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MHD Slip Flow and Heat Transfer on Stagnation Point of a Magnetite (Fe_3O_4) Ferrofluid towards a Stretching Sheet with Newtonian Heating

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ABSTRACT

Present paper investigates the flow and heat transfer of Magnetite (Fe3O4) water based nanofluid termed as ferrofluid on stagnation point past a stretching sheet with slip effect. The methodology starts with transforming the non-linear partial differential equations that governed the model to ordinary differential equations, then solved numerically by Runge-Kutta-Fehlberg (RKF45) method in Maple software. The influenced and characteristics of pertinent parameters which are the stretching parameter, magnetic parameter, velocity slip parameter and solid volume fraction for nanofluid are analyzed and discussed. It was found that the magnetite (Fe3O4) ferrofluid provided higher wall temperature and heat transfer capabilities compared to water. Meanwhile, the presence of slip effect had minimized the skin friction coefficient.

1. Introduction

Ferrofluid termed as a nanofluid with nanoscale ferromagnetic particles. It is strongly magnetized in the presence of magnetic field. It was first invented by NASA as a liquid rocket fuel that can be injected toward pump inlet in a weightless or no gravity situation in aero space [1]. Further, ferrofluid is also employed in field medicine as intelligent biomaterials for wound treatment and medicine drug targeting especially in cancer tumor treatment. Other applications involving this type of fluid includes in electrical instruments such as computer hard disks, heat controlling agents in electric motor and hi-fi speakers [2,3].

In considering the convective heat transfer boundary layer flow, Crane [4] pioneered this study in stretching surface. The study regarding this topic then extended with various type of fluid include the ferrofluid. In modelling the mathematical model of ferrofluid, Tiwari and Das [5] introduced a

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single-phase model with a specific case of nanofluid. This approaches is suitable to study the flow and heat transfer characteristic of specific ferrofluid. The coefficient physical properties like thermal conductivity, density and specific heat of nanofluid are taking account in computation.

Recent study focus on a stagnation point flow past a stretching sheet studied by Zokri *et al.,* [6], Kho *et al.,* [7] and Mohamed *et al.,* [8] whose considered the effects of MHD, thermal radiation, viscous dissipation and heat generation in viscous fluid, Jeffrey fluid and Williamson nanofluid.

Present paper extends the study on stagnation point flow past a stretching sheet in a magnetite (Fe_3O_4) ferrofluid with slip effects. Recent studies on ferrofluid includes the works from Ramli *et al.,* [9], Sheikholeslami [10], Jusoh *et al.,* [11] and Hussanan *et al.,* [12,13]. The Newtonian heating boundary conditions are more realistic to be taken account since it consider the proportional relation between the wall temperature and the heat transfer rate Merkin [14]. Results published here are important for researchers in this area either in numerical or experimental approach so it can be used as a comparison and reference in future. The fact that the problem considered here has never been studied before, thus the reported result in this paper is new.

2. Mathematical Formulation

Consider a steady incompressible ferrofluid on a stagnation point past a stretching sheet with ambient temperature T_∞. Assume that the free stream velocity U_∞ and stretching velocity $u_w(x)$ are in the forms of linear $u_w(x)=ax$ and $U_\infty = bx$ where a and b are positive constants [15]. Further, a uniform magnetic field of strength B_0 is assumed to be applied in the positive *y*-direction normal to the stretching sheet. The magnetic Reynolds number is assumed to be small, and thus the induced magnetic field is negligible. The physical model and coordinate system of this problem is shown in Figure 1. It is further assumed that the plate is subjected to a Newtonian heating as proposed by Merkin [14]. The boundary layer equations are [16,17,9]

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U_\infty\frac{dU_\infty}{dx}+\nu_{nf}\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_o^2(x)}{\rho_{nf}}(u-U_\infty), \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

subject to the boundary conditions

$$u=u_w+\lambda^*\frac{\partial u}{\partial y},\ v=0,\ \frac{\partial T}{\partial y}=-h_sT \text{ at } y=0,$$

$$u\to U_\infty,\ T\to T_\infty \text{ as } y\to\infty, \tag{4}$$

where, *u* and *v* are the velocity components along the *x* and *y* directions, respectively. Further, T is the ferrofluid temperature in the boundary layer, σ is the electrical conductivity, h_s is the heat transfer coefficient, ν_{nf} is the kinematic viscosity of ferrofluid, ρ_{nf} is the ferrofluid density and α_{nf} is the thermal diffusivity of ferrofluid, which can be expressed in terms of the properties of base fluid, ferroparticles and solid volume fraction φ as follows [17,18]

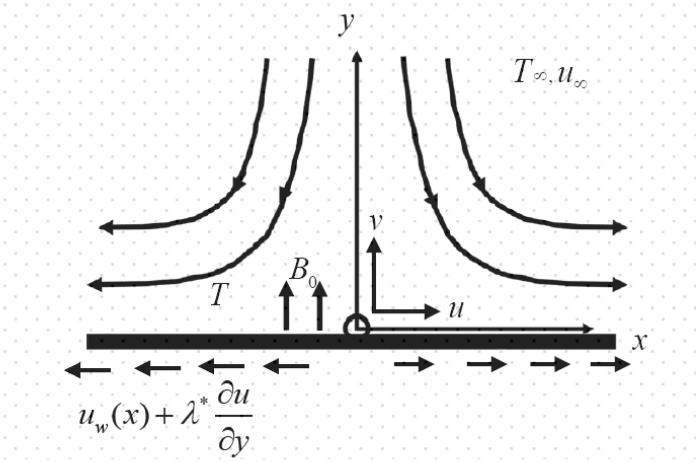

Fig. 1. Physical model and the coordinate system

$$\nu_{nf}=\frac{\mu_{nf}}{\rho_{nf}},\quad \rho_{nf}=(1-\varphi)\rho_f+\varphi\rho_s,\quad \alpha_{nf}=\frac{k_{nf}}{\rho_{nf}\left(C_p\right)_{nf}},\quad \mu_{nf}=\frac{\mu_f}{(1-\varphi)^{2.5}},$$
$$\left(\rho C_p\right)_{nf}=(1-\varphi)\left(\rho C_p\right)_f+\varphi\left(\rho C_p\right)_s,\quad \frac{k_{nf}}{k_f}=\frac{k_s+2k_f-2\varphi(k_f-k_s)}{k_s+2k_f+\varphi(k_f-k_s)}. \tag{5}$$

Note that k_{nf}, k_f and k_s are the thermal conductivity of the ferrofluid, base fluid and ferroparticles, respectively while $\left(\rho C_p\right)_{nf}$ is the heat capacity of ferrofluid.

The non-linear partial differential (Eq. 1-3) contains many dependent variables which in dimensional forms and difficult to solve. Therefore, the following similarity variables are applied

$$\eta=\left(\frac{b}{\nu_f}\right)^{1/2} y,\quad \psi=\left(b\nu_f\right)^{1/2} xf(\eta),\quad \theta(\eta)=\frac{T-T_\infty}{T_\infty}, \tag{6}$$

where η, θ and ψ are non-dimensional similarity variable, temperature and stream function. The Eq. 1 satisfied by definition $u=\dfrac{\partial\psi}{\partial y}$ and $v=-\dfrac{\partial\psi}{\partial x}$, respectively. Substitute the Eq. 5 and 6 into (Eq. 1-3), then the following ordinary differential equations are obtained

$$\frac{1}{(1-\varphi)^{2.5}\left[1-\varphi+(\varphi\rho_s)/(\rho_f)\right]} f'''+ff''-f'^2+\varepsilon^2-M(f'-\varepsilon)=0 \tag{7}$$

$$\frac{k_{nf}/k_f}{(1-\varphi)+\varphi(\rho C_p)_s/(\rho C_p)_f}\theta''+\Pr f\theta'=0, \tag{8}$$

where $\varepsilon = \frac{a}{b}, (\varepsilon > 0)$ is the stretching parameter, $M = \frac{\sigma B_o^2(x)}{b\rho_{nf}}$ is the magnetic parameter and $\Pr = \frac{\nu_f \left(\rho C_p\right)_f}{k_f}$ is the Prandtl number. The transformed boundary conditions are

$$f(0)=0,\ f'(0)=1+\lambda f''(0),\ \theta'(0)=-\gamma\left(1+\theta(0)\right),$$
$$f'(\eta)\to\varepsilon,\ \theta(\eta)\to 0 \text{ as } y\to\infty. \tag{9}$$

where, $\lambda = \lambda^*\left(\frac{b}{\nu_f}\right)^{1/2}$ and $\gamma = h_s\left(\frac{b}{\nu_f}\right)^{-1/2}$ is the velocity slip parameter and conjugate parameter, respectively. The physical quantity interest are the wall temperature $\theta(0)$, the heat transfer rate $-\theta'(0)$ and the skin friction coefficient C_f which given by

$$C_f = \frac{\tau_w}{\rho_f u_\infty^2}, \tag{10}$$

with surface shear stress $\tau_w = \mu_{nf}\left(\frac{\partial u}{\partial y}\right)_{y=0}$. Using the similarity variables in Eq. 6 gives

$$C_f \operatorname{Re}_x^{1/2} = \frac{f''(0)}{(1-\varphi)^{2.5}}, \tag{11}$$

where, $\operatorname{Re}_x = \frac{U_\infty x}{\nu}$ is the Reynolds number.

3. Results and Discussion

The system of ordinary differential Eq. 7 and Eq. 8 with boundary conditions Eq. 9 were solved numerically using the RKF45 technique in Maple. The boundary layer thickness η_∞ between 3 and 6 was used in the computation, depending on the values of the parameters considered so that the boundary condition at 'infinity' is achieved. The numerical results are obtained for wall temperature $\theta(0)$, the temperature gradient $-\theta'(0)$ and the reduced skin friction coefficient $C_f \operatorname{Re}_x^{1/2}$ for various values of pertinent parameter namely as the stretching parameter ε, magnetic parameter M, velocity slip parameter λ and solid volume fraction for nanofluid φ.

In order to validate the numerical results obtained, the comparison has been made. Table 1 shows the comparison values of $\theta(0)$ and $-\theta'(0)$ with previous published results by Salleh *et al.*, [19] and Sarif *et al.*, [20] for ordinary viscous fluid with various values of Pr. It is found that the results are in a good agreement.

Next, Table 2 shows data related to the thermophysical properties for water and magnetite (Fe_3O_4) applied by Khan *et al.,* [17] and Ramli *et al.,* [9]. Note that as water employed as a base fluid in this study therefore, the numerical computation set Pr as 6.2.

Table 1
Comparison results with previously published for $\Pr$ when $\varepsilon=\lambda=M=\varphi=0$ and $\gamma=1$.

Pr	Salleh *et al.*, [19]		Sarif *et al.,* [20]		Present	
	$\theta(0)$	$-\theta'(0)$	$\theta(0)$	$-\theta'(0)$	$\theta(0)$	$-\theta'(0)$
7	1.13511	2.13511	1.11816	2.11816	1.11689	2.11689
10	0.76531	1.76531	0.76507	1.76507	0.76456	1.76456
100	0.16115	1.16115	0.14757	1.14757	0.14781	1.14781

Table 2
Thermophysical properties of water base fluid and magnetite (Fe_3O_4) nanoparticles

Physical Properties	Water Base Fluid	Magnetite (Fe_3O_4) Nanoparticles
ρ (kg/m^3)	997	5180
C_p (J/kg·K)	4179	670
k(W/m·K)	0.613	9.7

Figures 2-5 show the temperature profiles for various values of ε, λ, M and φ. It is worth mentioning here that in Newtonian heating, the effect of heat transfer coefficient $-\theta'(0)$ is similar since this quantity is directly proportional to the wall temperature $\theta(0)$. In Figure 2, it is found that the increase of ε reduced the temperature as well as its thermal boundary layer thicknesses. The increase of stretching velocity over stream velocity has thinning the boundary layer thicknesses. Contrary with Figures 3-5, the increase of λ, M and φ gives rise on temperature. It is realistic since the increase volume of magnetite nanoparticles in the fluid had increase the ferrofluid capabilities in thermal conductivity hence increase the temperature as well as the heat transfer rate. Further, the increase in magnetic field had attract the ferrofluid particles stick to the surface which enhanced the efficiency of the heat transfer rate. Notice that in Figure 5, the boundary layer thicknesses increases more significantly with the increase in φ compared to the changes in M and λ from Figures 3 and 4.

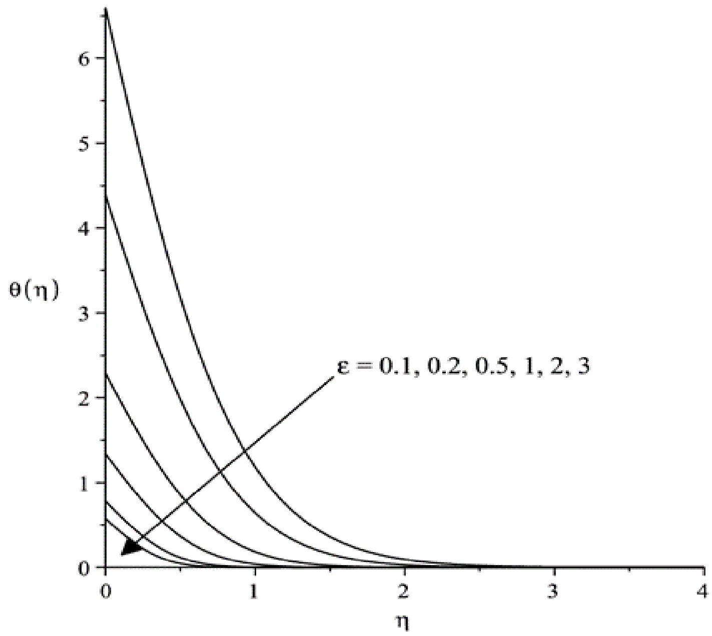

Fig. 2. Effect ε on $\theta(\eta)$ when $\Pr=6.2$, $\lambda=0.5, \gamma=M=1$ and $\varphi=0.1$.

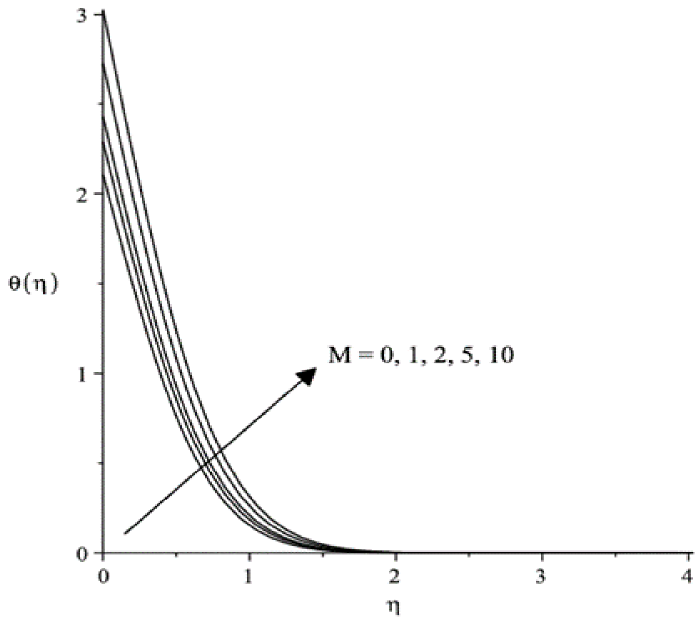

Fig. 3. Effect M on $\theta(\eta)$ when $\Pr = 6.2$, $\varepsilon = \lambda = 0.5$, $\gamma = 1$ and $\varphi = 0.1$.

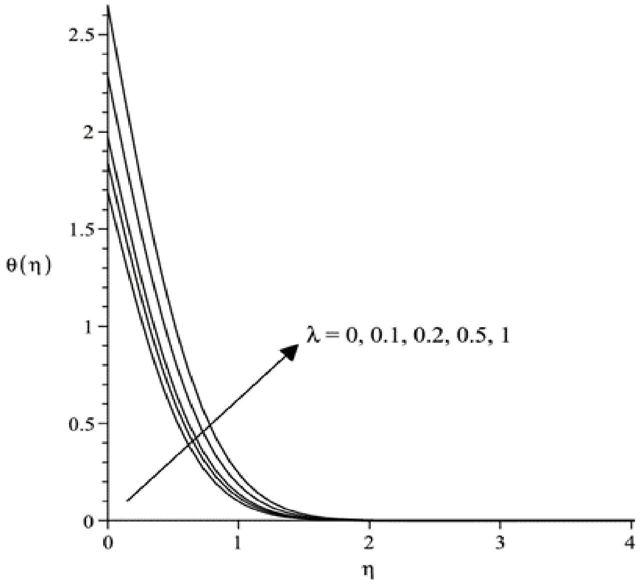

Fig. 4. Effect λ on $\theta(\eta)$ when $\Pr = 6.2$, $\varepsilon = 0.5$, $\gamma = M = 1$ and $\varphi = 0.1$.

Next, Figure 6 illustrates the velocity profiles for various values of ε. It is found that the presence of stretching effects for $\varepsilon \geq 1$ has increase the fluid velocity while reduced the thickness of velocity boundary layer. Thinning in velocity boundary layer thicknesses physically denoted to the increase in skin friction coefficient. For $\varepsilon = 1$, which imply to the case of stretching velocity is equal to the fluid external velocity outside the boundary layer, the velocity is constant which cause the zero skin friction coefficient, $C_f \operatorname{Re}_x^{1/2} = 0$. The velocity boundary layer thickness is increases as the stretching velocity is less than stream velocity. This is shown clearly at $\varepsilon \leq 1$. Further, the presents of slip velocity had reduced the fluid velocity and the velocity boundary layer thickness. Effect of λ on velocity profiles are shown clearly in Figure 7.

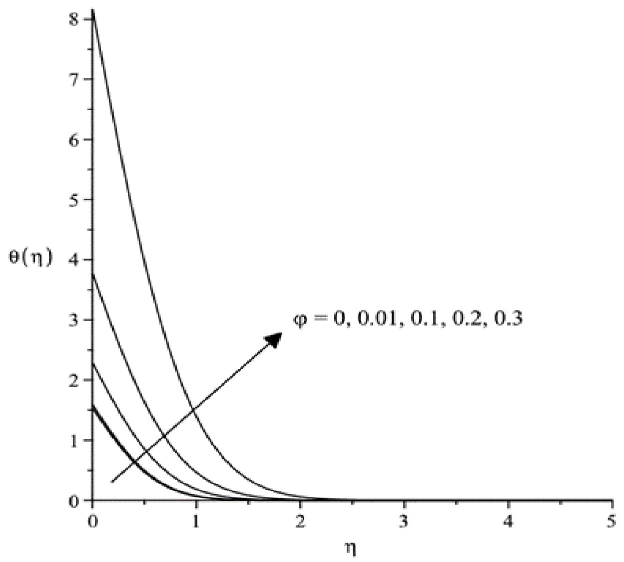

Fig. 5. Effect φ on $\theta(\eta)$ when $\mathrm{Pr}=6.2$, $\varepsilon=\lambda=0.5$ and $\gamma=M=1$.

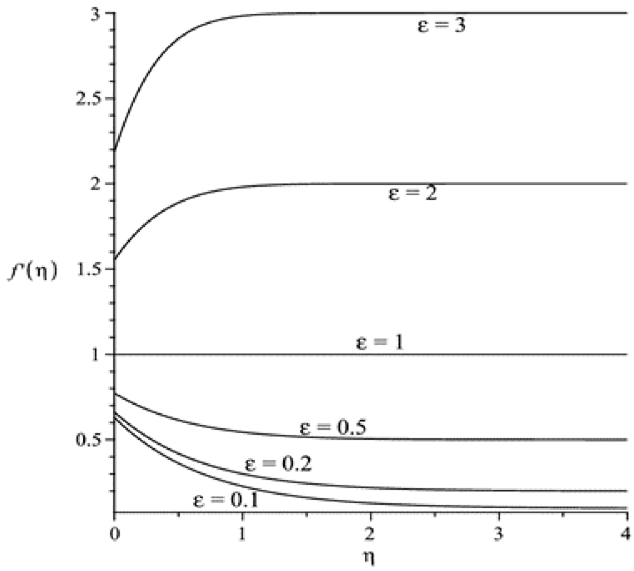

Fig. 6. Effect ε on $f'(\eta)$ when $\mathrm{Pr}=6.2$, $\lambda=0.5$, $\gamma=M=1$ and $\varphi=0.1$.

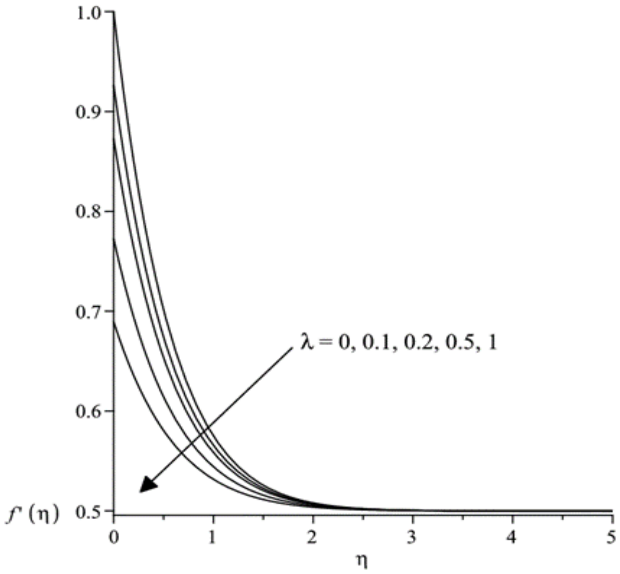

Fig. 7. Effect λ on $f'(\eta)$ when $\Pr = 6.2$, $\varepsilon = 0.5$, $\gamma = M = 1$ and $\varphi = 0.1$.

Figures 8 and 9 illustrate the effect of M and φ on variation of $\theta(0)$ and $C_f \operatorname{Re}_x^{1/2}$, respectively. It is found that the increase in M and φ had increase the wall temperature $\theta(0)$ as discussed in Figures 4 and 5. Notice that $\varphi = 0$ imply to the case that no magnetite nanoparticle or only water base fluid. From Figure 8, it is suggested that the ferrofluid $(\varphi > 0)$ provided higher $\theta(0)$ compared to the water. It is realistic due to the magnetite nanoparticles is very good in thermal conductivity as stated in Table 2. Blending this nanoparticle with water was enhanced the based fluid characteristic. It also had increase the fluid density and viscosity which reflects to a reducing in fluid velocity as well as the skin friction coefficient as shown in Figure 9. Thus, ferrofluid can be concluded had a low skin friction coefficient compared to its based fluid. Meanwhile, the value of $C_f \operatorname{Re}_x^{1/2}$ is decreases as M increases in Figure 9. The increase of M on a surface had attracted the magnetite particles in nanofluid which then reduced the fluid velocity. This reflects to the low in skin friction coefficient and the rise of temperature.

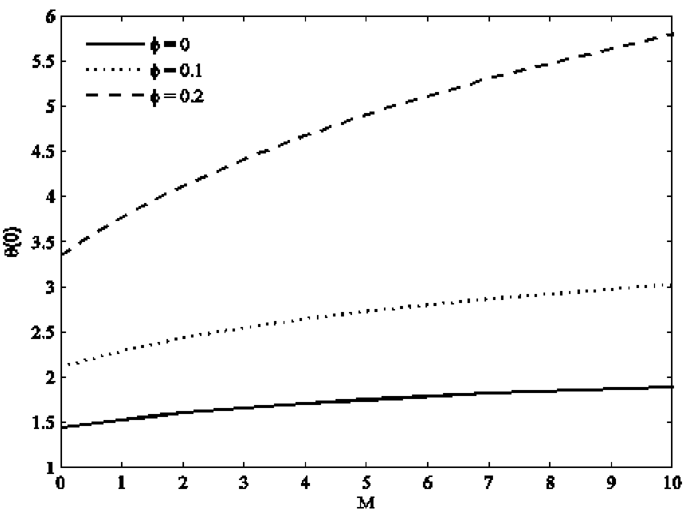

Fig. 8. Effect M and φ on variation of $\theta(0)$ when $\Pr = 6.2$, $\varepsilon = \lambda = 0.5$ and $\gamma = 1$.

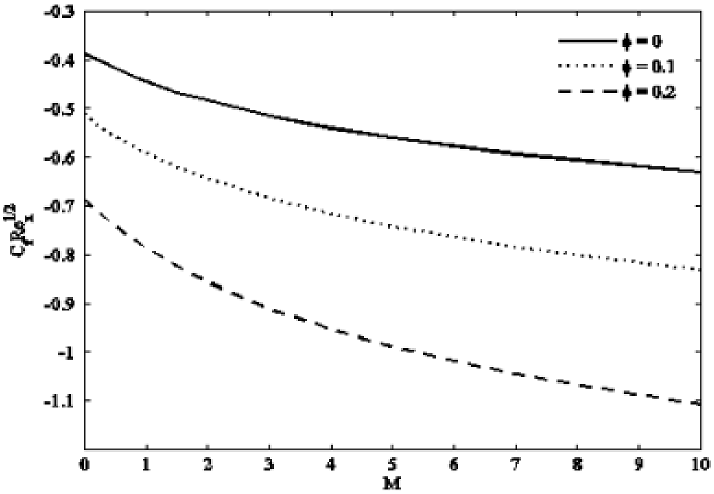

Fig. 9. Effect M and φ on variation of $C_f \operatorname{Re}_x^{1/2}$ when $\Pr = 6.2$, $\varepsilon = \lambda = 0.5$ and $\gamma = 1$.

Lastly, the variation of $C_f \operatorname{Re}_x^{1/2}$ with various values of ε and λ are presented in Figure 10. Generally, the increase of ε results to the increase in skin friction coefficient. For $\varepsilon < 1$, it is suggested that the value of $C_f \operatorname{Re}_x^{1/2}$ is negative due to the opposing direction as $\varepsilon < 1$ (refer to Eq. 9). The presence of slip effect $(\lambda = 0.5)$ has dominant the effects of this small stretching value and enhanced the value of $C_f \operatorname{Re}_x^{1/2}$. As $\varepsilon = 1$, the skin friction becomes zero and the effect of slip is negligible at this stage as the relations as described in boundary conditions (9). Further, for $\varepsilon > 1$, it is found that the presence of slip effect has opposed the stretching effects which then reduced the skin friction coefficient. Physically, the increase of stretching velocity over ambient velocity had rise the fluid flow velocity. Surprisingly, slip effect then minimize the speed difference between fluid flow and surface which reduced the value of the $C_f \operatorname{Re}_x^{1/2}$. From this figure, it is suggested that sufficient strength of slip effect may eliminated the stretching effects.

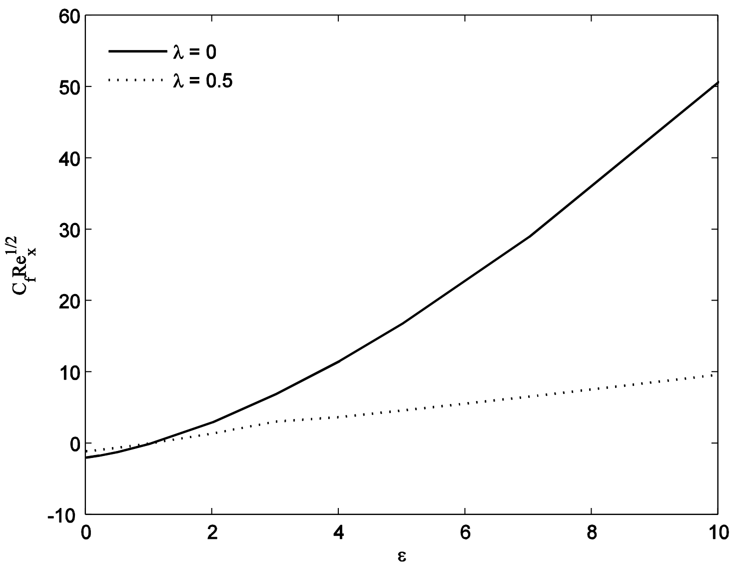

Fig. 10. Effect ε and λ on variation of $C_f \operatorname{Re}_x^{1/2}$ when $\Pr = 6.2$, $M = 1$, $\varphi = 0.1$ and $\gamma = 1$.

4. Conclusions

As a conclusion, the increase of magnetic parameter, velocity slip parameter and nanoparticles volume fraction results to a decrease of temperature profiles as well as the thermal boundary layer thicknesses while stretching parameter does contrary. The presence of slip effect has reduced the velocity boundary layer thickness.

Next, it is found that the magnetite (Fe_3O_4) ferrofluid provided higher wall temperature and heat transfer capabilities compared to water. The magnetite (Fe_3O_4) ferrofluid also results the low skin friction coefficient which physically reduce chances of erosion thus extending life-time of the surface components.

Lastly, it is worth to conclude that the presence of slip effect had opposed the stretching effect capabilities thus minimize the skin friction coefficient. Sufficient strength of slip effect may eliminated the stretching effects.

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