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A Stability Analysis of Boundary Layer Stagnation-Point Slip Flow and Heat Transfer towards a Shrinking/Stretching Cylinder over a Permeable Surface

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ABSTRACT

A stability analysis of dual solution for the problem of stagnation-point slip flow over a stretching or shrinking cylinder is studied. The partial differential equations governing will be transformed to a set of coupled nonlinear nonsimilar equations via similarity transformations. The transformed governing equations are solved numerically using the bvp4c function in MATLAB software. Numerical calculations exhibit the existence of dual solution and the implementation of stability analysis proved that the first solution is stable and physically realizable.

1. Introduction

Fuid flow and heat transfer over a shrinking or stretching surfaces, which occurs in several engineering processes have received great attention during the last decades. Crane [1] first attempted to solve the problem of boundary layer flow due to a stretching surface and Chiam [2] extended this problem to consider the stagnation-point flow. The stagnation flow towards a shrinking sheet was investigated by Wang [3] and he found that this problem has the dual solutions as well as unique solution for a specific value of the ratio of shrinking. After these pioneering works, the fluid flow and heat transfer over a shrinking or stretching surfaces has drawn considerable attention and a good number of literatures including nanofluids [4-8], ferrofluid [9], Williamson fluid [10] and magnetic flows [11, 12].

The effects of partial slip on the steady boundary layer stagnation-point flow of an incompressible fluid and heat transfer towards a shrinking sheet was analyzed by Bhattacharyya *et al.,* [13] , where

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the conditions of the non-existence, existence, uniqueness and duality of the solutions of self-similar equations has been numerically obtained. The boundary layer flow due to a vertical cylinder in a quiescent viscous and incompressible fluid have been considered in the papers by Ishak *et al.,* [14], Ishak [15] and Bachok and Ishak [16]. Mat *et al.,* [17] investigated the effects of partial slip on the boundary layer stagnation point flow and heat transfer towards a shrinking/stretching cylinder over a permeable surface and it is found that dual solutions exist for the shrinking cylinder.

The existence of dual solutions on the fluid flow become a question which solution is stable and vice versa. Recently, the implementation of the stability analysis has been the subject of interest to validate which solution is the physical solution. The earlier studies on mathematical formulation of stability analysis were examined by Merkin [18] and Weidman *et al.,* [19]. Many recent works also discussed the existence of dual solutions and emphasis on stability analysis (see Najib *et al.,* [20], Nazar *et al.,* [21], Hafidzuddin *et al.,* [22], Yahaya *et al.,* [23] and Saleh *et al.,* [24]). All the reported literatures implemented the bvp4c solver in the MATLAB software to examine the paired and stability solutions.

Inspired and motivated by the literatures above, the present work discusses in detail the stability analysis of dual solution for the problem of stagnation-point slip flow passing a stretching or shrinking cylinder over a permeable surface. Appropriate similarity transformation reduces the governing PDEs into a system of ODEs. The resulting equations are solved numerically using bvp4c function in MATLAB software. The present work is also concerned about the existence of dual solutions and the way of stability analysis is conducted to validate the physical solution.

2. Methodology

2.1 Mathematical Formulation

The steady stagnation-point flow passing stretching/shrinking cylinder with radius R immersed in an incompressible viscous fluid of constant temperature T_w is considered. Under the assumption of boundary layer approximation, the governing equations for this problem are:

$$\frac{\partial}{\partial x}(ru)+\frac{\partial}{\partial r}(rv)=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}=U\frac{dU}{dx}+\nu\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}\right) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial r}=\alpha\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right) \tag{3}$$

where x and r are coordinates measured along the surface of the cylinder and in the radial direction, respectively, with u and v being the corresponding velocity components. Further, $U(x)=ax$ represents the velocity of the straining stagnation-point flow, T represents the boundary layer temperature, ν is the coefficient of the kinematic viscosity and α is the thermal diffusivity. The boundary conditions of these equations are given by:

$$\begin{aligned} &v=v_w(x),\ \ u=cx+L(\partial u/\partial r),\ \ T=T_w+D(\partial T/\partial r)\ \text{ at }\ r=R \\ &u\to U(x)=ax,\ \ T\to T_\infty\ \text{ as }\ r\to\infty \end{aligned} \tag{4}$$

Here, $a(>0)$ and c are parameters of the straining rate and the shrinking/stretching rate (of the surface) where shrinking cylinder is $c<0$ and stretching cylinder is $c>0$, L and D is the velocity and thermal slip factor, respectively, $v=v_w(x)$ represents the velocity for mass transfer with $v_w(x)>0$ for suction and $v_w(x)<0$ for injection, T_w and T_∞ represents the surface and temperature for free stream, both are assumed to be constant with $T_w>T_\infty$.

Introducing the stream function ψ, which is defined as $u=r^{-1}\partial\psi/\partial r$ and $v=-r^{-1}\partial\psi/\partial x$. Then, we assume the similarity variables defined as

$$\psi=(a\nu)^{1/2}xRf(\eta),\ \ T=T_\infty+(T_w-T_\infty)\theta(\eta),\ \ \eta=\frac{r^2-R^2}{2R}(a/\nu)^{1/2} \tag{5}$$

We assume v_w have following expression:

$$v_w(x)=-\frac{1}{r}\left(\frac{\nu U}{x}\right)^{1/2}Rf_w \tag{6}$$

where $f_w=f(0)$ represents a non-dimensional constant determines the transpiration rate, with $f_w>0$ and $f_w<0$ are the constant suction and injection parameter. The surface is an impermeable if $f_w=0$. Substituting Eq. (5) into Eq. (2) and (3), we get as follows:

$$(1+2\lambda\eta)f'''+2\lambda f''+ff''+1-f'^2=0, \tag{7}$$

$$(1+2\lambda\eta)\theta''+2\lambda\theta'+\Pr f\theta'=0 \tag{8}$$

alongside boundary conditions as follows:

$$\begin{aligned}&f(0)=f_w,\ \ f'(0)=c/a+\delta f''(0),\ \ \theta(0)=1+\beta\theta'(0),\\&f'(\infty)\to 1,\ \ \theta(\infty)\to 0,\end{aligned} \tag{9}$$

Here, $\lambda=\left(\frac{\nu}{aR^2}\right)^{1/2}$ is the parameter for curvature, $\Pr=\frac{\nu}{\alpha}$ represents Prandtl number, c/a represents the parameter for the velocity ratio parameter, $\delta=L\frac{r}{R}(a/\nu)^{1/2}$ represents the velocity slip parameter and $\beta=D\frac{r}{R}(a/\nu)^{1/2}$ represents the thermal slip parameter.

The main physical quantities of interest are skin friction, c_f and local Nusselt number Nu_x are given by:

$$c_f=\frac{\tau_w}{\rho U_\infty^2}\quad\text{and}\quad Nu_x=\frac{xq_w}{k(T_w-T_\infty)} \tag{10}$$

where

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0} \quad \text{and} \quad q_w = -k\left(\frac{\partial T}{\partial u}\right)_{y=0}$$

Using similarity transforms in (5), we obtain

$$c_f Re_x^{1/2} = f''(0) \quad \text{and} \quad \frac{Nu_x}{Re_x^{1/2}} = -\theta'(0) \tag{11}$$

where $Re_x = \frac{U_x}{v}$ is a Reynolds number.

2.2 Stability Analysis

In this paper, we found that there are dual solutions. Following Weidman *et al.,* [19], a variable τ has to be introduced. We consider unsteady case for Eqs. (2) and (3), which are replaced by

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} = U\frac{dU}{dx} + \nu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right) \tag{12}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial r} = \alpha\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right) \tag{13}$$

where t indicates the time. Then, we introduce new similarity variables as below:

$$\psi = (a\nu)^{1/2} xRf(\eta,\tau), \; T = T_\infty + (T_w - T_\infty)\theta(\eta,\tau),$$
$$\eta = \frac{r^2 - R^2}{2R}(a/\nu)^{1/2}, \; \tau = at \tag{14}$$

We substitute Eq. (14) into Eqs. (12) and (13) and obtain equations as follow:

$$(1+2\lambda\eta)\frac{\partial^3 f}{\partial\eta^3} + 2\lambda\frac{\partial^2 f}{\partial\eta^2} + \frac{\partial f}{\partial\eta}\frac{\partial^2 f}{\partial\eta^2} + 1 - \left(\frac{\partial f}{\partial\eta}\right)^2 - \frac{\partial^2 f}{\partial\eta\partial\tau} = 0, \tag{15}$$

$$(1+2\lambda\eta)\frac{\partial^2\theta}{\partial\eta^2} + 2\lambda\frac{\partial\theta}{\partial\eta} + \Pr f\frac{\partial\theta}{\partial\eta} - \Pr\frac{\partial\theta}{\partial\tau} = 0 \tag{16}$$

with boundary conditions as follows:

$$f(0,\tau) = f_w, \; \frac{\partial f}{\partial\eta}(0,\tau) = c/a + \delta\frac{\partial^2 f}{\partial\eta^2}, \; \theta(0,\tau) = 1 + \beta\frac{\partial\theta}{\partial\eta},$$
$$\frac{\partial f}{\partial\eta}(\infty,\tau) \to 1, \; \theta(\infty,\tau) \to 0. \tag{17}$$

We determine the stability of dual solutions by adopting the analysis suggested by Weidman *et al.,* [19]:

$$f(\eta,\tau)=f_0(\eta)+e^{-\gamma\tau}F(\eta,\tau),\ \ \theta(\eta,\tau)=\theta_0(\eta)+e^{-\gamma\tau}G(\eta,\tau), \tag{18}$$

where γ denotes an unknown eigenvalue and it describes the growth or decay of the disturbance rate. Substituting Eq. (18) into Eqs. (15) and (16), we finally get:

$$(1+2\lambda\eta)F_0'''+2\lambda F_0''+f_0'F_0''-2f_0'F_0'+\gamma F_0'+f_0''F_0=0, \tag{19}$$

$$(1+2\lambda\eta)G_0''+2\lambda G_0'+\Pr fG_0'+\Pr\theta_0'F_0+\Pr\gamma G_0=0 \tag{20}$$

together with the new boundary conditions:

$$\begin{aligned}&F_0(0)=0,\ \ F_0'(0)=\delta F_0''(0),\ \ G(0)=\beta G_0'(0),\\&F_0'(\infty)\to 0,\ \ G_0(\infty)\to 0.\end{aligned} \tag{21}$$

The stability of the problem is determined by the smallest eigenvalue γ. The possible values of γ can be obtained by relaxing a boundary condition on $F_0'(\infty)$ or $G_0(\infty)$ (see Harris *et al.,* [25]). In this problem, we choose to relax the boundary condition $F_0'(\infty)\to 0$ and solve the system using the bvp4c function with the new boundary condition $F_0''(0)=1$.

3. Results and Discussion

To obtain the numerical results, we firstly transform the PDEs into nonlinear ODEs using similarity transformation (5), then the ODEs are reduced to a system of first order before being solved numerically using bvp4c solver in Matlab. To validate the accuracy of the present numerical method, the comparison of numerical results with the results described by Mat *et al.,* [17] has been done as shown in Table 1. It is found that excellent agreement exists. Therefore, we confident that the present results are accurate.

Table 1
Variations of $f''(0)$ with respect to δ and c/a when $\lambda=0,\ f_w=0, \beta=0.2$ and $\Pr=1$. ( ) is the second solution.

δ	c/a	Mat *et al.,* [14]	Present result
0	-1.20	0.932474	0.932473
		(0.233650)	(0.233649)
	-1.15	1.082232	1.082231
		(0.116702)	(0.116702)
0.1	-1.20	1.224941	1.224940
		(0.182621)	(0.182621)
	-1.15	1.306265	1.306264
		(0.100177)	(0.100177)

Figures 1 and 2 illustrate the skin friction coefficient and local Nusselt number coefficient with c/a. It can be seen in these Figures that the dual solutions are possible when $c/a > c/a_c$. The first and second solutions exist up to a critical value c/a_c and no solution remain for $c/a < c/a_c$. It means the the boundary layer separation may occur when $c/a < c/a_c$. From these figures, it is found that as velocity slip parameter, δ increases, the skin friction $f''(0)$ and local Nusselt number,

$-\theta'(0)$ increase. It is shown that the velocity slip parameter δ decreases the shear stress rate but increases the heat transfer rate at the surface. Figures 3 and 4 display the velocity and temperature profiles for several values of c/a with $\lambda = \delta = f_w = 0, \beta = 0.2$ and $\Pr = 1$. From these figures, it is shown that the curves approach the far field boundary conditions asymptotically. Further, these figures also support the existence of dual nature of the solutions as presented in Figures 1 and 2 where the boundary layer thickness for the first solution is thinner compared with the second solution.

A stability analysis is performed due to the presence of dual solutions in the present problem. The flow is unstable if the smallest eigenvalue γ is negative which indicates that an initial growth of disturbances occurs while positive value of γ implies that the flow is stable. The linear eigenvalue problems (17) and (18) with the boundary condition (19) were numerically solved using bvp4c function in MATLAB software to find the smallest eigenvalue γ for selected values of c/a when $\delta = 0$ and 0.1 and the results are shown in Table 2. From Table 2, it is seen that for the first solution, γ is positive whereas negative for second solution. Therefore, it is concluded that the first solution is stable and the second solution is unstable and not acceptable.

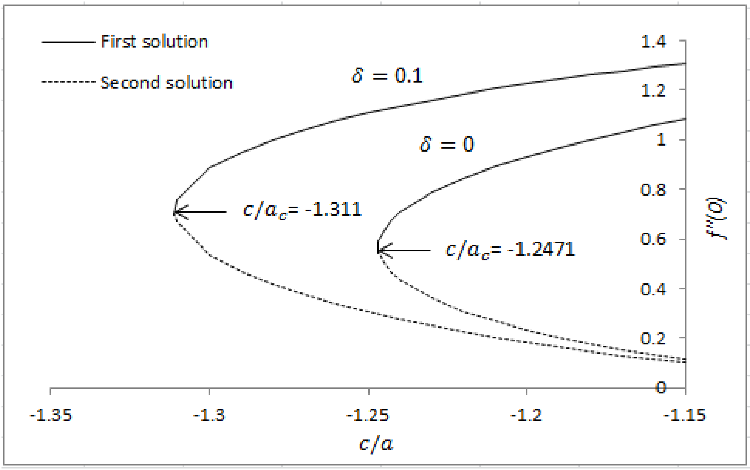

Fig. 1. Skin friction coefficient $f''(0)$ for several point of c/a

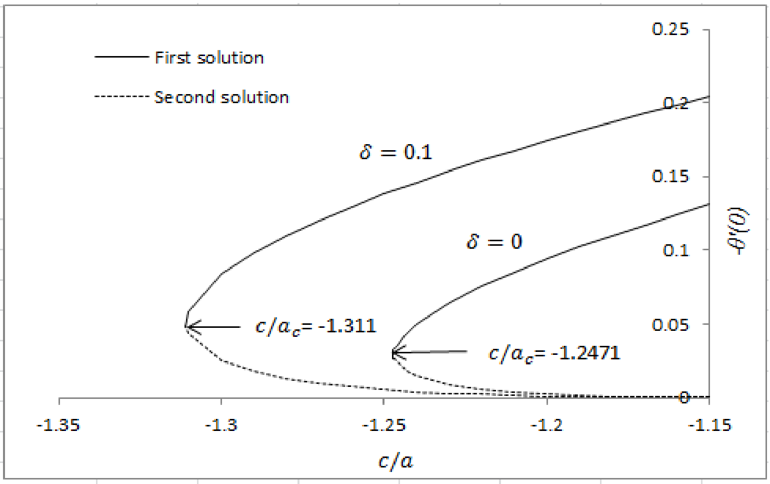

Fig. 2. Local Nusselt number coefficient $-\theta'(0)$ for several point of c/a

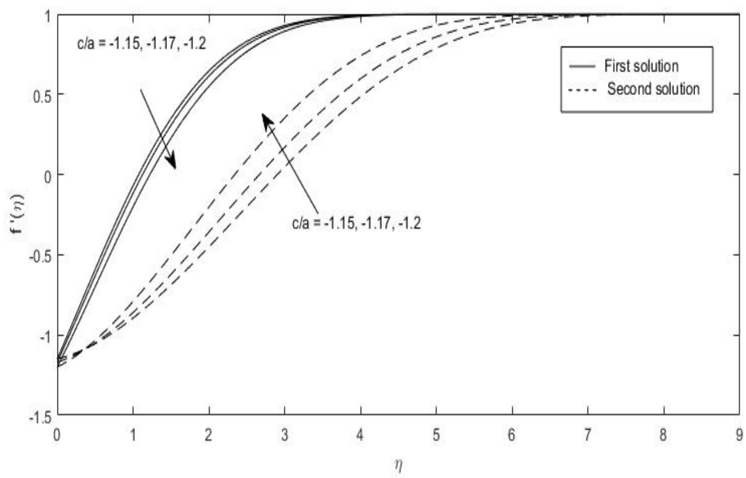

Fig. 3. Velocity profile $f'(\eta)$ for several values of c/a when $\lambda = \delta = f_w = 0, \beta = 0.2$ and $\Pr = 1$

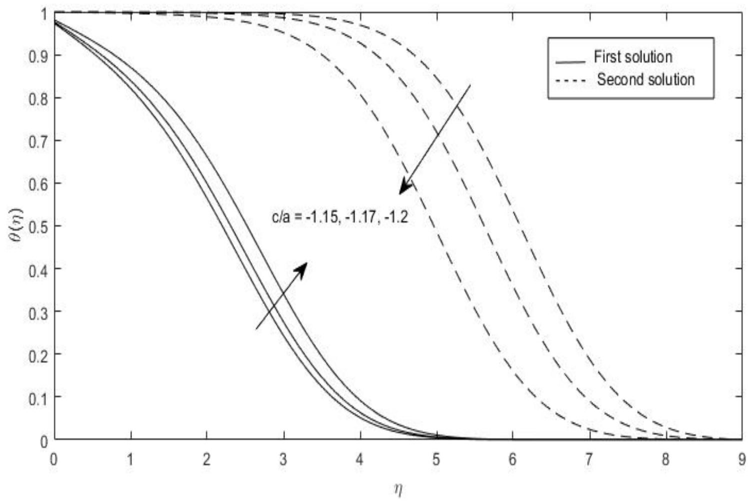

Fig. 4. Temperature profile $\theta(\eta)$ for several values of c/a when $\lambda = \delta = f_w = 0, \beta = 0.2$ and $\Pr = 1$

Table 2
Smallest eigenvalues γ for some values of δ and c/a

δ	c/a	Upper branch	Lower branch
0	-1.24	0.0240	-0.1209
	-1.20	0.2490	-0.1262
	-1.13	0.5339	-0.9583
0.1	-1.30	0.1447	-0.1317
	-1.28	0.2879	-0.1736
	-1.24	0.4974	-0.1819

4. Conclusions

The paper presents the numerical solutions and stability analysis of dual solution for the problem of stagnation-point slip flow passing a stretching or shrinking cylinder over a permeable surface. The numerical results and stability analysis were solved using bvp4c in Matlab. The first and second solutions exist up to a critical value c/a_c and no solution remains for $c/a < c/a_c$. A stability analysis has proven that there is an initial decay of disturbance for the first solution, while it is showed an initial growth of disturbance for second solutions, hence, the first solution is stable and thus physically reliable while the second solution is linearly in unstable state.

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