^{1/1.6}$ and for channel flow, $L/D = [(0.631)^{1.6} + (0.0442Re)^{1.6}]^{1/1.6}$ were used. The correlations were not suggested for $1 \leq Re \leq 100$.

Poole and Ridley [2] focussed on the non-Newtonian fluid to find its development length in case of laminar flow. They found out X_D/D ratios for $0 < Re < 1000$. The following correlation was proposed: $X_D/D \; = [(0.246n^2 - 0.657n + 1.03)^{1.6} + (0.0567Re)^{1.6}]^{1/1.6}$ where n is the power law index.

Fargie and Martin [3] found out a method for flow development studies. Combined differential and integral momentum equations were used. They proposed the below correlation for development of laminar flow in circular c/s: $L = 10.32(Re/X)^{0.278}$

Dombrowski *et al.,* [4] identified new behavior in the Newtonian fluid at Re < 50. They found out peaks in pressure at the entrance on the pipe thus proposed $Le/D = [(0.0575)^2 Re^2 + (0.655)^2]^{0.5} \pm 4\%$. The correlation was slightly modified with reference to previous available relations.

Poole and Chhabra [5] numerically investigated flow in a laminar pipe using the Bingham model. The investigation was valid for higher values Re and proved out to be weak for Re < 50. A non-dimensional length is plotted as a function of Re and Bingham numbers (1 < Bn. < 10). The results showed that convergence was not possible for higher Bn. And the results were indistinguishable for Bn. < 0.1.

Anil and Gregory [6] used power law and Newtonian fluids in laminar flow to find the magnitude of length of pipe. Numerical methods were used for studying circular pipes. Result graphs were plotted for both laminar and power-law fluids for Re > 200 and 0.75 < n < 1.5.

Mina *et al.,* [7] studied flow conditions from laminar to turbulent, i.e. transient conditions at larger values of Re. The development length was used by Durst *et al.,* [1] to plot the boundary conditions. Puffs and slugs were used for the creation of transition. The transition was visible at Re = 2495.

John and David [8] used two turbulent fluid flow for the break-up of fluid. Single drops were used for breakups. Initial stages used drops less than 0.1 cm diameter, and their significance was null. However, with the passage of time, these drops may increase.

Aihara *et al.,* [9] experimented heat transfer of dilute air suspensions in developing as well as developed flow. Heat flow was measured across various places and for $3 \times 10^4 < Re < 1.2 \times 10^5$. It was observed that behaviour changes at M = 1.2.

Salami [10] researched the possible existence of regimes in a flow that can occur at the entrance of a pipe of the circular cross-section. It was concluded that the laminar region has three regimes whereas turbulent flow has six regimes. For flat plate, a transition occurs at $Re = 3.15 \times 10^6$.

Reichert and Azad [11] carried out experiments on 56000 < Re < 153000 to obtain mean velocity curves. It was found that at the entrance of a pipe up to 70 times diameter, the turbulent flow was non-asymptotic in nature. Graphs of seven Re were plotted to support the research.

Wang and Derksen [12] created a modified model of k-Epsilon-Gamma $(k - \varepsilon - \gamma)$ type to predict the development of turbulent flow. This modified model proved out to be more useful in casthe e on internal flows. Also Re,ynolds shear stress can be accurately calculated using this modified model.

Lin and Chang [13] created a large Reynolds model and five small Reynolds model of k-Epsilon $(k-\varepsilon)$ type. Turbulent flow was developed in the initial region of a smooth pipe at Rea ynolds value of 194000. Simulations were also carried out at Reynolds value of 215000. Axial mean velocity was perfectly predicted using these models but turbulent kinetic energy and shear stress varied compared to experimental values.

Klein [14] presented a review of the development of turbulence in a pipe flow. He compared the blockage factors of various authors. Also, Reichert and Azad [11] were reported that proved peaks at $x/D = 32$ and minimum at $x/D = 60$.

Henry [15] analytically found out laminar entrance length of a pipe. He used momentum equation for his experiments and found out results after linearizing it. For laminar flow, he gave length (L) to diameter (D) ratio as 0.0575 Re.

Sparrow *et al.,* [16] did analytical studies for finding the length (L) to diameter (D) ratio in case of tubes and ducts. They introduced a new energy equation of mechanical energy and defined the ratio as 0.056 Re. For this purpose, they used a stretched axial coordinate system.

Lew and Fung [17] studied the entry of blood in blood vessels at various Reynolds number. They analytically used a pair of Eigenfunctions for this purpose. They gave two relations for laminar flow, $L/D = 0.065\, Re\ (Re < 1)$ and $L/D = 0.08\, Re\ (Re > 50)$.

Christiansen and Lemmon [18] found out a numerical solution by comparing it with momentum equation. Numerically it was found that laminar entrance length is $L = 0.0555 \times Re \times D$.

Vrentas *et al.,* [19] focussed on the development of vorticity in closed circular conduits. They numerically used a boundary value problem and found out that the length (L) to diameter (D) ratio of laminar flow can be given as 0.056 Re. Nikuradse [20] in his book Applied Hydro and Aerodynamics stated that the length (L) to diameter (D) ratio of laminar flow could be given as 0.0625 Re. He expressed this relation after experimentation in the year 1950.Bergman *et al.,* [21] defined laminar entry length as $L_{laminar} = 0.05 \times Re \times D$ whereas Cengel and Cimbala [22] defined turbulent length as $L_{turbulent} = 4.4 \times D \times (Re)^{1/6}$ in their books respectively.

Umair *et al.,* [23] studied pulse jet and found that type of augmentation totally depends on G no. Umair *et al.,* [24] also found that the critical range for secondary peaks in Nusselt distribution lies in the range of Reynold number of 176400 to 2205001. Pathan *et al.,* [25] investigated suddenly expanded flow using CFD. They calculated base pressure variations for internal and external expansions.

Beng *et al.,* [26] numerically analysed heat and fluid flow of micro channels consisting of triangular cavities. They concluded that channels with triangular cavities are better than straight channels because of high Nusselt number and lowest friction factor. Muhammed [27] investigated flow of turbulent magnetic Nano fluid inside square channel. It was proved that Nusselt number increases with increase in Re and volume, but friction factor decreased. Wong *et al.,* [28] studied effect of sudden expansion in pipe on turbulent flow. It was revealed that SST k-ω model predicts the best results compared to k-ε model.

In spite of the above research, the computational study of determination of developing length is not yet studied. For this reason a computational structure is proposed in the present study which can be widely used for studying the flow behavior for various variation in the parameters. This structure is applicable for a closed conduit only.

1.2 Objective

The principal objective of the current research lies in determining the minimum length at which the flow get develops. In the present work, two lengths are defined as one for laminar flow and other for turbulent flow. The lengths are determined against the diameter of closed conduit and Reynolds number using regression analysis.

2. Computational Methodology

In order to calculate the exact minimum length at which the flow profile gets developed, the length of pipe in the laminar region is varied from 2 – 3 meters, and for turbulent flow, the length is varied from 0.5 – 1.5 meters. In case of laminar flow, the Reynold number at the inlet was kept constant to 1000, while that for turbulent was kept to 5000. In all case, the diameter of the pipe used was 50 mm. The turbulence intensity and turbulent Prandtl number were taken a nominal value of 1% and 0.8. K- Epsilon turbulent model for laminar and SST for turbulent flow was incorporated. The solver used in the present study was CFX 18.1.

2.1 Computational Domain

In order to examine the development length, a computational domain consisting of a pipe of varying length and fixed diameter of 50 mm is computed for fluid flow (water as fluid). During the simulation, the Reynolds number for laminar simulation is kept to 1000, while that for turbulent is kept at 5000. The variation in L/D ratio for laminar is in the range of 40 -60, while turbulent is in range of 10-30. The computational domain (pipe) is also examined for the grid size dependency, and the most exceptional meshed computational domain is shown in Figure 1. Two initial boundary conditions are considered. The inlet where the velocity of water is given an outlet where the gauge pressure is defined. The walls of the pipe are assumed non-slip.

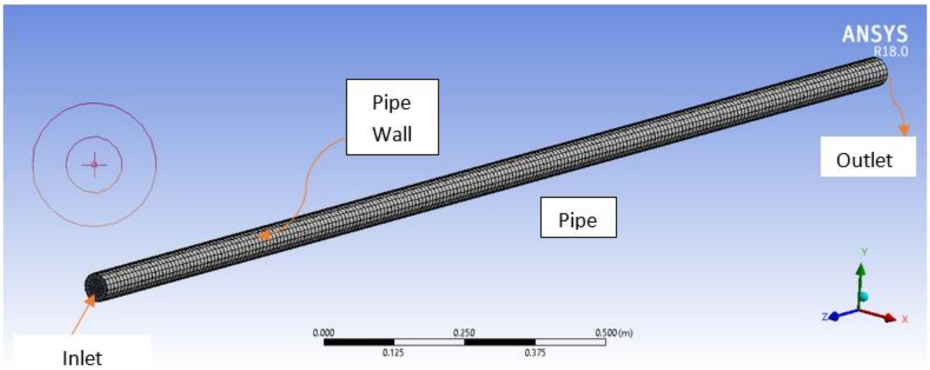

Fig. 1. Computational domain and boundary conditions

2.2 Grid Independence Study

In order to examine the dependence of grid size on flow profile at the outlet, various grid size in the range of 0.0001 – 0.1 are examined. For individual grid size, the velocity for various radial distance at the outlet is examined. For the purpose of non-dimensionality, a graph of local Reynolds number versus non-dimensional radial distance (r/d) is plotted. As seen in Figure 2 the grid size of 0.01 and 250000 number of nodes is optimum for achieving accurate results. Hence the grid size of 0.01 throughout the computational is optimistic.

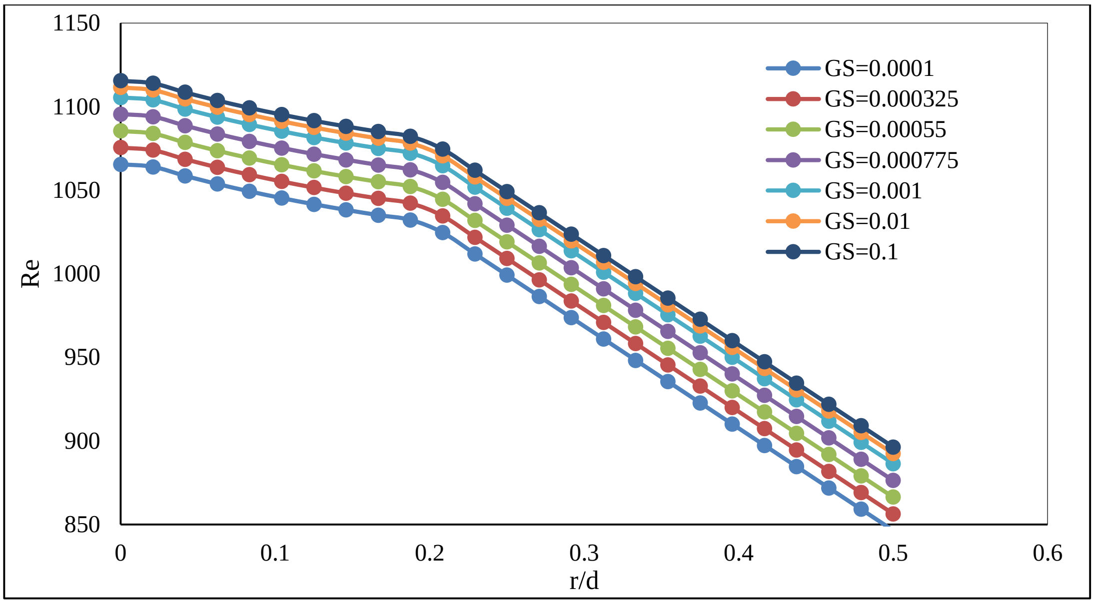

Fig. 2. Local Reynolds number versus non-dimensional radial distance for different grid size

2.3 Validation

Table 1 is clearly visible from the validation that percentage error for laminar and turbulent flow is within 2 %. Thus the source and computed values are validated.

Table 1
Validation of Turbulence model

Source	Average Reynold Number	Computed Average Reynold Number	Validation Agreement (Percentage Error)
Yunus Cengel [22]	1000	985	1.5
Yunus Cengel [22]	5000	5008	1.6

3. Results

In order to predict the developing length for laminar and turbulent flow, the local Reynolds number versus r/d and L/D versus Reynolds number for considering the peak velocity at the center are plotted. This is shown in Figure 3, 4, 5, 6. Considering different L/D ratios for different centered Reynolds number, a semi-empirical relation is constructed ($f_1(x)$) for laminar flow and turbulent flow (f_2(x)).

3.1 Laminar Flow

Figure 3 shows the deviation of velocity with radial distance. For laminar flow, the velocity profile converges when L/D ration 52. Considering the diameter of the pipe and Reynolds number in laminar condition, the developing round to 2.704 meters.

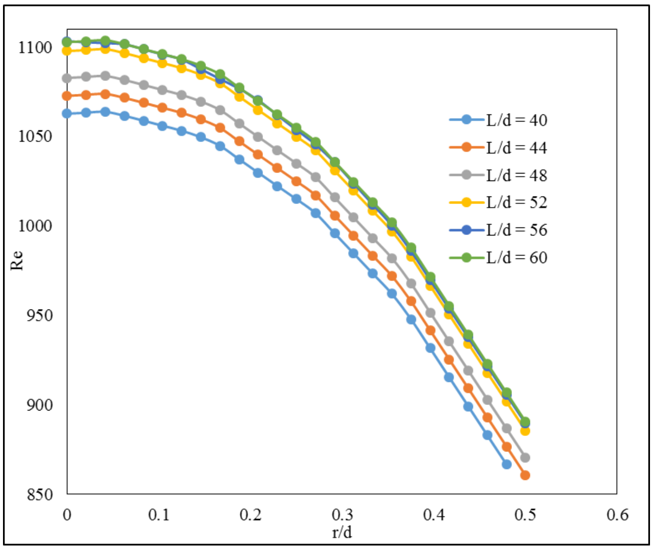

Fig. 3. Graph of Reynolds number versus r/d for various L/d ratios

As seen in Figure 4, with an increase in L/D the peak Reynolds number which is measured at the center of outlet increases initially. After the increase in L/D above 52 the peak Reynolds number becomes constant. This indicates the development of flow.

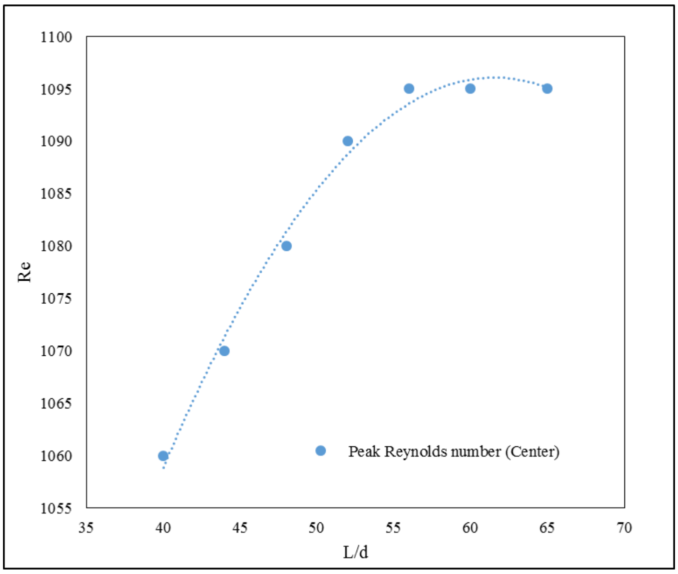

Fig. 4. Peak Reynolds number (center) versus L/d ratio

Considering the peak values of the Reynolds number at the center of outlet and corresponding L/D ratio, regression analysis is carried out between the two parameters. $f_1(x)$ represent the semi-empirical relation representing the developed length for different values of Reynolds number.

$$f_1(x) = \frac{L}{D} = 0.05 \times Re \times D \tag{1}$$

3.2 Turbulent Flow

Figure 5 shows the nature of velocity gradient across the radial distance for different L/D ratio ranging from 10-30. As seen from the graph the velocity profile coincides with an increase in L/D beyond 26. The corresponding developed length for this turbulent flow approximates to 1.3 meters. The Reynolds number used to predict the developed length was 5000 in this case.

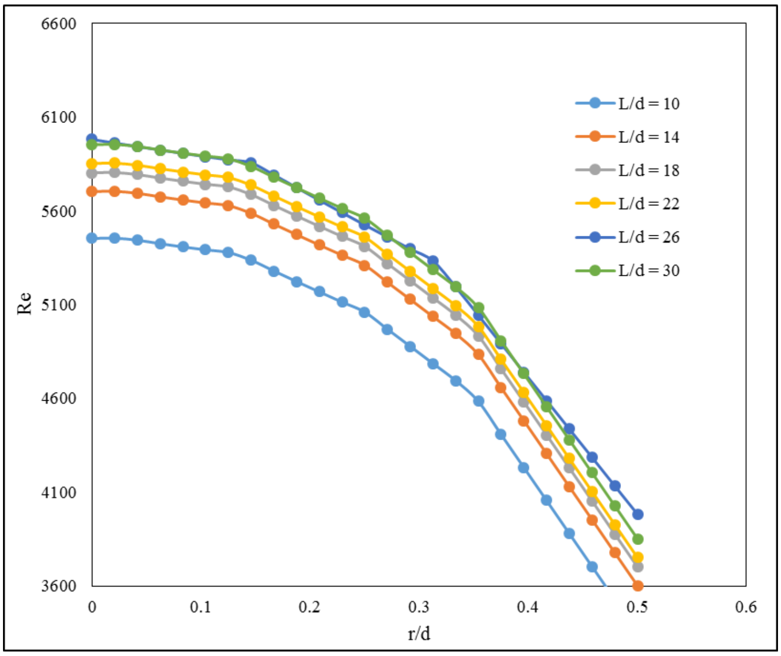

Fig. 5. Graph of Reynolds number versus r/d for various L/d ratios

In order to examine the developing length for turbulent flow. The peak value of Reynolds number versus L/D is plotted with a constant value of average Reynolds number as 5000. Figure 6 shows the variation in peak value of Reynolds number with L/D ratio. As seen the value of peak Reynolds number becomes stagnant at L/D = 26. The corresponding value of the length of pipe for turbulent flow, in this case, rounds to 1.3 meters.

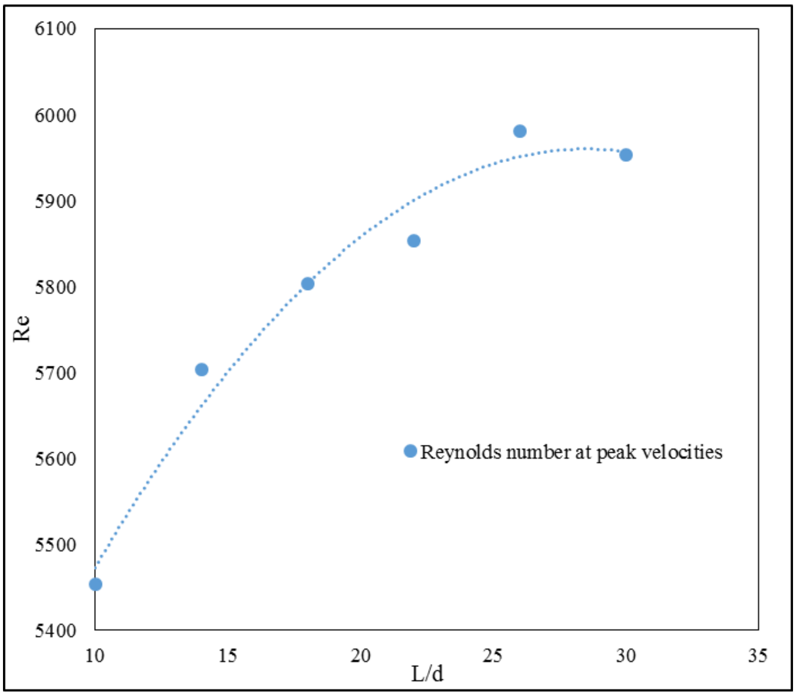

Fig. 6. Peak value of Reynolds number versus L/D ratio for turbulent flow

Considering the peak values of the Reynolds number at the center of outlet and corresponding L/D ratio, regression analysis is carried out between the two parameters. $f_1(x)$ represent the semi-empirical relation representing the developed length for different values of Reynolds number. This is shown in Eq. (2)

$$f_2(x) = L = 4.4 \times Re^{\frac{1}{6}} \times D \tag{2}$$

4. Conclusions

As seen from Eq. (1) and (2) the developing length semi-empirical relation is a strong function of Reynolds number in case of laminar flow, rather than that in a turbulent flow. Also, the developing length magnitude for turbulent flow is smaller than that compared to laminar flow, as the shear stress gets less opportunity to interact with a fluid. Hence, it can be concluded that shear stress at the wall plays a dominating role in the case of laminar flow rather than that in a turbulent flow.

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