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Carbon Nanotubes (CNTs) Nanofluids Flow and Heat Transfer under MHD Effect over a Moving Surface

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ABSTRACT

The 2D steady flow of CNTs nanofluids and heat transfer over the moving plate through a uniform free stream under the effect of magnetohydrodynamics (MHD) is studied. The movement of plate is presumed in the opposite or same direction. A mathematical modelling that is governed by a set of partial differential equations (PDEs) system subjected to boundary conditions is transformed into a system of nonlinear ordinary equations (ODEs). An attempt at finding the expected outcomes is successfully executed by solving ODEs system using MATLAB bvp4c solver. The effect of various parameters such as magnetic field, CNTs volume friction and moving parameter on velocity and temperature profile, skin friction, Nusselt number and heat transfer rate are investigated numerically. The results are illustrated using graphical approach. From the results, the increment of magnetic field into the flow will increase in both the skin friction and the heat transfer coefficient. Besides that, non-unique solutions are obtained when the plate move in opposite direction.

Keywords:
CNTs Nanofluids; Heat Transfer; Moving Plate; MHD

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1. Introduction

Heat transfer has been an important topic of debate in thermal engineering. Demand for enhanced thermal system has increased significantly over two decades and has generated many ideas including the use of nanofluids. The term nanofluids refers to the combination of any nanoscale materials in a base fluid [1]. Nanofluids are employed in various thermal applications such as the automotive industry [2]. Providing high thermal conductivity is key to the synthesising of high impact nanofluids. Carbon materials (such as carbon nanotubes (CNTs)), metal materials (such as copper (Cu)), semiconductor material (such as gallium nitride (GaN)) and polymer materials (such as polylactic acid (PLA)) have become favourable selection for preparing nanofluids [3]. Among them, CNTs nanofluids are considered as the best nanofluids as they have the highest thermal conductivity compared to other nanoparticles [4]. Jiang *et al.,* [5] suggested that CNT becomes one of the markedly effective materials as it provides higher thermal conductivity of nanofluids compared to other nanoparticles.

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CNTs consist of carbon nanoparticles with common range in diameter from 0.5nm to 1.5nm and above 100nm for single-walled carbon nanotubes (SWCNTs) and multi-walled carbon nanotubes (MWCNTs), respectively. Basically, CNTs are constructed and rolled into a cylindrical shape by using one-dimensional graphene structures [6]. To produce CNTs nanofluids, CNTs are immersed in a base fluid such as water, oil and ethylene glycol. The usage of CNTs is to meet the demand for various applications including bio sensors (mostly on electrochemical) [7–9], medical applications [10] and nanogenerators technology [11]. Increasing demand for CNTs with forecasting to elevate around 2,215.5 million by 2032 [12] is due to guarantee for providing high quantity of thermal conductivity, ranging from 0.1 W/mK to 6600 W/mK [13] and high current carrying capability [14]. The thermal conductivity acts a vital role in enhancing on the heat transfer coefficient by increasing the rate of heat transfer [15]. Besides that, CNTs have worked perfectly in nanotechnology by improving the performance of heat transfer with a great thermal and optical properties [16]. An addition, the application of nanomaterials has been involving with the low-cost production and making Ibrahim *et al.,* [17] classified it as an ideal working fluid. Ibrahim *et al.,* [17] concluded that nanofluids has been working at low-cost production but having a supreme property of specific heat and lower viscosity compare to the conventional fluids. These reasons have sufficiently caused many researchers such as Danish *et al.,* [14] to nominate CNTs as the best future transmission network of power system. CNTs have been getting attention from many researchers for over three decades since the first discovery of CNTs by Iijima in 1991 [18] for MWCNTs and followed by SWNCTs in 1993 [19]. Historically, Ferrier and Honeychurch [20] stated that the earliest development of CNTs was initiated by Radushkevich and Lukyanovich and in 1976 by Oberlin *et al.,* Based on [20], six methods can be performed for synthesising CNTs which are arc discharge, laser ablation, chemical vapour deposition, plasma enhanced, liquid electrolysis, and controlled flame environments.

The investigation of boundary layer flow and heat transfer of CNTs have been conducted by many researchers using different physical parameters and surfaces. Khan *et al.,* [21] was believed to be the first researcher to discover the fluid flow and heat transfer of CNTs with presence of Navier slip effect and invariable heat flux over the fixed plate. By taking into account of different parameters such as skin friction and Nusselt numbers they [21] found that a positively agreement of skin friction and Nusselt numbers with the available data. For the fluid flow over a moving surface, Norzawary *et al.,* [22] became one of first researchers to extend the study of boundary layer flow of CNTs passing this surface. To govern the model, Norzawary *et al.,* [22] inspired the work from Bachok *et al.,* [23]. From this analysis, a dual solution was found when the plate moved reversely to the free stream. In the term of skin friction coefficient and local Nusselt numbers, they also found that MWCNTs were less efficient than SWCNTs.

Magnetohydrodynamic (MHD) can represent as magneto-fluid dynamics where it can be used to investigate on electrically and conductivity accompanying fluids in magnetic field [24]. Magnetic field will create a positive response to the local skin friction where the local skin friction coefficient will rise significantly in fluid. Based on the experiment work completed by Masaaki *et al.,* [25], the magnetic field will give a positive impact to the coefficient of heat transfer. The heat transfer coefficient will increase when the magnetic field rises in fluid. Ferdows *et al.,* [26] shared the same view as Masaaki *et al.,* by concluding that the magnetic field can give the positive impact to the rate of heat transfer and temperature.

The boundary layer flow and heat transfer of CNTs nanofluids with the presence of MHD has been reviewed experimentally and theoretically by many investigators such as [27]. Mittal *et al.,* studied MHD flow with Cu-water nanofluid. Besides that, the numerical analysis on CNTs nanofluids flow in the existence of magnetic field have studied for various surfaces such as [28] for vertical plate, moving disk and stretching/shrinking sheet respectively. However, to date, we have found that no research has been conducted on exploring the boundary layer flow of CNTs past a moving surface in the presence of MHD. This gap allows us to extend the work from [22] and [23] to study the effect of MHD in CNTs nanofluids flow past the moving surface.

2. Mathematical Formulation

This model deals with two dimensional (2D) steady flow past a moving plate in the presence of magnetic field. This flow is categorised as incompressible and laminar flow that passes SWCNTs and MWCNTs. Water and kerosene are selected to be the base fluid. The thermal properties of CNTs and two types of base fluid are depicted in Table 1.

Table 1
Thermophysical properties of CNTs and base fluids [21]

Physical Properties	Nanoparticles		Base Fluid	
	SWCNTs	MWCNTs	Water	Kerosene
ρ (kg/m^3)	2,600	1,600	997	783
C_p (J/kg K)	425	796	4,179	2,090
k (W/m K)	6,600	3,000	0.613	0.145

By extending and modifying the works from [15] and [22], the boundary layer equations for this model can be constructed a

$$\frac{\partial(u)}{\partial x}+\frac{\partial(v)}{\partial y}=0, \tag{1}$$

$$u\frac{\partial(u)}{\partial x}+v\frac{\partial(v)}{\partial y}=\frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2(u)}{\partial y^2}+\frac{\sigma_{nf}}{\rho_{nf}}B^2(U-u), \tag{2}$$

$$u\frac{\partial(T)}{\partial x}+v\frac{\partial(T)}{\partial y}=\sigma_{nf}\frac{\partial^2(T)}{\partial y^2}. \tag{3}$$

The complete boundary conditions can be written as follows:

$$u=U_w=\lambda U, v=0, T=T_w \text{ at } y=0, u\rightarrow U_\infty, T\rightarrow T\infty, \text{as } y\rightarrow\infty. \tag{4}$$

The velocity components are represented by u and v where these components move along the coordinate cartesian (x, y) directions. The x–axis is determined along the plate and the y–axis is normal to the x–axis. The term of $\mu_{\mathrm{nf}}/\rho_{\mathrm{nf}}$ can be set as $\upsilon_{\mathrm{nf}}=\mu_{\mathrm{nf}}/\rho_{\mathrm{nf}}$, where υ_{nf} is the effective kinematic viscosity. The uniform velocity of free stream flow U can be written as $U=U_{\mathrm{w}}+U_\infty$. T represents the temperature of nanofluids and B is the magnetic field applied to the fluid flow such that $B=\beta_0/2x$. T_{w} defines as the constant value of temperature and T_∞ is assigned as the temperature in the ambient fluid. The term of λ and σ is the velocity parameter and the slip parameter, respectively. In this model, the subscripts f, nf and CNT can be referred as fluid, nanofluid and carbon nanotube, respectively. The thermal correlation of nanofluids for viscosity, μ_{nf}, density, ρ_{nf} and thermal diffusivity, α_{nf} can be written as follows:

$$\mu_{nf} = \frac{\mu_f}{(1-\varphi)^{(2.5)}}, \rho_{nf} = (1-\varphi)\rho_f + \varphi\rho_{CNT}, \alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}, \tag{5}$$

where φ is the nanoparticle volume fraction, $(\rho C_p)_{nf}$ is the heat capacity of nanofluids and k_{nf} is the thermal conductivity of nanofluids. From [29], $(\rho C_p)_{nf}$ and k_{nf} can be illustrated as:

$$k_{nf} = k_f \left(\frac{1-\varphi+2\varphi \frac{k_{CNT}}{k_{CNT}-k_f} \ln \frac{k_{CNT}+k_f}{2k_f}}{1-\varphi+2\varphi \frac{k_f}{k_{CNT}-k_f} \ln \frac{k_{CNT}+k_f}{2k_f}} \right), (\rho C_p)_{nf} = (1-\varphi)(\rho C_p)_f + \varphi(\rho C_p)_{CNT}, \tag{6}$$

where k_f and k_{CNT} are the thermal conductivity of fluids and carbon nanotubes, respectively, and $(\rho C_p)_f$ and $(\rho C_p)_{CNT}$ are the heat capacity of fluids and carbon nanotubes, respectively. The term of the kinematic viscosity of nanofluids, k_{nf} in Eq. (6) can be arranged as

$$\frac{k_{nf}}{k_f} = \left(\frac{1-\varphi+2\varphi \frac{k_{CNT}}{k_{CNT}-k_f} \ln \frac{k_{CNT}+k_f}{2k_f}}{1-\varphi+2\varphi \frac{k_f}{k_{CNT}-k_f} \ln \frac{k_{CNT}+k_f}{2k_f}} \right)$$

where $\frac{knf}{kf}$ is describing the effect of space distribution of CNTs on the thermal conductivity [30].

To reduce the partial differential equations (PDEs) Eq. (1 - 3) to the ordinary equations (ODEs), we will use the similarity variables as follows:

$$\eta = y\sqrt{\frac{U}{v_f x}}, \psi = \sqrt{v_f x U} f(\eta), \theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty}, \tag{7}$$

where ψ represents the stream function with $u = \frac{\partial\varphi}{\partial y}$ and $v = \frac{\partial\varphi}{\partial x}$

Utilizing the similarity variables (from Eq. (7)) to transform PDEs (Eq. (2) and (3) into ODEs with respect to η in the following form:

$$\frac{1}{(1-\varphi)^{(2.5)} + \left(1-\varphi+\frac{\varphi\rho_{CNT}}{\rho_f}\right)} f''' + \frac{1}{2} f f'' + M(1-f') = 0, \tag{8}$$

$$\frac{1}{\mathrm{P_r}} \frac{1}{\left[1-\varphi+\frac{\varphi(\rho Cp)_{CNT}}{(\rho Cp)_f}\right]} \theta'' + \frac{1}{2} f\theta' = 0, \tag{9}$$

where

$$M = U\frac{\sigma B^2}{\rho_{nf}}$$

defines as the magnetic field and Pr = $\frac{vf}{\alpha f}$ refers the Prandtl number and v_f is the effective kinematic viscosity of fluid, subject to the transformation boundary conditions as follows:

$$f'(0) = \lambda, f(0) = 0, \theta(0) = 1, f'(\eta) \to (1 - \lambda), \theta(\eta) \to 0, \text{as } \eta \to \infty. \tag{10}$$

As our main goal is to study the influence of different parameters on the performance of heat transfer such as velocity and temperature then we define the skin friction coefficient, C_f, and the local Nusselt number, Nu_x, as follows

$$C_f = \frac{\tau_w}{\rho_f U^2}, Nu_x = \frac{x}{k_f(T_w - T_\infty)}, \tag{11}$$

where τ_w and q_w is the surface shear stress and the heat flux, respectively. τ_w can be expressed as

$$\tau_w = \mu_{nf}\left(\frac{\partial u}{\partial y}\right) \text{ at } y = 0, q_w = -k_{nf}\left(\frac{\partial T}{\partial y}\right) \text{ at } y = 0. \tag{12}$$

From Eq. (7), we will set

$$\frac{\partial T}{\partial y} = \sqrt{\frac{U}{v_f x}}\theta' \text{ at } y = 0. \tag{13}$$

Substituting Eq. (12) and (13) into Eq. (11) resulting in

$$C_f \text{Re}_x^{1/2} = \frac{1}{(1 - \varphi)^{(2.5)}} f''(0), Nu_x \text{Re}_x^{-1/2} = -\left(\frac{k_{nf}}{k_f}\right)\theta'(0), \tag{14}$$

where $\text{Re}_x = \frac{Ux}{vf}$ is the local Reynold number.

3. Results and Discussion

Analytical solutions for ODEs system in Eq. (8) and Eq. (9) through a set of initial values and boundary conditions as mentioned in Eq. (10) are solved numerically using bvp4c MATLAB package. First, we observe the variation values of the reduced skin friction, $f''(0)$ and the reduced on the heat transfer coefficient, $-\theta'(0)$ with different values of magnetic field, M and moving parameter, λ. The value of $M = 0$ indicates that the flow is without the influence of magnetic field in the fluid flow. We also assume that the plate is moving to the free stream in opposite ($\lambda < 0$) or same ($\lambda > 0$) direction. These phenomena can be illustrated in Figure 1 and 2. From these figures, we can see the range of single solutions can be detected $\lambda > 0$ and $M > 0.1$ while for $\lambda < 0$ and $M < 0.1$ the non- unique solutions (dual solutions) will emerge. Besides that, the rise in M will increase and improve $f''(0)$ and $-\theta'(0)$.

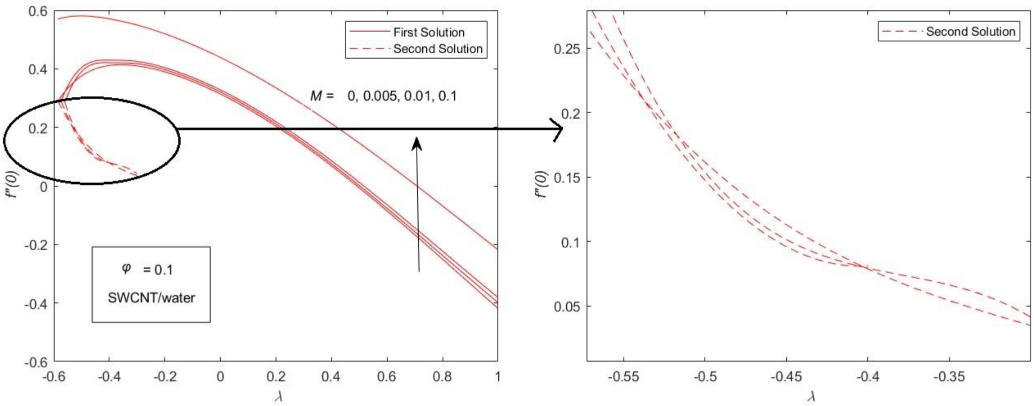

Fig. 1. Variation of $f''(0)$ with different M and λ for SWCNT/water

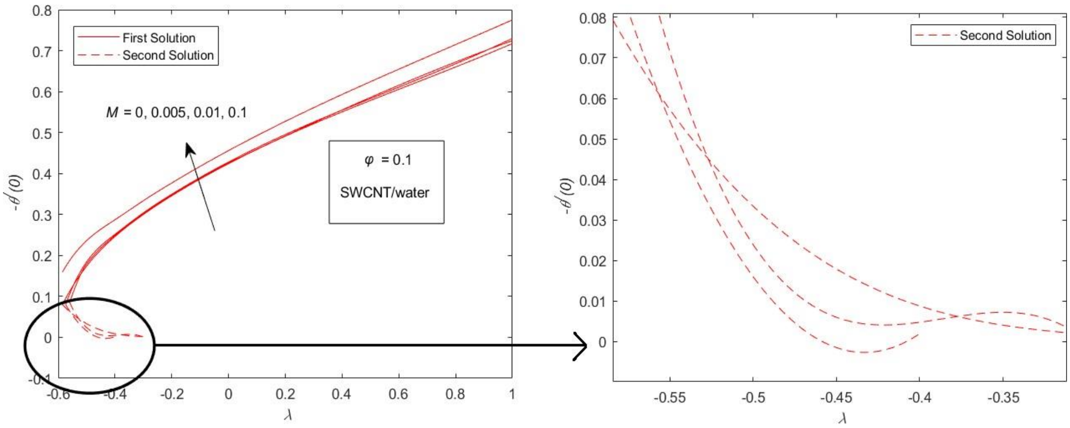

Fig. 2. Variation of $-\theta'(0)$ with different M and λ for SWCNT/water

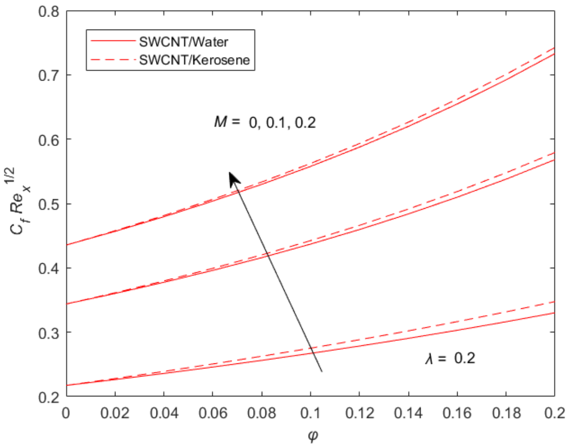

Fig. 3. Effect of different values of M on the skin friction coefficient using various φ for SWCNT/water and SWCNT/kerosene

To make a decision based on the type of base fluid, we do analysis on the effect of M for SWNCT in water and kerosene by applying a variety number of φ. By referring on Figure 3 and 4, we can see obviously that SWCNT/kerosene is more effective than SWCNT/water both in skin friction coefficient and local Nusselt number.

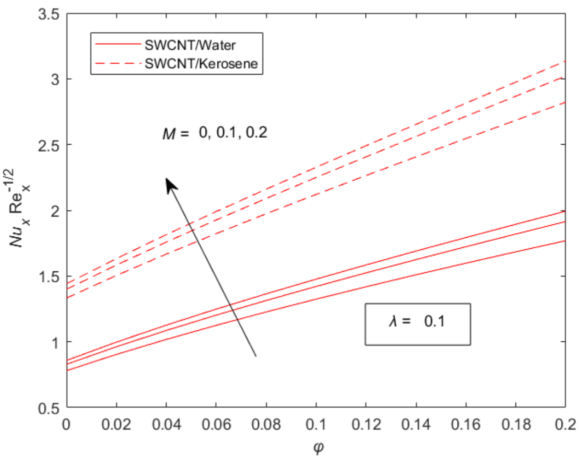

Fig. 4. Effect of different values of M on the Nusselt number using various φ for SWCNT/water and SWCNT/kerosene

The investigation of the velocity profile for SWCNT/water can be viewed in Figure 5. From this figure, when M is set at $M = 0.01$ and the range of λ is $-0.5 \leq \lambda \leq -0.3$, we can see that the thickness of momentum boundary and thermal boundary layer for second solution is bigger than first solution.

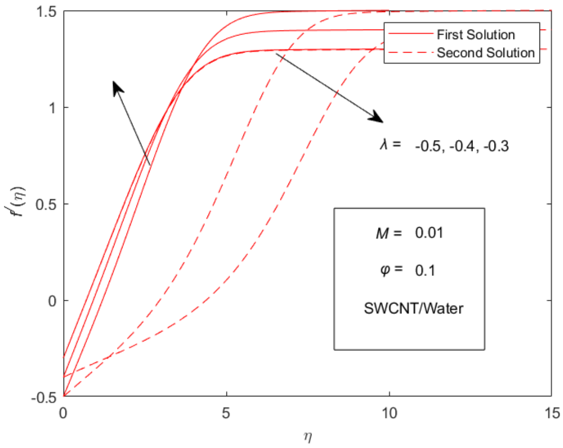

Fig. 5. Effect of different values of λ on the velocity profile using various η for SWCNT/water

4. Conclusion

From this present paper, we have investigated theoretically and numerically CNTs nanofluids flow and the properties of heat transfer over a moveable plate. From our analysis, we can summarise that

i. The duality solutions appear for the range of $\lambda < 0$ and at $M < 0.1$.
ii. Employing magnetic field into CNTs nanofluids can improve in both the skin friction and heat transfer coefficient.
iii. Kerosene will perform better as a base fluid compare to water in term of the skin friction coefficient and heat transfer coefficient.

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