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Magnetohydrodynamic (MHD) Slip Darcy Flow of Viscoelastic Fluid Over A Stretching Sheet and Heat Transfer with Thermal Radiation and Viscous Dissipation

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ABSTRACT

In this paper, we analytically study about the boundary layer flow and heat transfer on non-Newtonian fluid which in particular is the viscoelastic fluid. The magnetohydrodynamic (MHD) slip Darcy flow of viscoelastic fluid over a stretching surface in a porous medium with the presence of thermal radiation and viscous dissipation is examined. The results for two viscoelastic fluids which is elastico-viscous fluid and second grade fluid are obtained and compared. The governing partial differential equations are reduced to non-linear ordinary differential equations with the aid of similarity transformation, which are then solved analytically by using exact analytical method. The effects of the physical parameters on the velocity and temperature fields are presented through graphs and are discussed. Skin friction and heat transfer coefficients are computed and analysed.

Keywords:
Viscoelastic fluid; Velocity slip; Magnetohydrodynamic; Porous medium; Viscous dissipation; Thermal radiation

1. Introduction

The study towards the flow on boundary layer of non-Newtonian fluids has been the subject of great interest to investigators and researchers. Viscoelastic fluid is a common form of non-Newtonian fluid, which this fluid having the characteristic that shows both viscous and elastic properties under some circumstances. This viscoelastic reaction and the fluid flow past a stretching sheet are so significant in some real life applications especially in the engineering field, for instance, in the paper production, extrusion of plastic sheets, glass blowing, and metal spinning [1].

The flow of viscoelastic fluid past a stretching sheet has been studied by Rajagopal *et al.,* [2]. Later, Andersson [3] has added the magnetic effects towards the flow of a viscoelastic fluid past a stretching sheet and he obtained the exact analytical solution of the governing non-linear boundary

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layer equation. Consequently, Ariel [4] extended the problem by taking into account the flow with the existence of suction and found the exact solution. The investigation on the steady magnetohydrodynamic viscoelastic fluid flow over a semi-infinite, impermeable stretching sheet with the presence of thermal radiation and internal heat generation or absorption is investigated by Datti *et al.,* [5]. Moreover, Liu [6] presented the exact analytical solutions for the flow and heat transfer of second grade fluid over a stretching sheet with the presence of magnetic and viscous dissipation. Khan and Sanjayanand [7] has examined the viscoelastic boundary layer flow and heat transfer over an exponential stretching sheet. Viscous dissipation has been considered in the heat transfer and the obtained exact solution which then being compared with the numerical solution.

Later, Cortell [8] performed an analysis on the flow and heat transfer of viscoelastic fluid by considering the magnetic field parameter, viscous dissipation and suction parameters. Moreover, partial slip has been taken into account into the flow of viscoelastic fluid past a stretching sheet, where this analysis has been carried out by Ariel *et al.,* [9]. Next, closed form of analytical solution has been obtained by Khan [10] on the heat transfer in a viscoelastic fluid flow over a stretching surface with the consideration of heat source, suction and radiation. Hayat *et al.,* [11] then studied the magnetohydrodynamic flow of a second grade fluid with thermal radiation effect by the implementation of homotopy analysis method (HAM). Abel *et al.,* [12] studied the heat transfer in viscoelastic fluid over a stretching sheet by considering the effects of viscous dissipation and non-uniform heat source in both types of heating process.

Further, Abel and Manesha [13] have added the thermal radiation parameter as the extension of the analysis. The analytic solution of magnetohydrodynamic flow and heat transfer over a stretching sheet for both second grade and Walters' liquid B fluids has been solved analytically by Chen [14]. Turkyilmazoglu [15] studied the slip flow of viscoelastic fluid past a stretching surface in magnetic field and have found multiple analytical solutions. The analytical solution of magnetohydrodynamic viscoelastic fluid flow and heat transfer in a parallel plate channel with a stretching wall is obtained by employing homotopy analysis method [16]. Then, Turkyilmazoglu [17] has extended his work [15] by analyzing both types of viscoelastic fluids. Later, Nayak *et al.,* [18] have investigated the heat and mass transfer of the viscoelastic fluid bounded by a stretching sheet with consideration of various parameters and conditions. Furthermore, spectral homotopy analysis approach is being implemented by Fagbade *et al.,* [19] in their investigation on natural convection flow of viscoelastic fluid with thermal and heat source or sink. Nadeem *et al.,* [20] have considered Cattaneo Christov heat flux in the flow of viscoelastic fluid over a stretching surface with the presence of Newtonian heating and porosity parameter. Optimal homotopy analysis method has been used to solve the problem. Recently, the study about viscoelastic fluid is being investigated by Chen *et al.,* [21], which they have developed a double Maxwell model to examine the flow of magnetohydrodynamic viscoelastic fluid over a stretching sheet by using infinite difference method. So, through the review on these literatures, we want to extend the studies of the viscoelastic fluid (non-Newtonian fluid) with the consideration of various parameters and conditions. In order to solve the problem, we have implemented the exact analytical method.

From the literatures, the study about the effect of velocity slip parameter towards the boundary layer flow of viscoelastic fluid is rarely to be found. Several studies regarding the slip effects towards the other type of fluid also can be found recently as in some researches [22-24] to convince that the study towards the slip effects is significant. Thus, in this paper, we want to extend the previous study done by Nayak *et al.,* [18] by adding new additional parameter to the model which is the velocity slip parameter. So, the major purpose of this investigation is to formulate a mathematical model for magnetohydrodynamic slip Darcy boundary layer flow of viscoelastic fluid past a linear stretching surface with the presence of thermal radiation and viscous dissipation in porous medium. The analytical solutions for the velocity and the temperature distributions are obtained using the exact analytical method. The profiles are plotted and discussed for variations of pertinent parameters. The skin friction and heat transfer coefficients have been computed and analyzed through numerical data, and then being compared to other previous results to verify the calculation and the technique of the method that we used.

2. Modeling

We consider two-dimensional (2D) flow of viscoelastic fluid bounded by a linear stretching surface. The incompressible viscoelastic fluid saturates the porous space characterizing Darcy model. Here, x − axis is parallel to stretchable surface and we suppose $u_w(x) = cx$ defines the stretching velocity along the x – direction. Meanwhile, y − axis is perpendicular to x – axis. An applied uniform magnetic field of strength B_0 is encountered parallel to the y – axis. The flow geometry can be illustrated as shown in Figure 1. Thus, with all of these assumptions, the boundary layer equations governing the flow of viscoelastic fluid and heat transfer in the presence of magnetohydrodynamic, porous medium, velocity slip, viscous dissipation and thermal radiation can be written as Eqs. (1) to (3) which are the continuity equation, momentum equation, and energy equation, respectively. Also need to be mentioned that these equations are in the form of partial differential equations invariant in time.

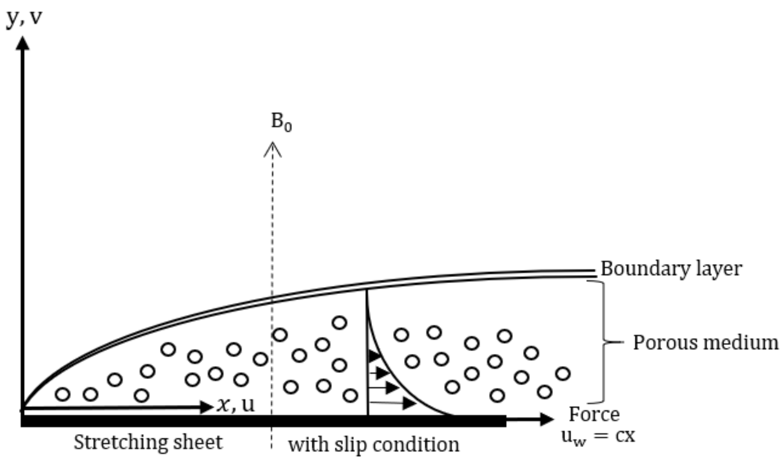

Fig. 1. Flow geometry

$$u_x + v_y = 0 \tag{1}$$

$$uu_x + vu_y = \nu u_{yy} + \frac{k_0}{\rho}\left(vu_{yyy} + uu_{xyy} + u_y v_{yy} + u_x u_{yy}\right) - \frac{\nu}{K^*}u - \frac{\sigma B_0^2 u}{\rho} \tag{2}$$

$$uT_x + vT_y = \alpha\left(T_{yy}\right) + \frac{\mu}{\rho C_p}\left(u_y\right)^2 + \frac{k_0}{\rho C_p}\left(u_y\left(uu_x + vu_y\right)_y\right) - \frac{1}{\rho C_p}q_{r\,y} + \frac{\sigma B_0^2 u^2}{\rho C_p} \tag{3}$$

subject to the associated boundary conditions:

$$u = u_w(x) = cx + l\frac{\partial u}{\partial y},\ v = 0,\ T = T_w(x) = T_\infty + A\left(\frac{x}{x_L}\right)^2 \quad at\ y = 0$$

$$u \to 0,\ \frac{\partial u}{\partial y} \to 0,\ T \to T_\infty,\ as\ y \to \infty \tag{4}$$

where u and v are the velocity components in $x-$ and $y-$ directions respectively, while $\nu = \frac{\mu}{\rho}$ is the kinematic viscosity, where μ is the coefficient of fluid viscosity and ρ is the fluid density. Then, k_0 is the modulus of viscoelastic fluid, σ is for fluid electrical conductivity, K^* for the permeability of porous media, B_0 for uniform magnetic field, T for temperature, α stands for thermal diffusivity and C_p is the specific heat at constant pressure. T_w and T_∞ stands for constant surface temperature and ambient fluid temperature respectively, c for positive stretching rate constant with T^{-1} as the dimension, A is a constant, l is the slip parameter and x_L is the characteristic length. Also, by applying Rosseland approximation for radiation we may expressed the radiative heat flux q_r as follows [25]:

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{5}$$

where σ^* and k^* are the Stefan–Boltzman constant and the absorption coefficient, respectively. We assume that the temperature difference within the flow is such that the term T^4, which can be expanded in a Taylor series about T_∞ can be presented as a linear function of temperature as shown below

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \tag{6}$$

By using the approximation as above, we have

$$q_r = -\frac{16\sigma^* T_\infty^3}{3k^*}\frac{\partial T}{\partial y} \tag{7}$$

By using the appropriate similarity transformation followed from the previous work by Nayak et al., [18]:

$$u = cxf'(\eta),\ v = -(c\nu)^{\frac{1}{2}}f(\eta),\ \eta = \left(\frac{c}{\nu}\right)^{\frac{1}{2}}y,\ \theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty} \tag{8}$$

the continuity equation in Eq. (1) is proven and the momentum equation in Eq. (2) and energy in Eq. (3) are reduced into ordinary differential equations as below respectively

$$f''' + ff'' + k_1^*\left(2f'f''' - f''^2 - ff^{iv}\right) - Kf' - f'^2 - Mf' = 0 \tag{9}$$

$$\left(1 + \frac{4}{3}R_d\right)\theta'' + Prf\theta' - 2Prf'\theta + PrEc\left(Mf'^2 + f''^2 + k_1^* f''(f'f'' - ff''')\right) = 0 \tag{10}$$

subject to the following transformed boundary condition

$$f = 0,\ f' = 1 + Lf''(0),\ \theta = 1,\ \text{at} \quad \eta = 0,$$

$$f' \to 0,\ f'' \to 0,\ \ \theta \to 0,\ \ \text{as} \quad \eta \to \infty \tag{11}$$

where f and θ is the dimensionless stream function and temperature function respectively, and the prime denotes differentiation with respect to η. Here, k_1^* is the viscoelastic parameter, K for porosity parameter, M for the magnetic parameter, R_d for the thermal radiation parameter, Ec for Eckert number, Pr is for the Prandtl number and L is the velocity slip parameter. Here $k_1^* < 0$ represents elastic-viscous fluid, $k_1^* > 0$ is for second grade fluid and $k_1^* = 0$ for Newtonian fluid. The definitions below will define the parameters specifically:

$$k_1^* = \frac{k_0 c}{\mu},\ K = \frac{\nu}{cK^*},\ M = \frac{\sigma B_0^2}{\rho c},\ R_d = \frac{4\sigma^* T_\infty^3}{k^*\, k},\ \mathrm{Ec} = \frac{c^2 x_L^2}{AC_p}, \mathrm{Pr} = \frac{\nu}{\alpha},\ L = l\sqrt{\frac{c}{\nu}} \tag{12}$$

Following [18], the skin friction coefficient is

$$Re_x^{\frac{1}{2}} C_f = -f''(0) \tag{13}$$

and the local Nusselt number is

$$Re_x^{\frac{1}{2}} Nu_x = -\theta'(0) \tag{14}$$

in which Re_x represents the local Reynolds number.

3. Exact Analytical Method

3.1 Velocity Field

We can assume that Eq. (9) possesses a solution of exponential type [26] expressed as

$$f(\eta) = \frac{1}{C}(1 - e^{-z\eta}) \tag{15}$$

where C is the slip condition produces the relation,

$$C = z(1 + Lz) \tag{16}$$

Consequently, by substituting Eq. (15) accordingly into Eq. (9), it gives another relation in the form of a cubic algebraic equation

$$Lz^3 + (1 + k_1^*)z^2 - (M + K)Lz - M - K - 1 = 0 \tag{17}$$

Then, solving the above algebraic equation, we obtain the positive value of root z which it contains all the involved parameters which need to be used in the profiles function. The velocity profile $f'(\eta)$ function is as shown below

$$f'(\eta) = \frac{1}{1+Lz} e^{-z\eta} \tag{18}$$

3.2 Temperature Field

By referring to Ebaid *et al.,* [27] and Ali *et al.,* [28], the exact solution for temperature field can be obtained. We suppose the following transformation, where $t = e^{-z\eta}$**.** Consequently, to apply the transformation, the governing equation need to be expressed in terms of variable t, thus the following relations between the derivatives with respect to η and the derivatives with respect to t is introduced, where it can be obtained by using the chain rule formula,

$$\frac{d}{d\eta}\theta = -zt\frac{d}{dt}\theta,\ \frac{d^2}{d\eta^2}\theta = z^2\left[t^2\frac{d^2}{dt^2}\theta + t\frac{d}{dt}\theta\right] \tag{19}$$

By applying the transformation to (10), we obtain

$$t\theta''(t) + (n - mt)\theta'(t) + 2m\,\theta(t) + P\ \mathrm{Ec}\left(\frac{z^2t}{C^2} + \frac{k_1^*z^3t}{C^3} + \frac{Mt}{C^2}\right) = 0 \tag{20}$$

with the following set of boundary conditions

$$\theta\,(0) = 0,\ \theta\,(1) = 1 \tag{21}$$

Therefore, solving the ordinary differential equation in Eq. (20), we obtain the exact solution for temperature field $\theta(\eta)$,

$$\theta(\eta) = -\frac{1}{2}\frac{(m^2(e^{-z\eta})^2 - 2m\,e^{-z\eta}(n+1) + n^2 + n)\mathrm{Ec}\,P(k_1^*\,z^3 + C\,z^2 + MC)}{C^3m^2(n+1)} + \frac{1}{2}\left(\frac{hypergeometric([-1-n],[-n+2],m\,e^{-z\eta})}{hypergeometric([-1-n],[-n+2],m)}\right)$$

$$\left(\frac{(e^{-z\eta})^{-n+1}(\mathrm{Ec}\,P\,k_1^*\,z^3 + \mathrm{Ec}\,P\,C\,z^2 + \mathrm{Ec}\,M\,P\,C + 2C^3n + 2C^3)}{C^3(n+1)}\right) - \frac{\left((z^2+M)C + k_1^*z^3\right)P\left(m\,e^{-z\eta} - \frac{1}{2}n\right)\mathrm{Ec}}{C^3m^2} \tag{22}$$

where the stated hypergeometric in Eq. (22) is the hypergeometric function, $n = 1 - \frac{P}{zC}$, $m = -\frac{P}{zC}$, and $P = \frac{\mathrm{Pr}}{(1+\frac{4}{3}R_d)}$.

4. Result and Discussion

In order to solve and analyze this analysis, we have used Maple software to facilitate the process. So, in this section, we will discuss and analyze the results that we obtained. The effects of various influential parameters on the non-dimensional velocity $f'(\eta)$ and temperature $\theta(\eta)$ distributions will be explored and discussed. Both elastico-viscous $(k_1^* < 0)$ and second grade fluids $(k_1^* > 0)$ are considered.

Impacts of porosity parameter K, magnetic parameter M and velocity slip parameter L on the non-dimensional velocity field $f'(\eta)$ are plotted in Figures 2, 3 and 4 respectively. Figure 2 presents the impact of porosity parameter K on velocity field $f'(\eta)$ for both elastico-viscous and second grade fluids which it can be concluded that the increment of porosity parameter K leads to decrement in the velocity field $f'(\eta)$ for both fluids. Physically, the existence of porous media is to increase the resistance to the fluid flow which causes decay in fluid velocity and related momentum layer thickness. Next, Figure 3 displays the change in velocity field $f'(\eta)$ for varying magnetic parameter M for both elastico-viscous and second grade fluids. It is found that the influence of magnetic parameter M slowers the velocity field $f'(\eta)$. The higher the value of magnetic parameter M, the slower the velocity field $f'(\eta)$. Meanwhile, Figure 4 shows the impact of velocity slip parameter L towards the velocity field $f'(\eta)$, which it depicts that the presence of slip parameter decays the velocity field $f'(\eta)$. This is because, the presence of slip allows more fluid to slip past the sheet causing the flow to slow down. We also found out that the second-grade fluid increases the velocity field $f'(\eta)$ as being compared to the elastic-viscous fluid.

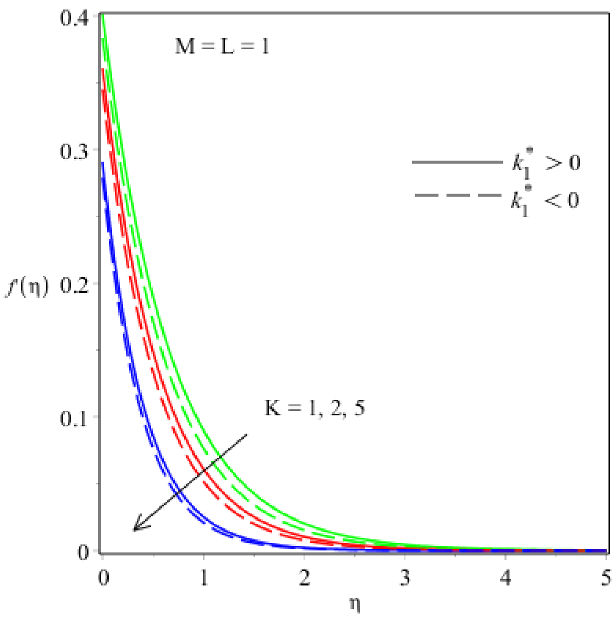

Fig. 2. Velocity profile for different values of K

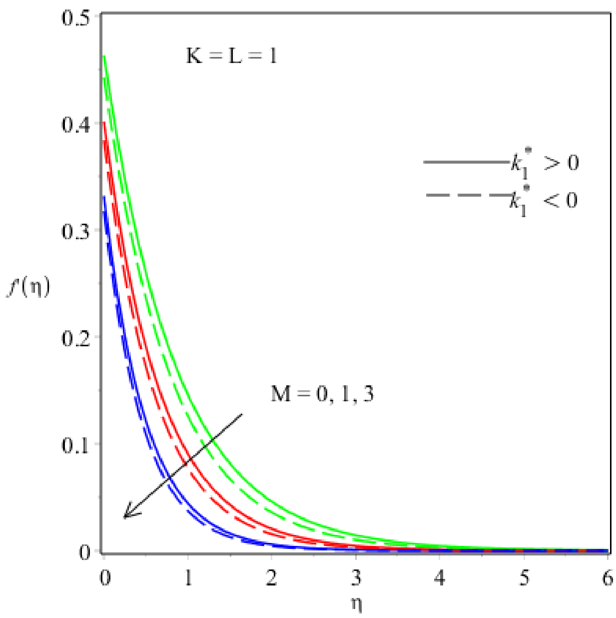

Fig. 3. Velocity profile for different values of M

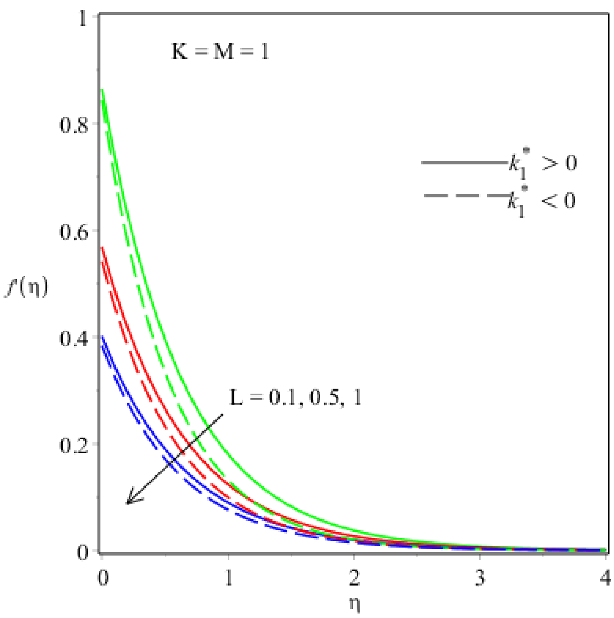

Fig. 4. Velocity profile for different values of L

Furthermore, Figures 5 to 10 are plotted to explore the impacts of porosity parameter K, magnetic parameter M, thermal radiation parameter R_d, Eckert number Ec, velocity slip parameter L and Prandtl number Pr on the non-dimensional temperature field $\theta(\eta)$. Figure 5 depicts the change in temperature field $\theta(\eta)$ for varying porosity parameter K. It is observed that the higher the value of porosity parameter K leads to an increment in temperature field $\theta(\eta)$ as well as more thermal layer thickness for both fluids. Then, Figure 6 reveals the effect of magnetic parameter M on the temperature field $\theta(\eta)$. It is noticed that the higher the magnetic parameter, the stronger the temperature field $\theta(\eta)$. Physically, the Lorentz force that is produced by the transverse magnetic field is opposed to the motion of the flow, which produces the resistance that enhances the temperature. The effect of thermal radiation parameter R_d on temperature field $\theta(\eta)$ is shown in Figure 7. The increasing values of thermal radiation parameter R_d leads to stronger temperature field $\theta(\eta)$ as well as the thermal layer thickness. This means that, the radiation should be minimum in order to initiate the cooling process. Same goes to Eckert number Ec which is the viscous dissipation parameter, where the increment of Eckert number enhances the temperature field $\theta(\eta)$ as shown in Figure 8. Also, as displays in Figure 9, the increases of velocity slip parameter L causing the increment in temperature field $\theta(\eta)$. Again, we also found out that the second-grade fluid decreases the temperature field $\theta(\eta)$ as being compared to the elastic-viscous fluid. Meanwhile, Figure 10 illustrates the impact of Prandtl number Pr on the temperature field $\theta(\eta)$. It is noted that the temperature field $\theta(\eta)$ is lower for increasing values of Prandtl number Pr for both fluids, elastico-viscous and second grade fluids respectively.

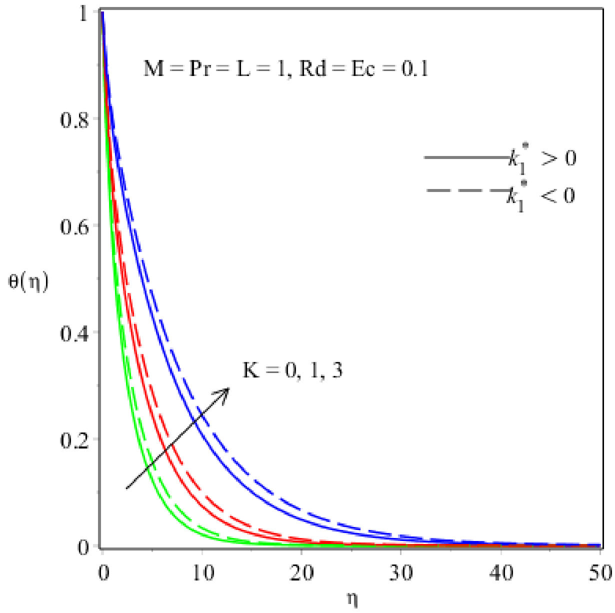

Fig. 5. Temperature profile for different values of $\boldsymbol{K}$

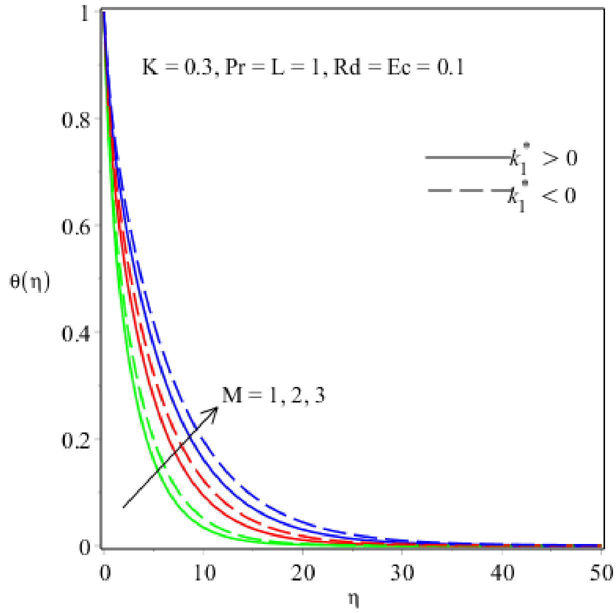

Fig. 6. Temperature profile for different values of $\boldsymbol{M}$

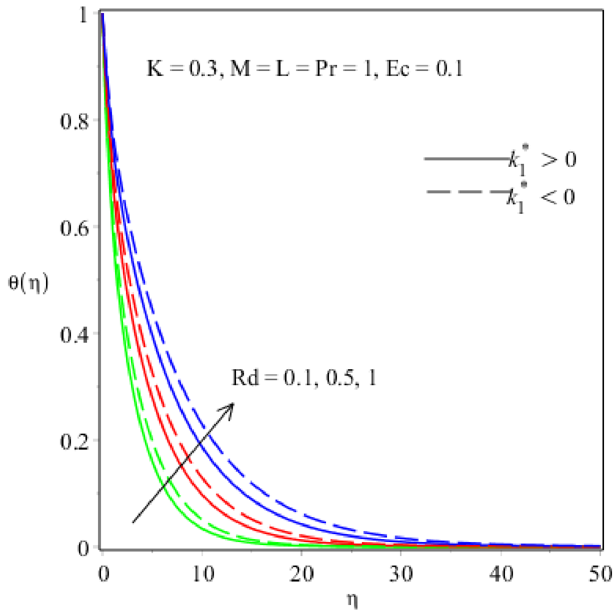

Fig. 7. Temperature profile for different values of $\boldsymbol{R_d}$

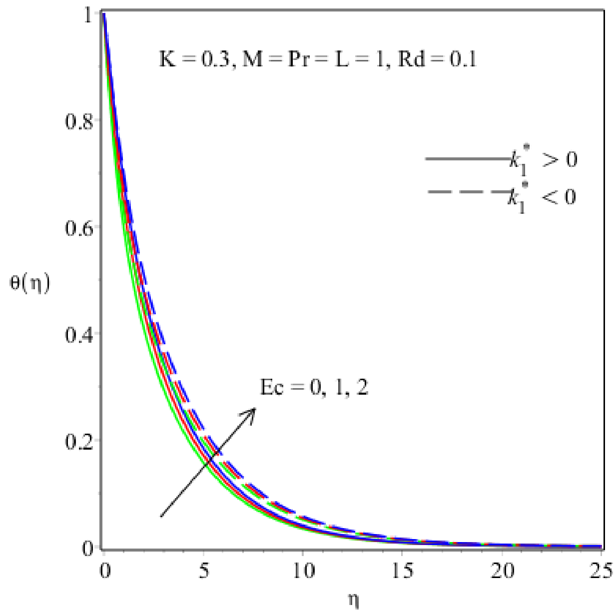

Fig. 8. Temperature profile for different values of **Ec**

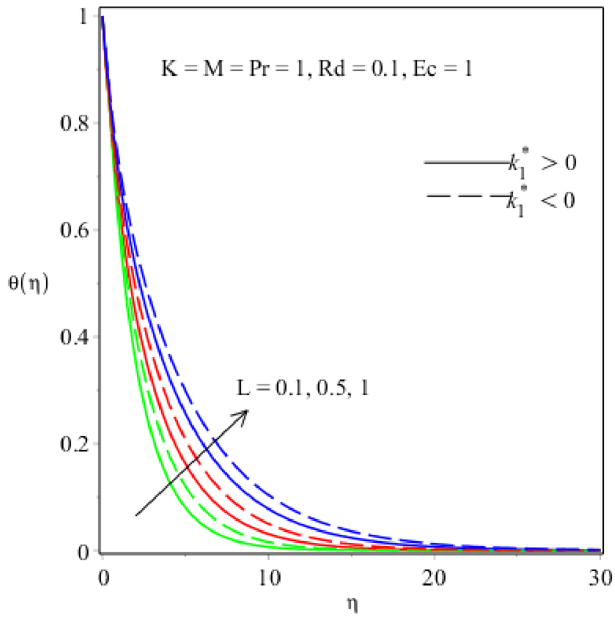

Fig. 9. Temperature profile for different values of ***L***

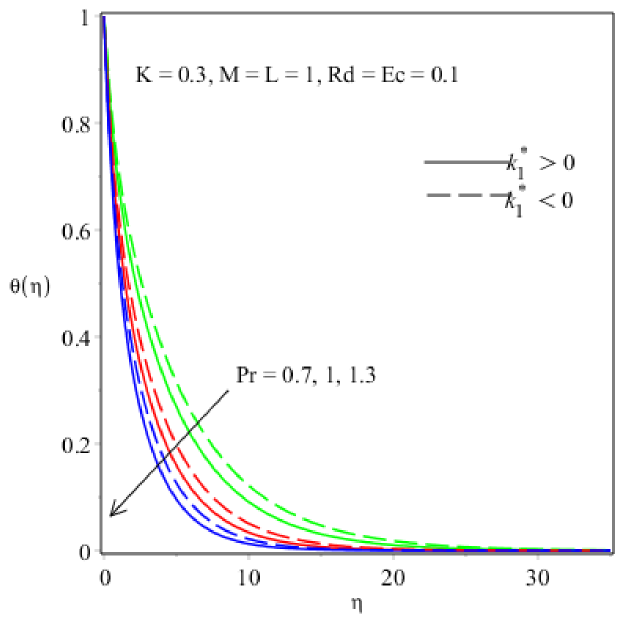

Fig. 10. Temperature profile for different values of **Pr**

Moreover, Table 1 is arranged to analyze the skin friction coefficient $-f''(0)$ for varying porosity parameter K, magnetic parameter M, and velocity slip parameter L. It is shown that the skin friction coefficient $-f''(0)$ value is increasing as the value of magnetic parameter M and porosity parameter K increasing, in the presence of both elastico-viscous and second grade fluids. Meanwhile differ for velocity slip parameter L, where the value of skin friction coefficient $-f''(0)$ is decreasing when the value of velocity slip parameter L increasing. We also have compared the value of skin friction coefficients $-f''(0)$ with the analytical results provide by Nayak *et al.,* [17] and it shows a good agreement.

Table 1
Comparison of skin friction coefficients

M	K	k_1^*	L	$-f''(0)$ Nayak *et al.,* [18]	$-f''(0)$ Present
1	0.01	-0.5	0	2.00499	2.00499
	2			2.82843	2.82843
0.5	0.01	-0.5	0	1.73781	1.73781
	2			2.64575	2.64575
			1	-	0.65017
			2	-	0.38889
		0.2	0.5	-	0.90628
			1	-	0.62063
1	0.01	0.5	0	1.15758	1.15758
	2			1.63299	1.63299
0.5	0.01	0.5	0	1.00333	1.00333
	2			1.52753	1.52753
1	0.01	1	0	1.00250	1.00250
	2			1.41421	1.41421
0.5	0.01	1	0	0.86891	0.86891
	2			1.32288	1.32288

Furthermore, Table 2 depicts the numerical data for local Nusselt number $-\theta'(0)$ of different values of thermal radiation parameter R_d, Eckert number Ec, and velocity slip parameter L for both elastico-viscous and second grade fluids. It is noted that the local Nusselt number $-\theta'(0)$ is reducing as thermal radiation parameter R_d, Eckert number Ec, and velocity slip parameter L increasing for both fluids. Also, in order to convince the validity of the method we used, we also have compared the value of Nusselt number $-\theta'(0)$ with the previous work done by Nayak *et al.,* [18], Liu [6] and Datti *et al.,* [4], and it can be seen that the values are almost the same as can be seen in Table 3 and 4. Table 3 displays the numerical data for local Nusselt number $-\theta'(0)$ for different values of magnetic parameter M and Prandtl number Pr for second grade fluid as to be compared with previous work by Nayak *et al.,* [18] and Liu [6]. The values of Nusselt number $-\theta'(0)$ are decreasing as the magnetic parameter M increasing, while the opposite trend occurs for Prandtl number Pr. Then, Table 4 shows the numerical data for local Nusselt number $-\theta'(0)$ for different values of magnetic parameter M and porosity parameter K for elastico-viscious fluid as to be compared with previous work by Nayak *et al.,* [18] and Datti *et al.,* [5]. The increment in porosity parameter K causes decrement in heat transfer coefficient which is the Nusselt number $-\theta'(0)$.

Table 2
Nusselt numbers for elastico-viscous and second grade fluids for *M* = 1, Pr = 1 and *K* = 0.01

k_1^*	$\boldsymbol{R_d}$	Ec	*L*	$\boldsymbol{-\theta'(0)}$
1	0.1	0.1	1	0.67753
	0.3	0.1	1	0.57966
		0.2		0.56643
		0.3	1	0.55320
			2	0.43419
-0.1	0.1	0.1	1	0.65394
	0.3	0.1		0.55788
		0.2	1	0.54566
		0.3	1	0.53344
			2	0.42072

Table 3
Comparison of Nusselt numbers for a second-grade fluid for $\boldsymbol{k_1^*}$ = 1.0, Ec = 0.2, $\boldsymbol{R_d}$ =0, ***L*** = 0 and ***K***= 0.01

M	Pr	$-\theta'(0)$ Liu [6]	Nayak *et al.,* [18]	Present
0	1	1.337265	1.331574	1.332375
	10	4.48696	4.478859	4.482695
1	1	1.13333	1.131589	1.132255
	10	3.74805	3.741224	3.744717

Table 4
Comparison of Nusselt numbers for a second-grade fluid for Ec = 0, $\boldsymbol{R_d}$ =0, Pr = 1 and ***L*** = 0

k_1^*	*M*	*K*	$-\theta'(0)$ Datti *et al.,* [5]	Nayak *et al.,* [18]	Present
0	0	0.01	1.3333	1.331932	1.331932
		2		1.126898	1.126898
	1	0.01	1.2158	1.214771	1.214771
		2		1.055361	1.055361
-0.1	0.1	0.01	1.3035	1.302129	1.302129
		2		1.093343	1.093343

5. Conclusion

The slip Darcy boundary layer flow of viscoelastic fluid and heat transfer over a linear stretchable surface with the presence of magnetohydrodynamic, porous medium, viscous dissipation and thermal radiation have been discussed. The governing partial differential equations are reduced to non-linear ordinary differential equations facilitate by appropriate similarity transformation. These non-linear ordinary differential equations were then solved analytically by using exact analytical method. The conclusion can be listed as follows:

I. Increment in the value of porosity parameter K, magnetic parameter M and velocity slip parameter L lead to decrement in velocity field $f'(\eta)$.
II. Increment in the value of porosity parameter K, magnetic parameter M, velocity slip parameter L, thermal radiation parameter R_d and Eckert number Ec (viscous dissipation parameter) lead to increment in temperature field $\theta(\eta)$.
III. Increment in the value of Prandtl number Pr, decays the temperature field $\theta(\eta)$.
IV. Both velocity $f'(\eta)$ and temperature $\theta(\eta)$ fields display opposite behavior for velocity slip parameter L.
V. The second-grade fluid $(k_1^* > 0)$ enhances the velocity field $f'(\eta)$ and reduces the temperature field $\theta(\eta)$ compared to elastico-viscous fluid $(k_1^* < 0)$.
VI. Elastico-viscous fluid $(k_1^* < 0)$ enhances the skin friction coefficient $-f''(0)$ and reduces the heat transfer coefficient $-\theta'(0)$ compared to second grade fluid.
VII. Skin friction coefficient $-f''(0)$ is increasing for larger magnetic parameter M and porosity parameter K, while decreasing for larger value of velocity slip parameter L for both fluids.
VIII. Local Nusselt number $-\theta'(0)$ for both fluids is higher for smaller porosity parameter K, magnetic parameter M, velocity slip parameter L, thermal radiation parameter R_d and Eckert number Ec, while the opposite trend for Prandtl number Pr.

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