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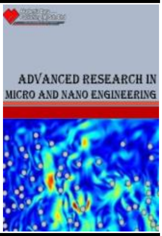

Natural Convection in Trapezoidal Cavity containing Hybrid Nanofluid

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ABSTRACT

In this paper, a numerical study of natural convection in a trapezoidal cavity with left heated wall, right cold wall and filled with $Cu - Al_2O_3$ and water hybrid nanofluid. The remaining walls of the cavity are kept at adiabatic. An enhancement in heat transfer is observed with the increase of three different parameters. Based upon the numerical predictions, the effect of Rayleigh number, inclination of the slopping wall, solid volume fraction of nanoparticles on flow and patterns as well tested. We found that acute slopping wall, high Rayleigh number and $Cu - Al_2O_3$ nanoparticles with high concentration are effective to enhance the rate of heat transfer. We also developed a new correlation for the average Nusselt number as a function of the angle, effective thermal conductivity and viscosity as well as Rayleigh number.

Keywords:
Hybrid nanofluid; natural convection; trapezoidal cavity; Rayleigh number

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1. Introduction

Numerical study of natural convection in a trapezoidal cavity with left heated wall, right cold wall and filled with $Cu - Al_2O_3$ and water hybrid nanofluid were studied. The remaining walls of the cavity are kept at adiabatic. An enhancement in heat transfer is observed with the increase of three different parameters. Based upon the numerical predictions, the effect of Rayleigh number, inclination of the slopping wall, solid volume fraction of nanoparticles on flow and patterns as well are tested. We found that acute slopping wall, high Rayleigh number and $Cu - Al_2O_3$ nanoparticles with high concentration are effective to enhance the rate of heat transfer. We also used a new correlation for the average Nusselt number as a function of the angle, effective thermal conductivity and viscosity as well as Rayleigh number. Natural convection is a sort of flow, or motion of a liquid such as water or a gas such as air in which the fluid motion is caused by certain sections of the fluid being heavier than others rather than by any external source such as a pump, fan or suction device. Because of the wide applications of natural convection in cooling industrial tools, this challenging topic has garnered the attention of many researchers. Natural convection has wide applications in

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the industry and technology such as growing crystals, cooling microchips, oil extraction, solar collectors and voltage increase transformers [1-5].

Natural convection problems of some water-based nanofluids flow in trapezoidal cavity were studied by Saleh *et al.,* [6], Aminossadati and Ghasemi [7] and Mahmoudi *et al.,* [8]. Hossain *et al.,* [9] analyzed MHD natural convection within trapezoidal cavity having circular block. Esfe *et al.,* [10] studied natural convection in a trapezoidal enclosure filled with carbon nanotube–EG–water nanofluid. Effect of magnetic field on natural convection flow in a prism shaped cavity filled with nanofluid was analyzed by Parvin and Akter [11]. Al-Weheibi *et al.,* [12] investigated unsteady natural convection flow and heat transfer inside a trapezoidal enclosure filled with nine different types of nanofluids having various shapes of the nanoparticle. Alhashash and Saleh [13] investigated natural convection induced by undulated surfaces in a porous cavity containing nanofluid. Later, Hossain *et al.,* [14] analyzed numerical simulation of MHD Natural convection flow within porous trapezoidal cavity with heated triangular obstacle. Recently, the idea of hybrid nanofluids has emerged, aiming to further enhance thermal properties which two or more different kinds of nanoparticles are dispersed into the base fluid. Rashad *et al.,* [15] studied numerically the convective heat transfer in a Triangular Cavity filled with hybrid nanofluid in the presence of magnetohydrodynamics and internal heat generation. Chamkha *et al.,* [16] studied the unsteady conjugate natural convection inside a semi-circular enclosure filled with hybrid nanofluid. They found that the use of hybrid nanofluid led to higher values of thermal conductivity and Rayleigh number. Asmadi *et al.,* [17] studied the effect of nanoparticle shape on natural convection heat transfer in U shape cavity containing hybrid nanofluid.

From the literature review, the study of natural convection with left heated wall, right cold wall and filled with $Cu - Al_2O_3$ and water hybrid nanofluid has not been studied yet. Hence, the present study examines the fluid flow and heat transfer rate in the cavity. We have chosen the aluminum oxide and copper nanoparticles in this work because copper and aluminum oxide are high in thermal conductivity and cheaper as compared to other types.

2. Mathematical Formulation

Consider a trapezoidal cavity model that involves a sliding bottom lid and a heat source at the left wall, T_h is demonstrated in Figure 1. The right wall surface is kept cold, T_c while the other walls are maintained adiabatic. The side walls' declination angle is set to 5°. This double lid-driven cavity is occupied with a base fluid of water, copper and alumina-nanoparticles. The left and right surfaces are moving upwards and downwards, respectively.

The following dimensional system of equations for hybrid nanofluid:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho_{hnf}}\frac{\partial p}{\partial x} + \frac{\mu_{hnf}}{\rho_{hnf}}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right), \tag{2}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \frac{1}{\rho_{hnf}}\frac{\partial p}{\partial x} + \frac{\mu_{hnf}}{\rho_{hnf}}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + \frac{(\rho\beta)_{hnf}}{\rho_{hnf}} g(T - T_c), \tag{3}$$

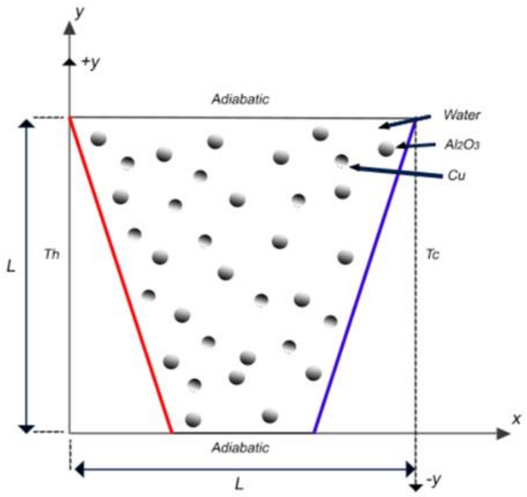


Fig. 1. Dynamic representation regarding convection into a trapezoidal cavity inducing a heat source and coordinate system

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y} = \alpha_{hnf}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right), \tag{4}$$

where u and v are velocity components in the x-direction and y-direction, respectively T is temperature, ρ is density, α is thermal diffusivity, β is thermal expansion coefficient and subscript hnf refers to hybrid nanofluid.

The boundary conditions

On left inclined wall: $u = 1, v = 0, T = Th, y = 1.$ (5)

On right inclined wall: $u = 0, v = 0, T = Tc, y = 0.$ (6)

On top and bottom walls: $u = v = T = 0.$ (7)

The thermal properties of base fluid and empirical correlation for hybrid nanofluid is given in Table 1 and the thermophysical properties of water and solid hybrid nanoparticles Alumina (Al_2O_3) and Copper (Cu) are stated in Table 2 (Ghalambaz *et al.,* [18]).

Consider the following parameters

$$(X,Y) = \frac{(x,y)}{L}, (U,V) = \frac{(uL,vL)}{\alpha_{hnf}}, P = \frac{pL^2}{\rho_f\alpha_{hnf}^2}, (T-T_c) = (T_h-T_c)\theta. \tag{8}$$

From Eq. 8, the governing Eq. (1) to (3) are reduced to the following set of dimensionless equation

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y} = 0, \tag{9}$$

Table 1
Thermophysical Properties Correlations

Properties	Hybrid Nanofluid correlations
Density	$\rho_{hnf} = \varphi_{Al_2O_3}\rho_{Al_2O_3} + \varphi_{Cu}\rho_{Cu} + (1-\varphi)\rho_f$, where $\varphi_{hnf} = \varphi_{Al_2O_3} + \varphi_{Cu}$
Heat Capacity	$(\rho C_p)_{hnf} = \varphi_{Al_2O_3}(\rho C_p)_{Al_2O_3} + \varphi_{Cu}(\rho C_p)_{Cu} + (1-\varphi)(\rho C_p)_f$
Dynamic Viscosity	$\mu_{hnf} = \mu_f(1 - Al_{Al_2O_3} - \varphi_{Cu})^{-2.5}$
Thermal diffusivity	$\alpha_{hnf} = \frac{k_{hnf}}{(\rho C_p)_{hnf}}$
Thermal Conductivity	$\frac{k_{hnf}}{k_f} = \frac{\left[\left(\frac{\varphi_{Al_2O_3}k_{Al_2O_3} + \varphi_{Cu}k_{Cu}}{\varphi}\right) + 2k_f + 2(\varphi_{Al_2O_3}k_{Al_2O_3} + \varphi_{Cu}k_{Cu}) - 2\varphi k_f\right]}{\left[\left(\frac{\varphi_{Al_2O_3}k_{Al_2O_3} + \varphi_{Cu}k_{Cu}}{\varphi}\right) + 2k_f - (\varphi_{Al_2O_3}k_{Al_2O_3} + \varphi_{Cu}k_{Cu}) + \varphi k_f\right]}$
Thermal expansion	$(\rho\beta)_{hnf} = \varphi_{Al_2O_3}(\rho\beta)_{Al_2O_3} + \varphi_{Cu}(\rho\beta)_{Cu} + (1-\varphi)(\rho\beta)_f$

Table 2
Thermophysical Properties for Hybrid Nanofluid

Properties	Water (f)	Alumina (Al_2O_3)	Copper (Cu)
$\rho(kg/m^3)$	997.1	3970	8933
C_p	4179	765	385
$k(W/mK)$	0.613	40	400
$\beta(1/K)$	21x10^2	0.85x10^{-5}	1.67x10^{-5}
Pr	6.2		
α	1.47x10^{-7}		

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + Pr\frac{\upsilon_{hnf}}{\upsilon_f}\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right), \tag{10}$$

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial X} + Pr\frac{\upsilon_{hnf}}{\upsilon_f}\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) + \frac{(\rho\beta)_{hnf}}{\rho_{hnf}\beta_f}PrRa\theta, \tag{11}$$

$$U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{\alpha_{hnf}}{\alpha_f}\left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right), \tag{12}$$

where $Pr = \frac{\mu_f(\rho C_p)_f}{\rho_f k_f}$ is the Prandtl number and $Ra = \frac{g(\rho\beta)_f(T_h - T_c)(\rho C_p)_f L^3}{\mu_f k_f}$ is the Rayleigh number.

The corresponding non-dimensional boundary conditions are

On left inclined wall: $U = 1, V = 0, \theta = 1, Y = 1.$ (13)

On right inclined wall: $U = V = 0, \theta = 0, Y = 0.$ (14)

On top and bottom walls: $U = V = 0,\ \theta = 0.$ (15)

The important physical quantity in this model is the average Nusselt number, Nu_{avg} along the left heated wall of the enclosure which is calculated from the following expression

$$Nu_{avg} = -\frac{k_{hnf}}{k_f}\frac{1}{S}\int_0^S \frac{\partial\theta}{\partial N} dN, \tag{16}$$

where $\frac{\partial\theta}{\partial N} = \sqrt{\left(\frac{\partial\theta}{\partial X}\right)^2 + \left(\frac{\partial\theta}{\partial Y}\right)^2}$ and S, N are the non-dimensional length and coordinate along the inclined heat surface respectively. If the left heated wall of the cavity is vertical and has the aspect ratio 1, the Eq. (16) can be written as

$$Nu_{avg} = -\frac{k_{hnf}}{k_f}\int_0^1 \frac{\partial\theta}{\partial X} dY. \tag{17}$$

3. Numerical Method and Validation

The finite element approach and the Galerkin weighted residual methodology are used to explore the control equations of Eq. (1)-(4) and Eq. (12) that are governed by the boundary conditions of Eq. (13)-(15). A Newton-Raphson iteration approach was used to simplify the nonlinear elements in the momentum equations. The result is permitted to converge if the relative error to any of the variables meets the resulting convergence criteria

$$\left|\frac{\Gamma^{m+1} - \Gamma^m}{\Gamma^{m+1}}\right| \leq 10^{-5}.$$

The governing Eq. (8) to Eq. (11) along with the associated boundary condition (12) is solved using the Galerkin weighted residual along with finite element method. Figure 2 displays the problem's finite element mesh. Five different finite element meshes have been considered and the value of average Nusselt number are carried out and presented in Table 3. Therefore, all the simulations have been carried out at mesh size extra fine with 16009 elements grid system. The iteration is reported until the normalized residual of the governing equations less than 10^{-6}.

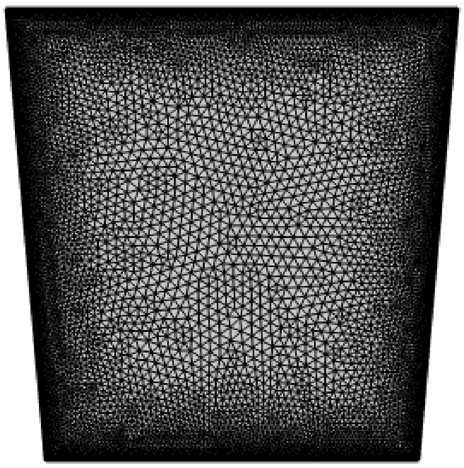


Fig. 2. Example of mesh generation

Table 3
Comparison of Nu_{avg} for Different Grid Resolution at $Ra = 10^4$, $\varphi_{hnf} = 0.01, \phi = 5°$ and Pr = 6.2.

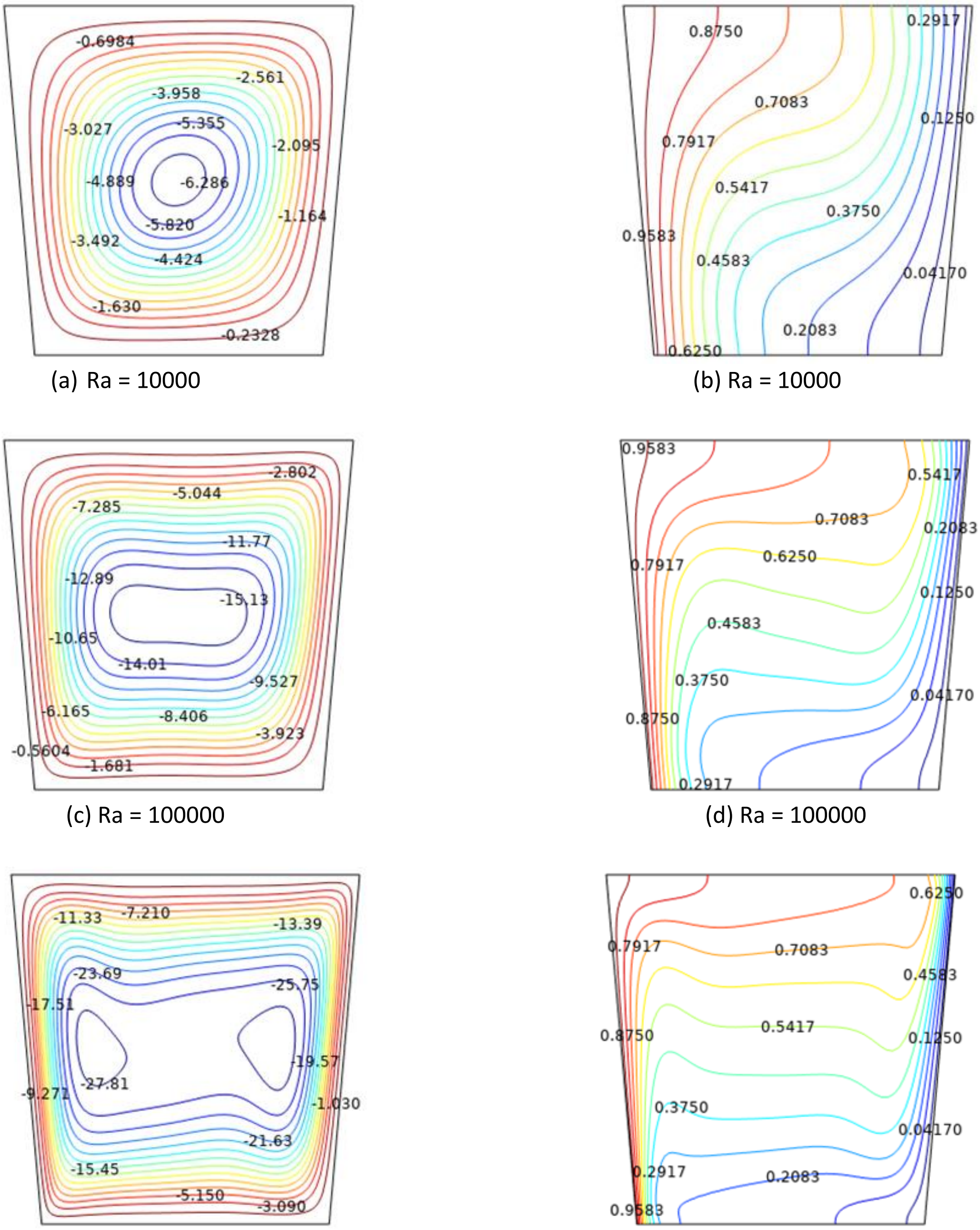
Predefined mesh size	Domain elements	Boundary elements	Nuavg	CPU time (s)
Normal	1173	111	3.3316	1
Fine	2024	138	3.3328	2
Finer	5661	297	3.3388	6
Extra fine	16009	573	3.3421	17
Extremely fine	24229	573	3.3422	21

4. Results and Discussion

In this section, the effects of key parameters on the fluid flow and temperature distribution within the trapezoidal cavity have been illustrated using streamlines, isotherms, average Nusselt number scatter charts. The cavity was filled with hybrid nanofluid containing copper and alumina nanoparticles of uniform size and concentration. Here, natural convection is resulted from heated left wall. The simulation has been performed for the variation of Rayleigh number ($10^4 - 10^6$), solid volume fraction (0%- 5%) and inclination angle of slopping wall ($5° - 25°$) at fixed Prandtl number (6.2).

Figure 3 depicts the influence of the Rayleigh number on isotherms and streamlines for $\phi = 5°$ and $\varphi_{hnf} = 0.01$ of hybrid nanofluid in the current configuration. The flow field inside the trapezoidal enclosure was formed by the difference in temperature between the hot left wall and the cold right wall. As a result, the flow vortices begin on the heated left wall due to its high temperature and move to the insulated bottom and top walls of the trapezoidal cavity due to the action of buoyancy force. Then, they shift their orientation from the insulated bottom to the top wall influenced by the cold right wall. This cyclic movement resulted in the convection vortex inside the enclosure. Density and temperature disparities between the cold right and heated left walls trapezoidal enclosure generate vortex that imitate natural convection currents. The streamline distributions also reveal that conduction is the most common mechanism of heat transfer. The buoyant force caused by the temperature differential between the hot surface and the cold fluid is insufficient, which signifies in terms of physics. As can be observed from this figure by raising $Ra(\geq 10^5)$, the convection mode became more noticeable than conduction. This indicates that the warmed fluid adjacent to both walls is well accelerated when buoyancy effect is strengthened due to the fact that the streamlines become more extremely compact at the left heated and right cold walls of the cavity. We further observe that as Ra increases, the streamlines' density inside the enclosure increases and the vortex transforms into an elliptic shape. An increase in Rayleigh number enhances thermal energy transfer from the hot surface to the fluid, forcing decelerating fluid particles to migrate away from the boundary layer**.**

The effectiveness of heat transfer in a fluid can be determined using isotherm contours, which also reveal whether conduction or convection is the dominant mechanism of heat transfer. Figure 4.1(b), (d) and (e) present isotherm contours for various values of the Rayleigh number ($Ra = 10^4 - 10^6$). From these figures, we observe that the isotherms are almost parallel to the bottom wall and dense close to the left and right walls of the cavity due to convection. The isotherms became parallel to the cavity's cold right wall with lower Rayleigh numbers. The isotherms are dispersed across the enclosure when the Rayleigh number is relatively higher. The density of the isotherms increased significantly close to the heated cavity wall as the Rayleigh number was increased as a result of the enhanced convection effects. At higher Ra value, adding more nanoparticles to be base fluid enhances convective heat transfer mechanisms.



(a) Ra = 10000 (b) Ra = 10000

(c) Ra = 100000 (d) Ra = 100000

(e) Ra = 1000000 (f) Ra = 1000000

Fig. 3. Rayleigh number effect on streamlines and isotherms respectively on the left and right with $\varphi_{hnf} = 0.01$ and $\phi = 5°$.

The variations of average Nusselt number, Nu_{avg} with respect to the solid volume fraction of hybrid nanofluid for different Rayleigh numbers at fix value of inclination angle of slopping wall, $\phi = 5°$ are presented in Figure 4. This figure demonstrates that for each value of Rayleigh number, the average Nusselt number increases with increasing φ_{hnf}. As expected with increasing φ_{hnf}, the viscosity increases more than thermal conductivity. Therefore, the flow strength is sufficient to generate stronger vortex and consequently higher natural convection heat transfer since the value the value of Rayleigh number is increasing.

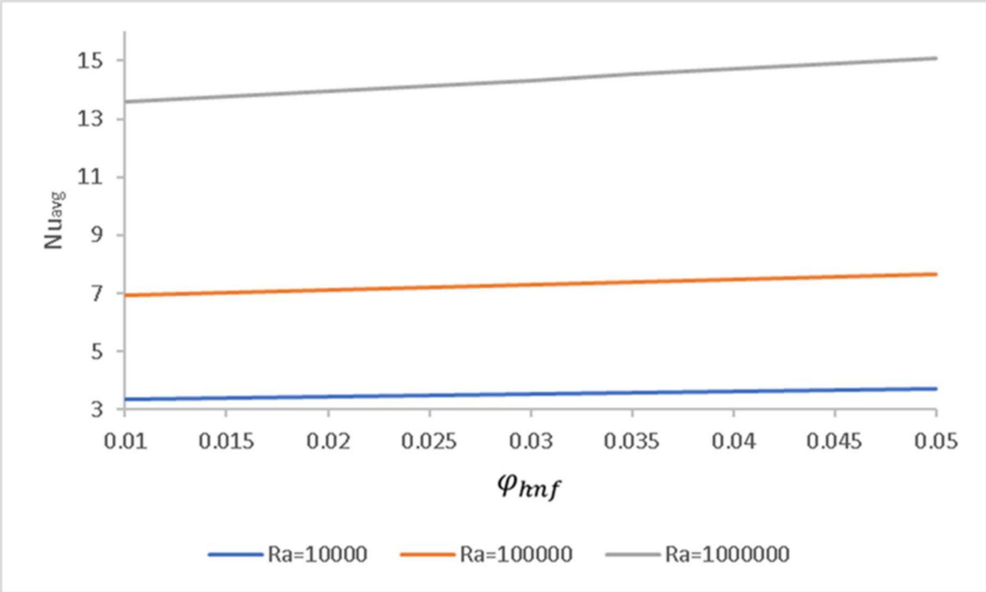
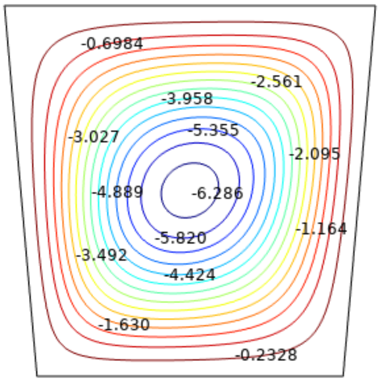
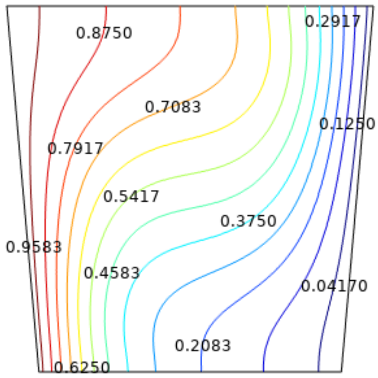


Fig. 4. Plots of the average Nusselt number, Nu_{avg}, against φ_{hnf} for the values of Ra labelled on the figure with $\phi = 5°$ for $Cu - Al_2O_3$ and water hybrid nanofluid.

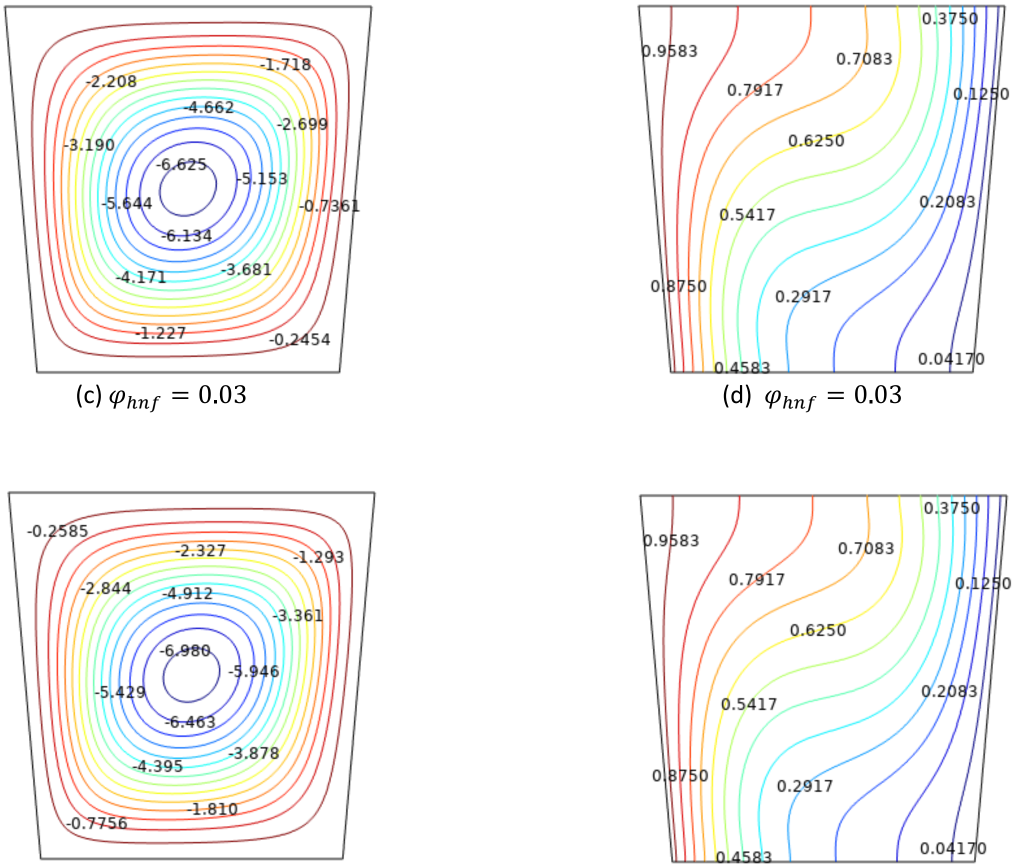
Figure 5 presents the streamline and the isotherms patterns for $Cu - Al_2O_3$ and water hybrid nanofluid with different solid volume fractions of the hybrid nanofluid which is $\varphi_{hnf} = 0.01$, 0.03 and 0.05. Numerical experiments show that the type and concentration level of nanofluids have no effect on the shape of the whole cavity. Increasing φ_{hnf} has the effect of enhancement both the flow strength and the average Nusselt number. This is because a greater concentration of solid nanoparticles results in a higher ratio thermal conductivity, which eventually leads to more energy, which speeds up the flow



(a) $\varphi_{hnf} = 0.01$



(b) $\varphi_{hnf} = 0.01$



(c) $\varphi_{hnf} = 0.03$ (d) $\varphi_{hnf} = 0.03$

(d) $\varphi_{hnf} = 0.05$ (e) $\varphi_{hnf} = 0.05$

Fig. 5. Solid volume fraction effect on streamlines and isotherms respectively on the left and right with $Ra = 10000$ and $\phi = 5°$.

Figure 6 demonstrates a relationship between the solid volume fraction of the nanoparticles, φ_{hnf} ,the average Nusselt number, Nu_{avg} and the Rayleigh number, Ra for the $Cu - Al_2O_3$ and water hybrid nanofluid on an angle of the slopping wall, ϕ = 5°. Heat transfer performance improves as φ_{hnf} increases. At a high Ra value, the enhancement is more obvious. When the buoyancy force is strong, this feature related to solid concentration of nanoparticles has a significant influence. We discover that Nu_{avg} intensifies as φ_{hnf} increases. The reason for this is because when the base fluid's nanoparticle concentration is higher, the fluid's energy exchange rates rise as well because of the nanoparticles' irregular and random movements. Thus, heat transfer performance is enhanced with the increase of φ_{hnf} as well as with the Rayleigh number, Ra.

The effects of sidewall inclination angle, ϕ on the flow field and temperature distributions are shown in Figure 7 with $Ra = 10000$ and $\varphi_{hnf} = 0.03$. The impact of different inclination angles on the flow pattern is seen in Figure 7 (a), (c) and (e). These illustrations demonstrate that a major cell fills the whole enclosure for all inclination angles, with one inner cell whose size decreases with increasing inclination. As a result, one detachment zone can be seen in the top-right corner when the flow separates from the walls at large values of inclination angle like $\phi = 25°$. It's interesting to note that due to the minimal difference at low inclination degrees, this phenomenon eventually disappears as the inclination angle lowers.

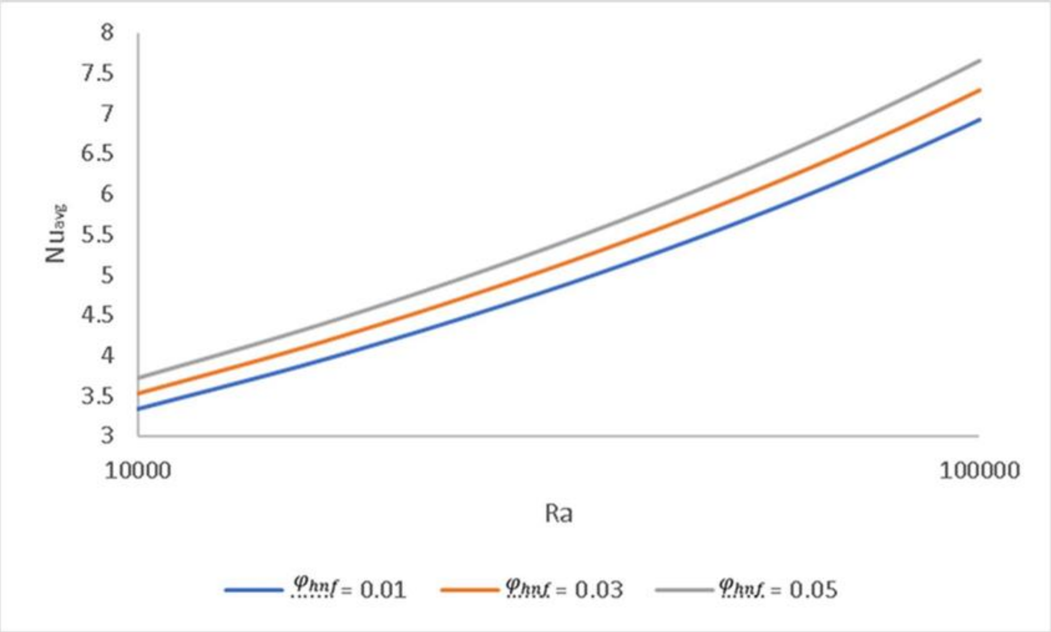
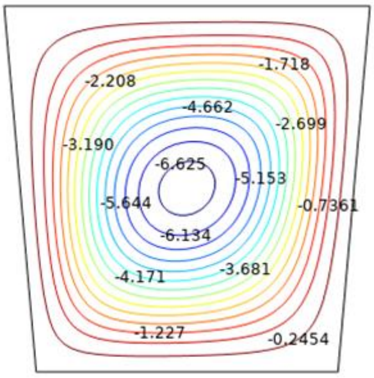
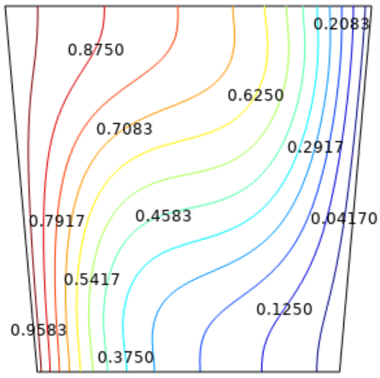


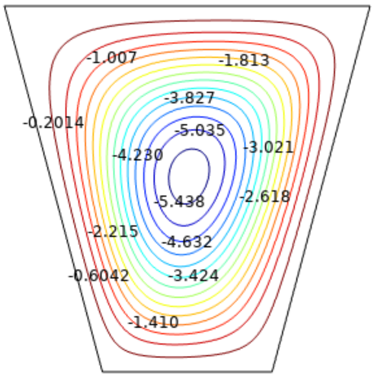
Fig. 6. Plots of the average Nusselt number, Nu_{avg}, against Ra for the values of φ_{hnf} labelled on the figure with $\phi = 5°$ for $Cu - Al_2O_3$ and water hybrid nanofluid.



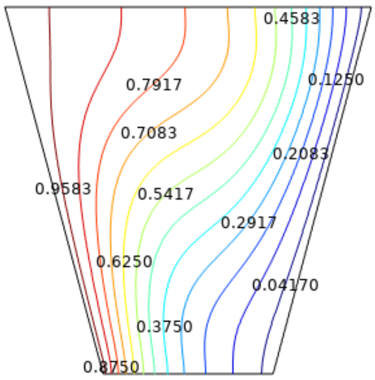
(a) $\phi = 5°$



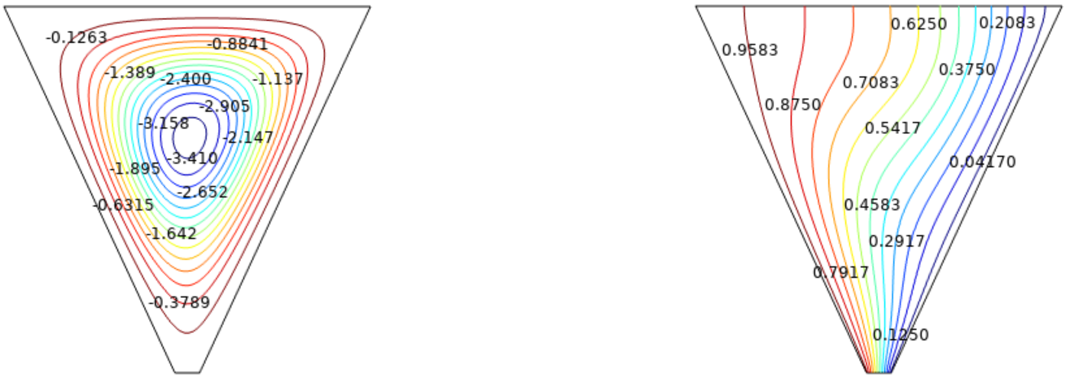
(b) $\phi = 5°$



(c) $\phi = 15°$



(d) $\phi = 15°$



(e) $\phi = 25°$ (f) $\phi = 25°$

Fig. 7. Inclination angle of slopping wall effect on streamlines and isotherms respectively on the left and right with $Ra = 10000$ and $\varphi_{hnf} = 0.03$.

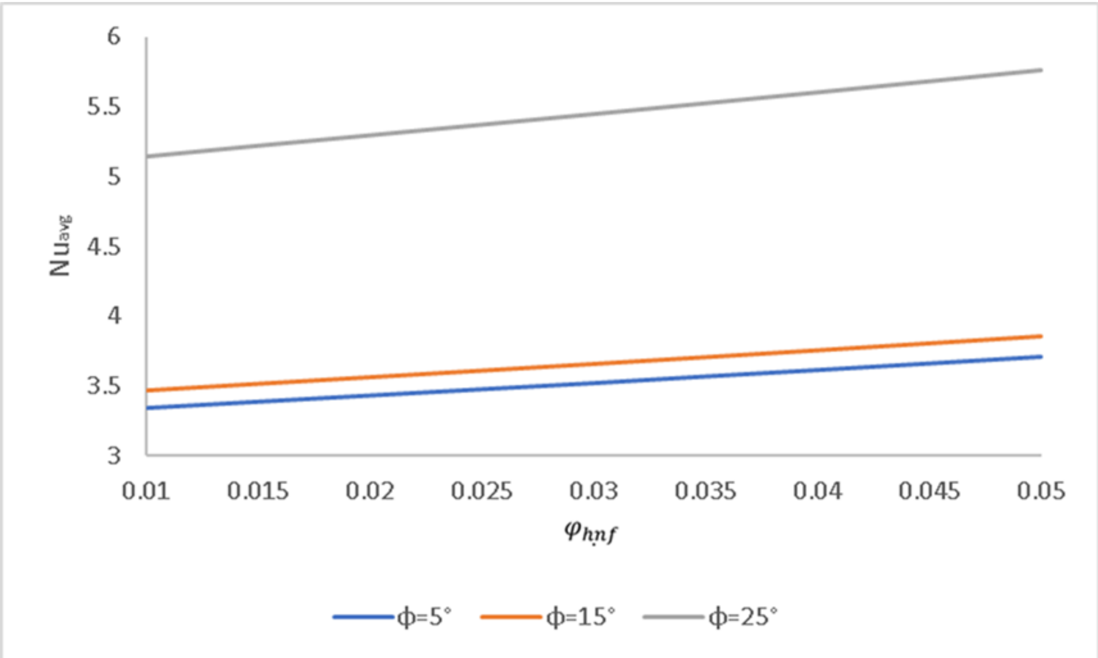


Fig. 8. Plots of the average Nusselt number, Nu_{avg}, against φ_{hnf} for the values of ϕ with $Ra = 10000$ for $Cu - Al_2O_3$ and water hybrid nanofluid

Figure 8 demonstrates a relationship between the different inclination angle of slopping wall, ϕ, the average Nusselt number, Nu_{avg} and the solid volume fraction, φ_{hnf} for the $Cu - Al_2O_3$ and water hybrid nanofluid on a fix Rayleigh number, $Ra = 10000$. As seen from the figure, the average Nusselt number, Nu_{avg} increases with increasing inclination angle due to decreasing distance between hot and cold walls. It causes buoyancy force increasing, so heat transfer inside trapezoidal cavity also increases.

5. Conclusions

In this research, we investigated the effect of hybrid nanofluid in a trapezoidal cavity for the heat transfer numerically. We also constructed mathematical modelling for the problem natural convection of hybrid nanofluid in trapezoidal cavity with adiabatic walls between hot and cold temperature. The numerical solutions for various parameters from the related formulation research were derived to get the value of average Nusselt number. Therefore, we can summarize that

i. The average Nusselt number, Nu_{avg} is increasing as the Rayleigh number parameter, Ra increases.
ii. The average Nusselt number, Nu_{avg} shows an increasing behaviour alongside the solid volume fraction parameter, φ_{hnf} increases.
iii. The average Nusselt number, Nu_{avg} increases when the inclination angle of slopping wall parameter, ϕ increases.

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