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Chaotic Convection in a Ferrofluid with Internal Heat Generation

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ABSTRACT

The nonlinear stability analysis of a ferrofluid layer system is formulated mathematically. This system considered the upper and lower free isothermal boundary with the system heated from below. A mathematical formulation is produced to study the behaviour of the chaotic convection in a ferrofluid layer system using Galerkin truncated expansion. The Boussinesq approximation is opted with the existence of internal heating and the magnetic number. It is found that the transition to chaos in this present study is identical to the Lorenz attractor and thus validate the method and analysis of this study. The impact of elevating the internal heat generation is found to hasten the instability of the system and as for the magnetic number, at M_1 = 2.5 the homoclinic bifurcation occurs and thus accelerates the convection process.

1. Introduction

The chaotic study has a vital role in both laboratory and nature. It gains much attention from the researcher because of the potential application in an engineering field. This topic started to evolve after the work of Lorenz [1], who discussed the Rayleigh-Benard model in the process of understanding the weather.

Much work of the chaotic convection in a different type of fluid was carried out before with various additional effects. Vincent and Yuen [2] studied high-Prandtl numbers in chaotic convection. Aside from this, Kiran *et al.,* [3] examined the impact of through flow in chaotic convection. Gupta *et al.,* [4] had studied the chaotic convection in the existence of a rotating effect. In the literature, there are a few examples of chaos in double-diffusive convection, such as by Abu-Zaid and Ahmadi [5] in the presence of noise and Narayana *et al.,* [6], who studied the external magnetic field in a viscoelastic fluid. Chaotic convection of a porous medium model in viscoelastic fluid had been carried out by Sheu *et al.,* [7]. Besides that, Abu-Ramadan *et al.,* [8] used a four-dimension nonlinear system

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to examine the chaotic convection in a viscoelastic fluid. Other than that, Bhadauria and Kiran [9] examined binary viscoelastic fluid in g-Jitter.

Walden *et al.,* [10] had done a chaotic convection study in a binary fluid layer system in the presence of traveling waves. In contrast, Deane [11] had done the studied in the thermosolutal convection. Chaotic convection in a porous medium had been reviewed by Sheu [12] by using the autonomous system. The learned of chaotic in a porous medium with an additional effect of the magnetic field had been presented by Idris and Hashim [13]. Roslan *et al.,* [14] added the effect of feedback control in a porous medium in the study of chaotic convection. Apart from that, Zhao *et al.,* [15] discussed the impact of gravity modulation in chaotic convection of a porous medium while Bhadauria and Kiran [16] added temperature modulation in the system. Bhadauria and Singh [17] examined the model of an anisotropic porous medium in the presence of through flow effect and g-jitter in chaotic convection. Chaotic convection in an electroconductive fluid in the existence of rotating effect had been demonstrated by Kopp *et al.,* [18]. Recently, the impact of rotation and gravity modulation in chaotic convection had been examined by Kiran [19]. Chaotic convection in a ferrofluid layer system had been discussed by Laroze *et al.,* [20], just to present a few examples.

The studied of ferrofluid convection with various effect had been discussed by Senin *et al.,* [21] with additional effect of gravitational field in an anisotropic porous medium. The internal heating effect is a well-known effect that had been discussed in a different kind of convection. This effect is usually studied in Rayleigh-Benard or Marangoni-Benard convection. Recently, Mokhtar and Hamid [22] studied the Marangoni convection with internal heating effect and deformable surface. Furthermore, Marangoni convection in the presence of internal heating in a ferrofluid layer system had been done by Nanjundappa *et al.,* [23]. Chaotic convection in the existence of internal heating had been discussed by Jawdat and Hashim [24] in the porous medium. Bhadauria [25] also examined chaotic convection with internal heating in a viscoelastic system's porous medium. Another studied of chaotic convection with internal heating effect had been discussed by Kiran [26] with a vibrational effect in a porous medium.

The present work aims to study the route to the chaos of a ferrofluid layer system in internal heating. In order to control heat transfer, it is vital to understand the effect of internal heating on chaotic convection as it plays a significant role in the use of ferrofluid technology. This motivated us to mathematically contribute by creating a mathematical model which can boost the impact of internal heating in chaotic convection of a ferrofluid layer system. We presume that the system is heated from below and the upper-lower boundary is known to be a free isothermal boundary. Galerkin truncated expansion was used to deduce a three-dimensional system to be represented as a Lorenz like model. Detailed analysis of the effect of magnetic number and internal heating on the system has been studied in detail

2. Methodology

2.1 Problem Formulation

We considered a Boussinesq ferrofluid, which fills a horizontal layer of thickness, d with an imposed spatially magnetic field, H_0 in a vertical direction, as in Figure 1. The upper and lower boundaries are maintained at a constant temperature where $T(z=0)=T_0+\Delta T$ and $T(z=d)=T_0$. By referring to Laroze *et al.,* [20], the dimensionless equations can be written as

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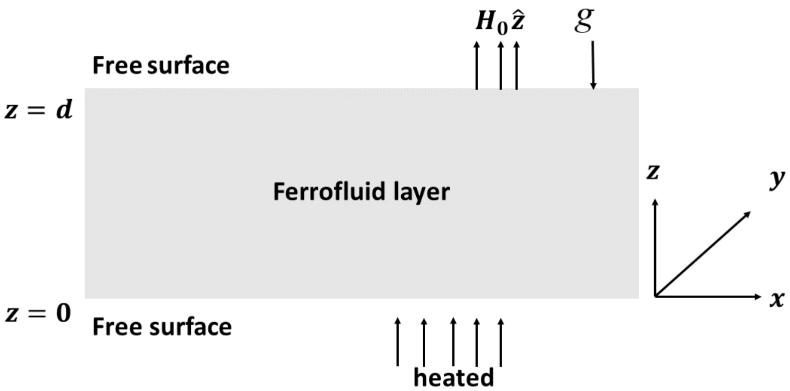


Fig. 1. The physical configuration of the ferrofluid layer system

$$\nabla\cdot\vec{V}=0 \tag{1}$$

$$\frac{1}{\Pr}\left(\frac{\partial}{\partial t}\vec{V}+\vec{V}\cdot\nabla\vec{V}\right)=-P+\nabla^2\vec{V}+\left[\left(Ra+RmF\right)T-RmF\frac{\partial\phi}{\partial z}\right]\hat{z}+RmF\nabla T\frac{\partial\phi}{\partial z}, \tag{2}$$

$$\frac{\partial T}{\partial t}+\vec{V}\cdot\nabla T=\nabla^2 T+w\cdot F+Ns\cdot T, \tag{3}$$

$$\frac{\partial T}{\partial z}=\left[\frac{\partial^2}{\partial z^2}+M_3\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\right]\phi, \tag{4}$$

where $\vec{V}=(u,v,w)$ are the velocity vector, T is the temperature, ϕ is the magnetic potential, Ns is the internal heating, $Ra=\frac{\alpha_t g\beta d^4}{\nu\kappa}$ is the Rayleigh number, $\Pr=\frac{\nu}{\kappa}$ is the Prandtl number, $Rm=Ra\cdot M_1$ is the magnetic Rayleigh number, $M_1=\frac{\mu_0 K^2\beta}{1+\chi}$ is the magnetic number, $M_3=\frac{1+M_0/H_0}{1+\chi}$ is the non-linearity of fluid magnetization, and $F=-\frac{\sqrt{Ns}\cos\sqrt{Ns}}{\sin\sqrt{Ns}}$ is the nondimensional basic temperature field. Laroze *et al.,* [20] stated that the suitable value for Pr is $10-10^3$ with M_1 from 10^{-4} till 10^2, and M_3 is only a weak function of the magnetic field.

The derivation of the partial differential equation started by applying the curl operator on Eq. (2) to eliminate the pressure. For simplification, the stream function is introduced to limit the study in two-dimensional flow. The stream functions defined by $u=-\frac{\partial\psi}{\partial z}$ and $w=\frac{\partial\psi}{\partial x}$ are substituted into Eqs. (2)-(4) and can be written as follows

$$\frac{1}{\Pr}\left[\frac{\partial}{\partial t}\nabla_+^2\psi+J(\psi,\nabla_+^2\psi)\right]-\left(Ra+Rm\cdot F\right)\frac{\partial T}{\partial x}+Rm\cdot F\frac{\partial^2\phi}{\partial z^2}-\frac{\partial T}{\partial z}\frac{\partial^2\phi}{\partial x\partial z}-\nabla_+^4\psi=0, \tag{5}$$

$$\frac{\partial T}{\partial t}+J\left(\psi,\theta\right)-\frac{\partial\psi}{\partial x}F-Ns\cdot T-\nabla_+^2=0, \tag{6}$$

$$\frac{\partial^2\phi}{\partial z^2}+M_3\frac{\partial^2\phi}{\partial z^2}-\frac{\partial T}{\partial z}=0, \tag{7}$$

where $\nabla_+^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}$ and J stands for the Jacobian. The boundary condition considered in the study are

$$T=\psi=\frac{\partial^2}{\partial z^2}=\frac{\partial\phi}{\partial z}=0 \text{ at } z=0,1. \tag{8}$$

In order to solve the nonlinear partial differential Eqs. (5)-(7), the stream function, temperature and magnetic potential are represent in the following form (Laroze *et al.,* [20])

$$k\psi(t,z,x)=-a_1(t)\sin(\pi z)\sin(kx), \tag{9}$$

$$T(t,z,x)=a_2(t)\sin(\pi z)\cos(kx)+a_3(t)\sin(2\pi z), \tag{10}$$

$$\phi(t,z,x)=a_4(t)\cos(\pi z)\cos(kx)+a_5(t)\cos(2\pi z), \tag{11}$$

where k is the wavenumber and $a_n,(n=1,2,3)$ is a constant coefficient with $a_4(t)=-\frac{\pi a_2(t)}{k^2M_3+\pi^2}$ and $a_5(t)=-\frac{a_3(t)}{2\pi}$. Substituting Eqs. (9)-(11) into Eqs. (5)-(7), then multiplying the equations by the orthogonal function and integrating them in space over the wavelength convection $\int_0^1\int_{\frac{\pi}{k}}^{-\frac{\pi}{k}} dxdz,$ it yields a set of the ordinary differential equation for the time evolution of the amplitudes as follows

$$\frac{d}{dt}a_1(t)=\Pr[-qa_1(t)-q^2R_y\alpha a_2(t)+q^2R_yM_{13}a_2(t)a_3(t)], \tag{12}$$

$$\frac{d}{dt}a_2(t)=\left(-q+Ns\right)a_2(t)-\pi a_1(t)a_3(t)+\frac{F}{2}a_1(t), \tag{13}$$

$$\frac{d}{dt}a_3(t)=\frac{\pi}{2}a_1(t)a_2(t)+\left(-4\pi^2+Ns\right)a_3(t), \tag{14}$$

where $\alpha=\frac{FM_1M_3k^2}{2\left(k^2M_3\left(1+M_1\right)+\pi^2\right)}+\frac{M_3k^2+\pi^2}{k^2\left(1+M_1\right)M_3+\pi^2},\qquad M_{13}=\frac{F\pi k^2M_1M_3}{2\left(\pi^2+k^2\left(1+M_1\right)M_3\right)},$ $q=\pi^2+k^2,\ R_y=\frac{Ra}{Ra_s}$ with $Ra_s=\frac{q^3\left(k^2M_3+\pi^2\right)}{k^2\left[k^2\left(1+M_1\right)M_3+\pi^2\right]}$ (Finlayson [27]). It is convenient to introduce new notation and to rescale the amplitude as follows

$$\tau = qt, \quad X(\tau) = \frac{\pi a_1 t}{q\sqrt{2}}, \quad Y(\tau) = -\frac{\pi R_y a_2(t)}{\sqrt{2}}, \quad Z(\tau) = -\pi R_y a_3(t). \tag{15}$$

Substitute Eq. (15) into Eqs. (12)-(14) and simplified, thus yields

$$X'(\tau) = -\Pr X(\tau) + \Pr \alpha Y(\tau) + Q_{13} Y(\tau) Z(\tau), \tag{16}$$

$$Y'(\tau) = R_y X(\tau)\frac{F}{2} - X(\tau)Z(\tau) - \beta Y(\tau), \tag{17}$$

$$Z'(\tau) = X(\tau)Y(\tau) - \gamma Z(\tau), \tag{18}$$

where $Q_{13} = \frac{\Pr M_{13}}{\pi R_y}$, $\beta = \frac{-Ns + q}{q}$, and $\gamma = \frac{4\pi^2 - Ns}{q}$. Eqs. (16)-(18) are equivalent to the Lorenz equation as stated in Lorenz [1] but with different coefficient and noted that when M_1 or M_3 approaching zero, it will cause the Q_{13} approaching zero as well.

2.2 Stability Analysis

The fixed points for the Eqs. (16)-(18) can be obtained by setting the derivative equal to zero, which are

$$-\Pr X(\tau) + \Pr \alpha Y(\tau) + Q_{13} Y(\tau) Z(\tau) = 0, \tag{19}$$

$$R_y X(\tau)\frac{F}{2} - X(\tau)Z(\tau) - \beta Y(\tau) = 0, \tag{20}$$

$$X(\tau)Y(\tau) - \gamma Z(\tau) = 0. \tag{21}$$

There is one trivial solution that is origin in the phase space, that are

$$X_1 = Y_1 = Z_1 = 0, \tag{22}$$

which correspond to the motionless solution, and the other two fixed points are

$$X_{2,3} = \pm\frac{1}{2}\frac{\sqrt{-2\Pr\left(K_1\left(\pm\sqrt{K_2}\right)\right)\gamma}}{\Pr},$$
$$Y_{2,3} = \pm\frac{R_y\sqrt{2}\sqrt{-\Pr K_1 \pm \sqrt{K_2}}}{\left(\frac{F}{2}\right)^2 Q_{13}R_y^{\ 2} + \frac{F}{2}\alpha\Pr R_y \pm K_2}, \tag{23}$$
$$Z_{2,3} = \pm\frac{K_1 \pm \sqrt{K_2}}{\left(\frac{F}{2}\right)^2 Q_{13}R_y^{\ 2} + \frac{F}{2}\alpha\Pr R_y \pm K_2},$$

with

$$K_1 = -\left(\frac{F}{2}\right)^2 Q_{13}R_y^{\ 2} - \frac{F}{2}\alpha\Pr R_y + 2\beta\Pr,$$
$$K_2 = \left(\frac{F}{2}\right)^4 Q_{13}^{\ 2}R_y^{\ 4} + 2\left(\frac{F}{2}\right)^3 \alpha Q_{13}\Pr R_y^{\ 3} + \left(\frac{F}{2}\right)^2 \alpha^2\Pr^2 R_y^{\ 2} - 4\beta\left(\frac{F}{2}\right)^2 Q_{13}\Pr R_y^{\ 2}. \tag{24}$$

The Jacobian matrix of the Eqs. (16)-(18) can be written as

$$J = \begin{bmatrix} -\Pr & \Pr\alpha + Q_{13}Z & Q_{13}Y \\ R_y\frac{F}{2} - Z & -\beta & -X \\ Y & X & -\gamma \end{bmatrix}. \tag{25}$$

The stability of the fixed points corresponds to the motionless solution $\left(X_1 = Y_1 = Z_1 = 0\right)$ is controlled by the zeros of the following characteristic polynomial equation for the eigenvalues, σ

$$\sigma^3 - \left(-\beta - \gamma - \Pr\right)\sigma^2 - \left(\frac{F}{2}\alpha\Pr R_y - \beta\gamma - \beta\Pr - \gamma\Pr\right)\sigma - R_y\frac{F}{2}\Pr\alpha\gamma + \beta\gamma\Pr = 0. \tag{26}$$

After solving Eq. (26), we will have

$$\sigma_1 = -\gamma, \tag{27}$$

$$\sigma_{2,3} = \frac{1}{2}\left(\beta + \Pr\right) \pm \frac{\sqrt{2F\alpha\Pr R_y + \beta^2 - 2\beta\Pr + \Pr^2}}{2}. \tag{28}$$

The first eigenvalue is always negative as $\gamma > 0,$ and the other two eigenvalues are always real. Eq. (28) is solved to obtain the critical value of R_y as follows

$$R_{cr} = \frac{2\beta}{F\alpha}. \tag{29}$$

At R_{cr}, the motionless solution loses stability, and the convection solution takes over. The following equation controls the stability of fixed points $\left(X_{2,3}, Y_{2,3}, Z_{2,3}\right)$ that is

$$\sigma^3 + L_2\sigma^2 + L_1\sigma + L_0 = 0, \tag{30}$$

where

$$\begin{aligned}
L_0 &= -\frac{F}{2}Q_{13}\gamma R_y Z_{2,3} - \frac{F}{2}Q_{13}R_y X_{2,3}Y_{2,3} - \beta Q_{13}Y_{2,3}{}^2 - \frac{F}{2}\alpha\gamma R_y + Q_{13}\gamma Z_{2,3}{}^2 + 2Q_{13}X_{2,3}Y_{2,3}Z_{2,3} + \\
&\quad \beta\gamma\,\mathrm{Pr} + \alpha\gamma Z_{2,3} + \alpha X_{2,3}Y_{2,3} + \mathrm{Pr}\,X_{2,3}{}^2, \\
L_1 &= \left(-\frac{F}{2}R_y Z_{2,3} - Y_{2,3}{}^2 + Z_{2,3}{}^2\right)Q_{13} + \alpha Z_{2,3} - \frac{F}{2}\alpha R_y + \left(\gamma + \mathrm{Pr}\right)\beta + \gamma\,\mathrm{Pr} + X_{2,3}{}^2, \\
L_2 &= \beta + \gamma + \mathrm{Pr}.
\end{aligned} \tag{31}$$

The stability of the stationary solution depends on the σ in Eq. (30). In the case of (30), the eigenvalues are much more complicated; thus, the analytical prediction is not possible, as in Laroze *et al.,* [20]. By referring to Lorenz [1], it is possible to obtain a critical equation if the eigenvalue is pure imaginary. Idris and Hashim [13] said that when the complex eigenvalues cross the imaginary axis, and Hopf bifurcation occurs, the eigenvalue is purely imaginary. This reflects the fixed point convection loss of stability or the critically modified Rayleigh number, R_{c2}. The Hopf bifurcation point is obtained with the continuation package of MatLab, MatCont. Eq. (30) has three eigenvalues, where the first eigenvalue is always real and negative for any value of the parameters. Simultaneously, the other two are complex eigenvalues, where the real part is negative at slightly higher R_y. As in Figure 2, these two roots are moving towards their origin and become equal at $R_y \approx 1.066$ for the case of $Ns = 5, M_1 = 1$, and $M_3 = 1.1$. As we increase R_y value, at Ry=17.315129 the imaginary and real part of the eigenvalue continue to increase and cross the imaginary axis, Hopf bifurcation occurs at that point and chaotic convection takes over.

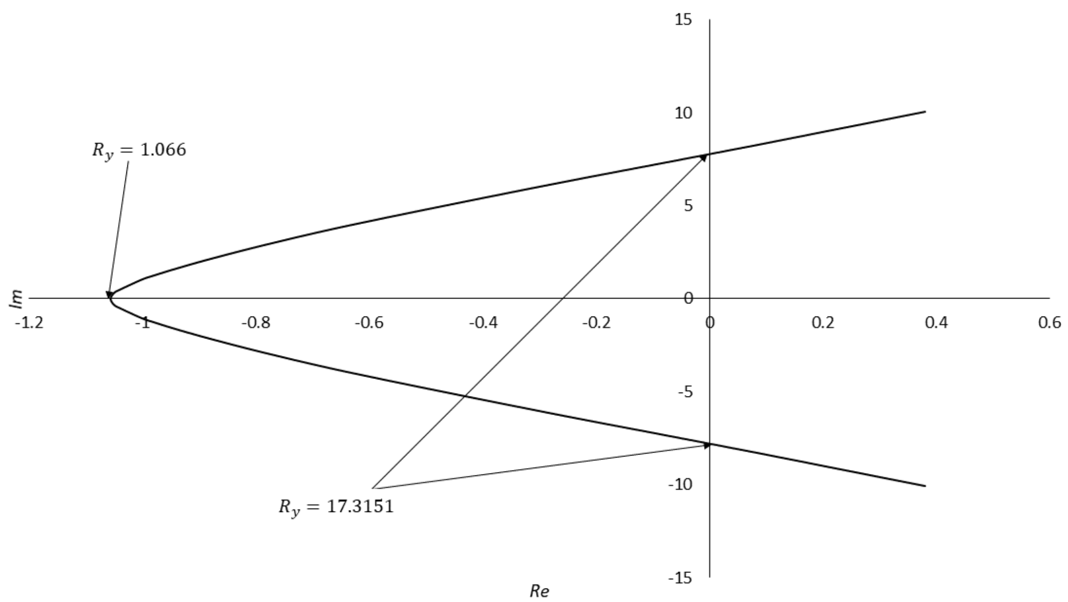


Fig. 2. The evolution of complex eigenvalue with the increasing of R_y

3. Results

In the previous section, we already obtained the set of Lorenz like model as in Eqs. (16)-(18) with the effect of internal heating in a ferrofluid layer system. Eqs. (16)-(18) are solved using the MATLAB R2018B built-in ODE45 method. Laroze *et al.,* [20] stated that the suitable value for Pr is $10 - 10^3$ with M_1 from 10^{-4} till 10^2, and M_3 is only a weak function of the magnetic field. The value of Ns is refered from the previous study of Nanjundappa *et al.,* [23]. The values of Pr and k used in all computations are 10 and $\frac{\pi}{\sqrt{2}}$, respectively. All solutions are obtained using the same initial conditions, which were selected to be in the neighborhood of the positive convection fixed points. The initial conditions are $X = Y = Z = 0.9$ with the time domain (τ) is taken from 0 to 210, and the step size $\Delta\tau = 0.001$. This section demonstrated the effect of internal heating and the magnetic number of the ferrofluid layer system in the projection of the trajectories onto the dimensional plane.

Table 1 presents the modified Rayleigh number, R_y, where the eigenvalue crosses the imaginary axis, and the convection fixed point loses its stability. The values of a magnetic number, M_1, are varied with constant values of $Ns = 5$ and $M_3 = 1.1$. As shown clearly in Table 1, an increase of M_1 decreased the value of the modified Rayleigh number and made the system become unstable. The decline of R_y values is also reported by Nanjundappa *et al.,* [28]. They stated that the increment of M_1 leads to an increase in the destabilized magnetic force that can cause the system's destabilization.

Table 1
Value of magnetic number and modified Rayleigh number when Hopf bifurcation occurs

M_1	R_y
1	17.315128
2	14.856765
3	13.526235
4	12.694626
5	12.126367
6	11.713749

Figure 3 shows the projection of the solution data point on the increasing Rayleigh number $0.780 < R_y < 18$ on the $Y - X$ plane with the value of $M_1 = 1, Ns = 5, M_3 = 1.1$. For a Rayleigh number slightly above the loss of stability of the motionless solution, which $R_{cr} = 0.78004$ in Figure 3(a), the trajectory moves to the steady-state convection points on a straight line. At $R_y = 4.743$, the trajectories approach the fixed point on a spiral as shown in Figure 3(b). Figure 3(c) shows the homoclinic bifurcation pattern by increasing the value of the modified Rayleigh number. By referring to Bhadauria [25], he stated that this bifurcation is known as global bifurcation, and the pattern could not be traced through a local stability analysis. By increasing the value of Rayleigh number, $R_y = 17.3151$ (as obtained in Table 1), the flow becomes complete chaos, as shown in Figure 3(d). The transition to chaos in this present study is similar to the Lorenz attractor, as in Lorenz [1].

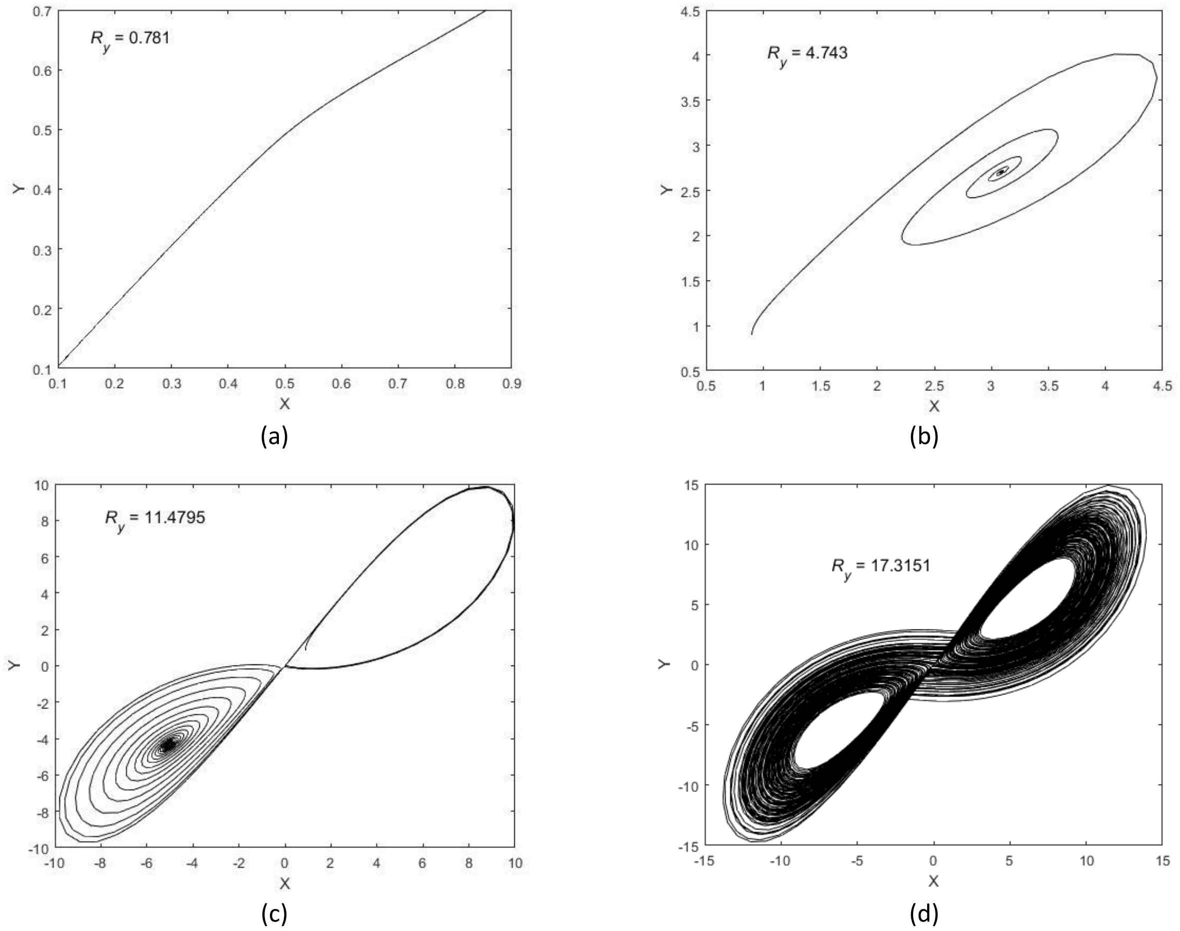
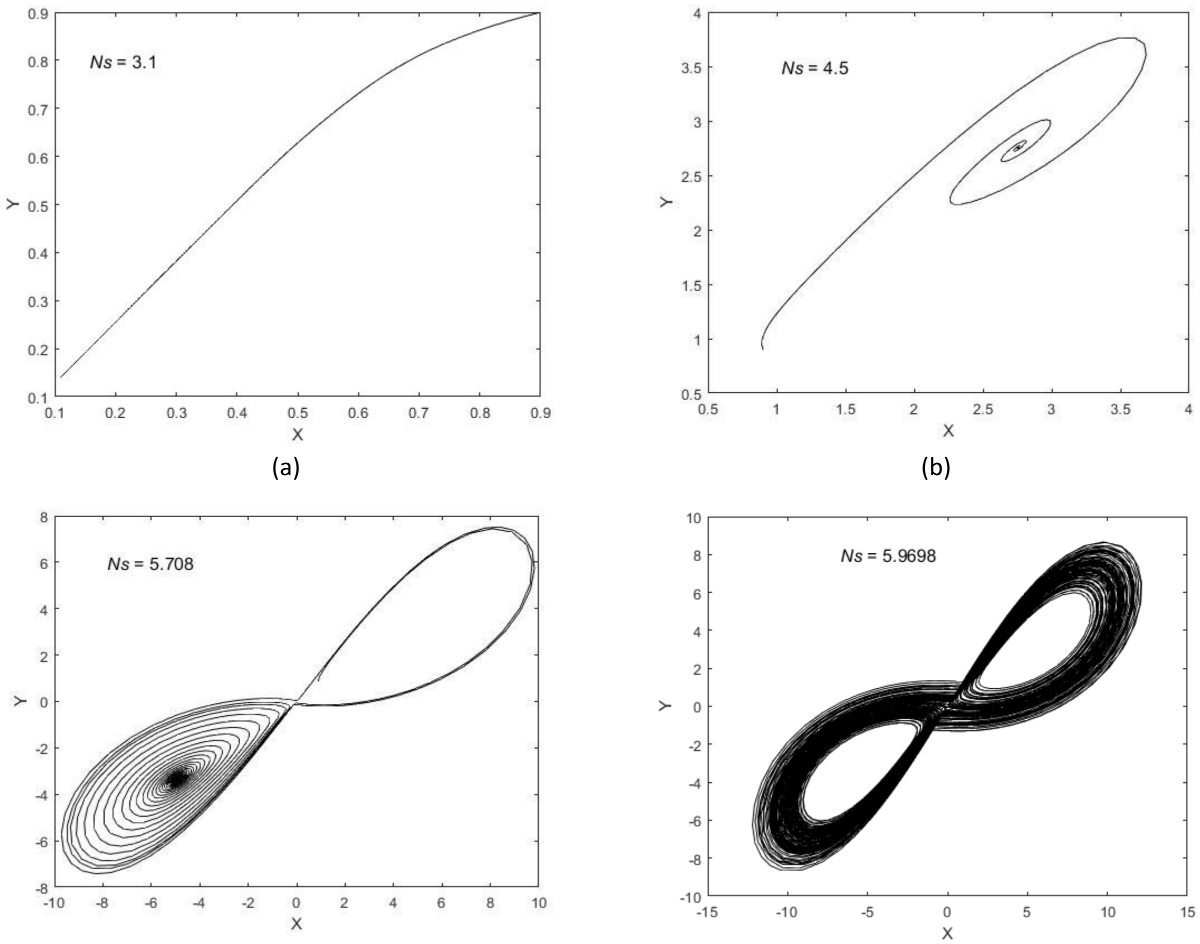


Fig. 3. The evolution of time in the state space for the various value of modified Rayleigh number in terms of R_y

The impact of internal heating on the onset of chaotic convection in the ferrofluid layer system can be seen in Figure 4. Ns values are varied while the other parameters are kept constant at $R_y = 6$, $M_1 = 1$, and $M_3 = 1.1$. Figure 4(a) shows that the trajectory moves towards the steady-state convection on a straight line for the value of internal heating recorded at $Ns = 3.1$. By increasing the effect of internal heating, the phase spiral trajectory exhibit at the value of $Ns = 4.5$, as presented in Figure 4(b). For $Ns = 5.708$, the homoclinic pattern of flow is seen to be appeared in Figure 4(c). In the case of strong internal heating $Ns = 5.9698$, the convection becomes complete chaos, and the fixed point lose their stability as presented in Figure 4(d) at the value of $R_y = 6$. This figure shows that the increase of Ns will enhance the chaos of the ferrofluid layer system. This scenario happened because of the increment of energy supply towards the system that disturbs and caused the destabilization of the convection (Khalid *et al.,* [29]).



(a) (b) (c) (d)

Fig. 4. The evolution of trajectories in the state space for the different values of Ns

Figure 5 demonstrates the effect of the magnetic number, M_1, on the chaotic convection in the existence of internal heating. For the lower value of the magnetic number, $M_1 = 1$, the trajectories move in the spiral phase approaching the steady-state for R_y=11. From Figure 5(b), the homoclinic bifurcation occurs for the $M_1 = 2.5$. Further increase of the magnetic value, M_1, caused the transition to chaos as presented in Figure 5(c) at a value of $M_1 = 8.7656$. As stated earlier, the increase of M_1 will reduce the value of R_y and making the ferrofluid layer system unstable.

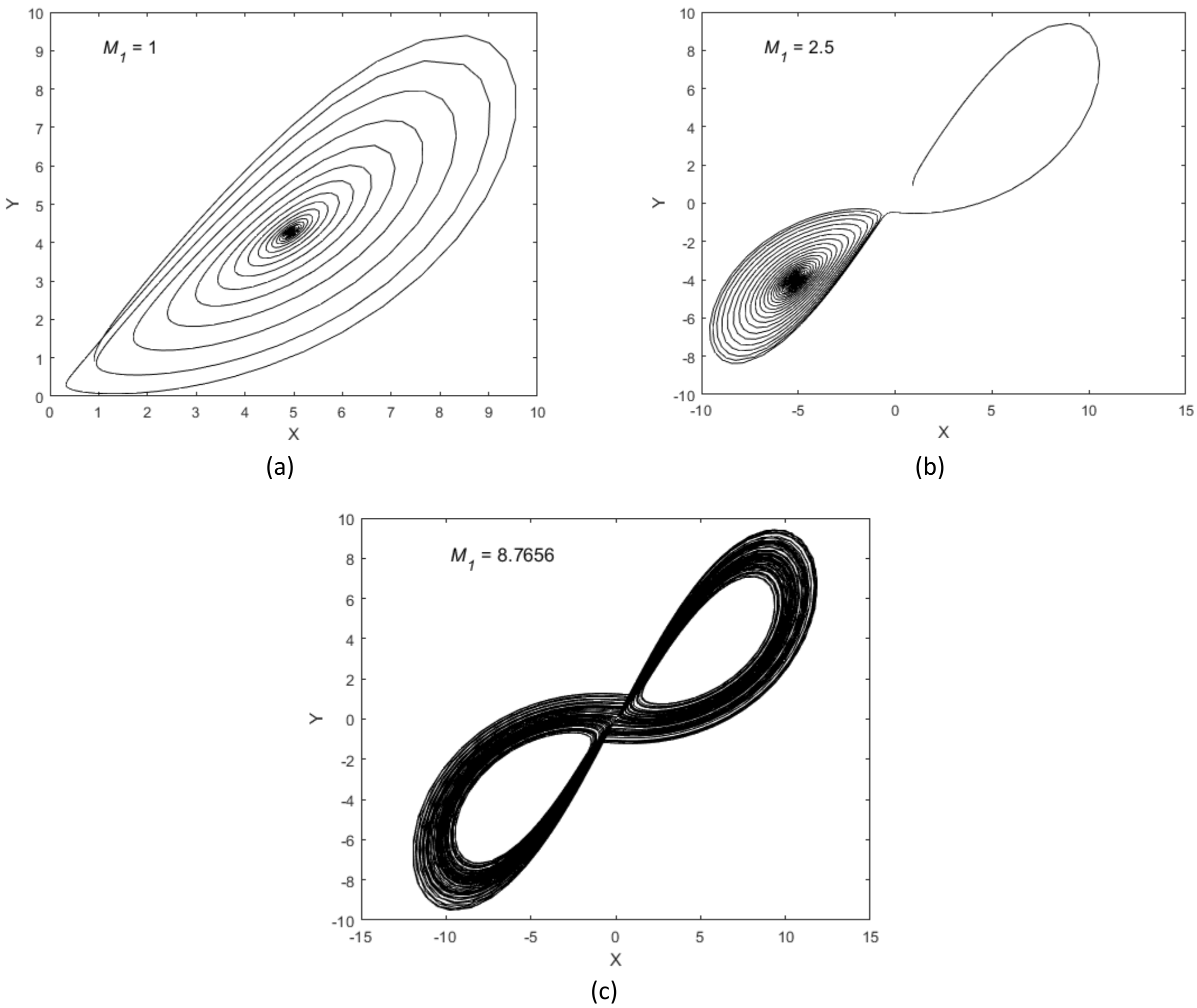


Fig. 5. The evolution of trajectories in the state space for the different values of M_1

4. Conclusions

In this paper, the chaotic convection in a ferrofluid layer system in the existence of internal heating is analyzed. The partial differential equation is deduced by using the Fourier series to obtain the Lorenz like model. Without the presence of magnetic, the classical Lorenz model is recovered. The effect of internal heating and magnetic number are investigated on dynamic convection. An increment of magnetic number and internal heating are found to enhance the chaotic convection, thus destabilizing the ferrofluid layer system. Whereas, the non-linearity of fluid magnetization does not affect the convection of the ferrofluid system.

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References
[1] Lorenz, Edward N. "Deterministic nonperiodic flow." *Journal of the atmospheric sciences* 20, no. 2 (1963): 130-141. https://doi.org/10.1175/1520-0469(1963)020<0130:DNF>2.0.CO;2
[2] Vincent, Alain P., and David A. Yuen. "Thermal attractor in chaotic convection with high-Prandtl-number fluids." *Physical Review A* 38, no. 1 (1988): 328.

https://doi.org/10.1103/PhysRevA.38.328
[3] Geethanjali, Palle Kiran1 K., and Y. Narasimhulu. "Chaotic Convection in the Presence of Throughflow." *International Journal of Pure and Applied Mathematics* 117, no. 11 (2017): 357-367.
[4] Gupta, Vinod K., B. S. Bhadauria, I. Hasim, J. Jawdat, and A. K. Singh. "Chaotic convection in a rotating fluid layer." *Alexandria Engineering Journal* 54, no. 4 (2015): 981-992. https://doi.org/10.1016/j.aej.2015.09.002
[5] Abu-Zaid, Sameer A., and Goodarz Ahmadi. "Chaos in a Double-Diffusive Convection Model in the Presence of Noise." *Applied Mathematical Modelling* 13 (1989): 291–97. https://doi.org/10.1016/0307-904X(89)90072-3
[6] Narayana, M, S N Gaikwad, P Sibanda, and R B Malge. "Double Diffusive Magneto-Convection in Viscoelastic Fluids." *International Journal of Heat and Mass Transfer* 67 (2013): 194–201. https://doi.org/10.1016/j.ijheatmasstransfer.2013.08.027
[7] Sheu, Long Jye, Lap Mou Tam, Juhn Horng Chen, Hsien Keng Chen, Kuang Tai Lin, and Yuan Kang. "Chaotic Convection of Viscoelastic Fluids in Porous Media." *Chaos, Solitons and Fractals* 37, no. 1 (2008): 113–24. https://doi.org/10.1016/j.chaos.2006.07.050
[8] Abu-Ramadan, Ehab, Jim M Hay, and Roger E Khayat. "Characterization of Chaotic Thermal Convection of Viscoelastic Fluids." *Journal Non-Newtonian Fluid Mech* 115 (2003): 79–113. https://doi.org/10.1016/j.jnnfm.2003.07.001
[9] Bhadauria, B. S., and Palle Kiran. "Chaotic and Oscillatory Magneto-Convection in a Binary Viscoelastic Fluid Under g-Jitter." *International Journal of Heat and Mass Transfer* 84 (2015): 610–24. https://doi.org/10.1016/j.ijheatmasstransfer.2014.12.032
[10] Walden, R.W, Paul Kolodner, A. Passner, and C.M Surko. "Travelling Waves and Chaos in Convection in Binary Fluid Mixtures." *Physical Review Letters* 55, no. 5 (1985): 496–99. https://doi.org/10.1103/PhysRevLett.55.496
[11] Deane, A.E. "Traveling Waves and Chaos in Thermosolutal Convection." *Physical Review A* 36, no. 6 (1987): 2862–69. https://doi.org/10.1103/PhysRevA.36.2862
[12] Sheu, Long-jye. "An Autonomous System for Chaotic Convection in a Porous Medium Using a Thermal Non-Equilibrium Model." *Chaos, Solitons and Fractals* 30 (2006): 672–89. https://doi.org/10.1016/j.chaos.2005.11.080
[13] Idris, R., and I. Hashim. "Effects of a Magnetic Field on Chaos for Low Prandtl Number Convection in Porous Media." *Nonlinear Dynamics* 62, no. 4 (2010): 905–17. https://doi.org/10.1007/s11071-010-9773-8
[14] Roslan, R., M. N. Mahmud, and I. Hashim. "Effects of Feedback Control on Chaotic Convection in Fluid-Saturated Porous Media." *International Journal of Heat and Mass Transfer* 54, no. 1–3 (2011): 404–12. https://doi.org/10.1016/j.ijheatmasstransfer.2010.09.031
[15] Zhao, Moli, Shaowei Wang, S. C. Li, Q. Y. Zhang, and U. S. Mahabaleshwar. "Chaotic Darcy-Brinkman Convection in a Fluid Saturated Porous Layer Subjected to Gravity Modulation." *Results in Physics* 9 (2018): 1468–80. https://doi.org/10.1016/j.rinp.2018.04.047
[16] Bhadauria, B. S., and Palle Kiran. "Chaotic and Oscillatory Magneto-Convection in a Binary Viscoelastic Fluid Under g-Jitter." *International Journal of Heat and Mass Transfer* 84 (2015): 610–24. https://doi.org/10.1016/j.ijheatmasstransfer.2014.12.032
[17] Bhadauria, B. S., and Ajay Singh. "Throughflow and G-Jitter Effects on Chaotic Convection in an Anisotropic Porous Medium." *Ain Shams Engineering Journal* 9, no. 4 (2018): 1999–2013. https://doi.org/10.1016/j.asej.2016.08.024
[18] Kopp, M. I., A. V. Tur, and V. V. Yanovsky. "Chaotic magnetoconvection in a non-uniformly rotating electroconductive fluids." *arXiv preprint arXiv:1805.11894* (2018).
[19] Kiran, Palle. "G-Jitter Effects on Chaotic Convection in a Rotating Fluid Layer." In *Advances in Condensed-Matter and Materials Physics-Rudimentary Research to Topical Technology*. IntechOpen, 2020. https://doi.org/10.5772/intechopen.90846
[20] Laroze, David, P. G. Siddheshwar, and Harald Pleiner. "Chaotic convection in a ferrofluid." *Communications in Nonlinear Science and Numerical Simulation* 18, no. 9 (2013): 2436-2447. https://doi.org/10.1016/j.cnsns.2013.01.016
[21] Nor Halawati Senin, Nor Fadzillah Mohd Mokhtar, and Mohamad HasanAbdul Sathar. "Ferroconvection in an Anisotropic Porous Medium with Variable Gravity." *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences* 71, no 2 (2020): 56-68.

https://doi.org/10.37934/arfmts.71.2.5668
[22] Mokhtar, N. F. M. and Nur Zarifah Abdul Hamid. "Influence of Internal Heating on Surface Tension Driven Convection in Deformable Binary Fluid Layer." In *AIP Conference Proceedings*, Vol. 1974, 2018. https://doi.org/10.1063/1.5041623
[23] Nanjundappa, C. E., Shivakumara, I. S. and Arunkumar, R. "Onset of Benard-Marangoni Ferroconvection with Internal Heat Generation." *Microgravity Science and Technology* 23, no. 1 (2011): 29–39. https://doi.org/10.1007/s12217-010-9218-5
[24] Jawdat, J. M., and I. Hashim. "Low Prandtl Number Chaotic Convection in Porous Media with Uniform Internal Heat Generation." *International Communications in Heat and Mass Transfer* 37, no. 6 (2010): 629–36. https://doi.org/10.1016/j.icheatmasstransfer.2010.03.011
[25] Bhadauria, B. S. "Chaotic Convection in a Viscoelastic Fluid Saturated Porous Medium with a Heat Source." *Journal of Applied Mathematics* 2016 (2016): 1–18. https://doi.org/10.1155/2016/1487616
[26] Palle Kiran. "Vibrational Effect on Internal Heated Porous Medium In The Presence Of Chaos." *International Journal of Petrochemical Science & Engineering Research* 4, no. 1 (2019): 13-23.
[27] Finlayson, B. A. "Convective Instability of Ferromagnetic Fluids." *Journal of Fluid Mechanics* 40, no. 4 (1970): 753–67. https://doi.org/10.1017/S0022112070000423
[28] Nanjundappa, C. E., I. S. Shivakumara, and R. Arunkumar. "Onset of Benard-Marangoni Ferroconvection with Internal Heat Generation." *Microgravity Science and Technology* 23, no. 1 (2011): 29–39. https://doi.org/10.1007/s12217-010-9218-5
[29] Khalid, I. K., N.F.M. Mokhtar, I. Hashim, Z.B. Ibrahim, and S. S.A. Gani. "Effect of Internal Heat Source on the Onset of Convection in a Nanofluid Layer with Feedback Control Strategy." *Advances in Mathematical Physics* 116, no. 1 (2017): 1827–32. https://doi.org/10.1155/2017/2789024