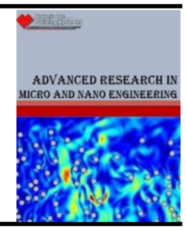
Journal of Advanced Research in
Micro and Nano Engineering

Journal homepage: https://akademiabaru.com/submit/index.php/armne
ISSN: 2756-8210



Effects of The Optimal Imposition of Viscous and Thermal Forces on Spectral Dynamical Features of Swimming of a Microorganism in Nanofluids

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ABSTRACT

This paper applies a system identification approach to introduce external forces to time series data. Specifically, a two-equation dynamical system is utilized to replicate the time evolution of experimental parallel velocity values of a bull spermatozoa's head during circular swimming. The differential system accurately models the sperm's planar movement in three different scenarios. Additionally, the time marching of the differential system employs a least-squares analysis on a system with a larger amount of sampled data and fewer data points, demonstrating its reliability in approximating the actual pattern. Furthermore, a linearized model of the system is explored near its equilibrium points. The study also demonstrates the application of this straightforward system when dominant external viscous forces arise from swimming in a nanofluid. Finally, the paper discusses the optimal modes with the highest velocity magnitude when oscillating forces come into play.

Keywords:
system identification; nanofluids; micro swimmers; optimal frequency

1. Introduction

In the so-called system identification approach the governing equations of the systems are reconstructed by a proper selection of a mathematical form from the experimental time series. Albeit, over-fitting the data is often inevitable [Perona] which isone of the challenges dealing with high-dimensional disturbances. The resulting equations need to contain sufficient dynamical features of the system [Porporato]. If the low-dimensional components are adequate the system identification algorithm is reliable even for noisy systems [Perona, Crutchfield, Parlitz].

hIn the current work ,we have analyzed the optimal oscillation in the forcing scenarios discussed in [Sharafatmandjoor] in which, the system identification idea to the experimental time series of an experiment on the bull sperm swimming phenomenon is analyzed .Our motivation was the fact that viscous effects are the essential factor in migration of a successful sperm through the female tract .The complex physics of highly viscous invaginated media limits the number of sperms ever to succeed to only tens .Presenting an ordinary differential equation (ODE) model for the

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problem of sperm movement is as we can build a basis for more complex systems with added spatial degrees of freedom by assessing the change in viscosity of the surrounding flow .After presenting the ODE ,the next part of the paper is devoted to studying the effect of changing the parameters of the linearized equation near the equilibrium points.

Also, we can model the movement of the sperm in a nanofluid by modeling the added particles as a homogeneous single-phase suspension [Brinkman]. As the nanoparticles are very small in size and the volume fractions are in the low range ,the suspension has no erosion ,sedimentation, pressure change or non-Newtonian side effects [Bachok]. For some relevant applications of the ideasee [Mehrabi, Azwadi, Kefayati] .Now, the system feels the effect of external forcing with a proper forcing term borrowed from resistive force theorem (RFT) .Forcing the dynamical systems with physical external excitations has been proposed and verified by [Porporato].

2- Mathematical Model

The classical two-equation differential system is used in a way that it represents the instantaneous velocity components of the head of a bull sperm .The instantaneous velocities measured in a plane that are extracted from [Friedrich] and are induced by the flagellar beat through circulating swimming in pure water. It is assumed that the regime of the flow field is creeping flow. Here ,as discussed in [Sharafatmandjoor] we propose a polynomial-based coupled model with ten coefficients designed to get minimized cost functions associated with the error of the model [Porporato].

$$\frac{dy}{dt} = v \quad (1a)$$

$$\frac{dv}{dt} = c_1 + c_2 y + c_3 v + c_4 y^2 + c_5 yv + c_6 v^2 + c_7 y^3 + c_8 y^2 v + c_9 y v^2 + c_{10} v^3 \quad (1b)$$

where y and v values denote the displacement and velocity values respectively and the constants $c_1 : c_{10}$ are to be obtained .The acceleration-type forcing terms are added to the to the right-hand side of equation (1b). The four test cases discussed in [Sharafatmandjoor] are revisited. The equations 1 is written for each point for cases 1 ,2 and 3 the algebraic system is composed by writing as:

$$\begin{bmatrix} 1 & \cdots & v^3_{(1)} \\ \vdots & \ddots & \vdots \\ 1 & \cdots & v^3_{(10)} \end{bmatrix} \times \begin{bmatrix} c_1 \\ \vdots \\ c_{10} \end{bmatrix} = \begin{bmatrix} \frac{dv}{dt}(1) \\ \vdots \\ \frac{dv}{dt}(10) \end{bmatrix} \quad (2)$$

Now, we call the coefficient matrix, unknown vector and the known vector are denotedas $A_{10\times10}, C_{10\times1}$ and $B_{10\times1}$respectively .In system 2 the subscripts $(i), i \in 1:10$ mean the corresponding values at points 1 to 10 .We obtain the approximated displacement y from the given velocity values by employing the trapezoidal rule. Also we calculate the differentiations in vector Bvia a second-order differencing directly from the time series .Then we apply an LU decomposition routine to find the ten unknowns .

Here we use a Runge-Kutta time step that is ten times smaller than the original sampling time step . Firstly, we can observe how the reconstructed ODE is consistent with the experimental values for the finest time sampling on the left panel of figure 1. See the middle and right panel of figure1 for the negative effect of rather large time samplings .The ODE estimations are weaker for larger time samplings, the ODE estimations are poorer for larger time samplings however, the dominant trend of the data is still captured .

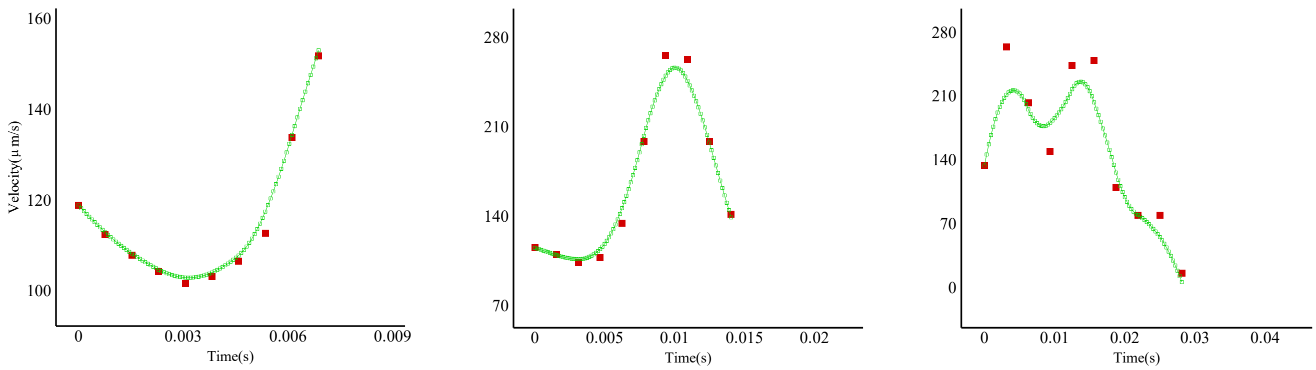

Fig. 1. Extracted Experimental Velocity Time Series (red) and the Modeled Values Obtained by Solving the System 1a,b (green) .Left, middle and right panels correspond to cases 1, .2 and 3 respectively

The over-determined problems could be dealt with by using more than 10 data points with one single set of c_1 to c_{10}, in a least-squarescontext. For an-data point system $(n > 10)$ either sides of equation 2 are premultiplied by the transpose of the coefficient matrix namely A^T to get:

$$A^T_{10\times n} A_{n\times 10} C_{n\times 1} = A^T_{10\times n} B_{n\times 1} \tag{3}$$

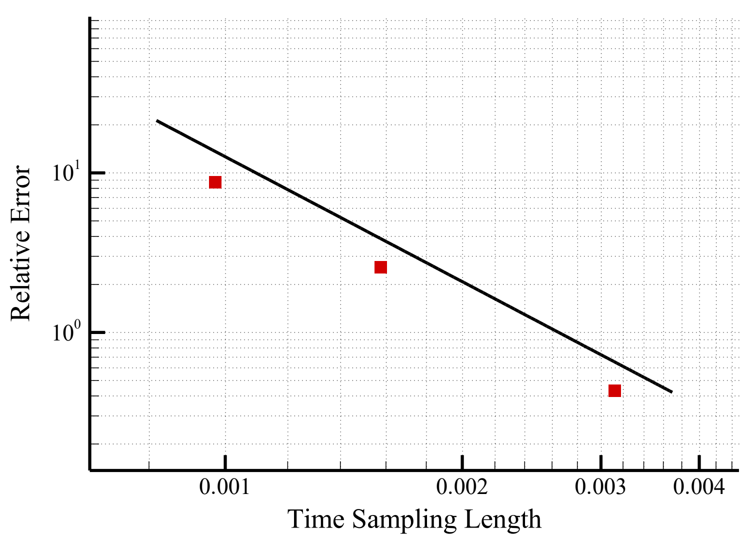

Fig. 2. The Relative Error versus Different Sampling Times .cases 1 , and 3 correspond to $2\delta t = 9.7E-4, 1.6E-3$ and$3.1E-3$ respectively. Slope of the Solid line is- .2.6

A new 10 equation and 10 unknown system is obtained from the above system and it can be solved similar to system 2.

Here, we define the relative error with respect to the experimental velocity values $u_{(i)}^{ex}$ as:

$$E = \sum_i \frac{\left|u_{(i)} - u_{(i)}^{ex}\right|}{\left|u_{(i)}^{ex}\right|} \times 100\% \tag{4}$$

The relative error values E for cases 1 ,2 and 3 are depicted in figure 2. The relative errors decay at least with order2.6- . So, the time convergence of the method is guaranteed .On the other hand, we have found that the gain saturates for very fine Runge-Kutta time steps.

Figure 3 shows the Fourier space representation of the velocity magnitudes in different mode numbers. Other than the first mode which represents the mean values of the field, the modes correspond to the highlighted values with blue circle are the excitation modes which an optimally effective oscillation could mimic.

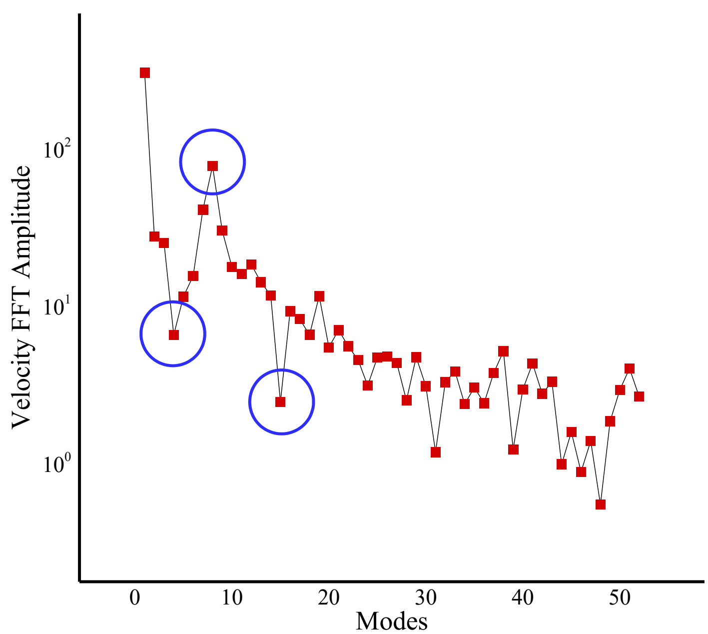

Fig. 3. Magnitude of the Velocities in Fourier space versus Mode Numbers.

3- Analysis of The Linearized Model and the External Forcing

In order to effectively model the basic dynamics of a system the linearization concept is followed [Hirsch] to enjoy the useful properties of linear systems. As nonlinear systems behave similar to their linear correspondents close to the equilibrium points, here a non-homogeneous linearized model of equation 1 is derived and used .

$$\frac{d^2y}{dt^2} - c_3\frac{dy}{dt} - c_2 y = c_1 \tag{5}$$

For the external forcing analysis one would observe that the RFT method is itself a low-dimensional simplification to the slender body theory where slender filaments are approximated in the zero-Reynolds number limit .RFT is first presented by [Gray] and its relations are well described in [Lauga] for a helical pattern that is the common movement path for a typical sperm.

As per this theory elements travelling with velocity $\boldsymbol{u} = u_{\parallel}\hat{\eta} + u_{\perp}\hat{\zeta}$ where $u_{\parallel}$ and $u_{\perp}$are respectively the velocities parallel and normal to the tangential vector with unit vectors $\hat{\eta}$ and $\hat{\zeta}$ .Also ,the local viscous forces per unit mass are associated with the local drag coefficients $c_{\parallel}$ and $c_{\perp}$ .Provided the total acceleration tangent to the movement direction can be written as $\mathbf{a}_{\parallel} = -c_{\parallel}u_{\parallel}\hat{\eta}$, the parallel drag force is associated with $\hat{\eta}$ .If we assume a far from a solid body, for a filament moving in a plane through a liquid with viscosity μ RFT approximates the $c_{\parallel}$ is written as:

$$c_{\parallel} = \frac{2\pi\mu}{ln(\frac{2\lambda}{r}) - \frac{1}{2}} \tag{6}$$

where λ is the wavelength of the flagellar bending waves and r is the approximate radius of the filament$\lambda = 66 \pm 8\mu m$ for water at 36^0C . .The head of sperm is approximated also about $10\mu m$ .In the lack of inertia the only governing force of the system is the viscous forcing inthe creeping flow regime. Therefore, the remarkable extra viscous effects due to the added nanoparticles are introduced as an additive acceleration term to the second equation of system 1.

The experimental relation of Brinkman [Brinkman] is used to model the contribution of adding nanoparticles to change in the viscosity of the base flow .The formula is valid only for spherical nanoparticles [Brinkman] .Based on the relation we can find which is the cause of the extra viscous forcing.

$$\delta\mu = \mu_{bf}(1 - \frac{1}{(1-\varphi)^{2.5}}) \tag{7}$$

In the above relation $\delta\mu$ is the change in the viscosity of the medium, μ_{bf} is the viscosity of the base flow and φ is the volume fraction. For water at 36^0C , $\mu_{bf} = 0.7\ mPas$. If figure 4 we have shown the solution space of the system corresponding to case 3 for two typical values $\varphi = 0.1, 0.2$ where the vertical axis is the percentage of deviation of the acceleration of the sperm and the horizontal axis represents its base velocity values where the acceleration is triggered mostly for some unique velocity values. We can also see that increasing the volume fraction leads to relatively higher discrepancy in acceleration values.

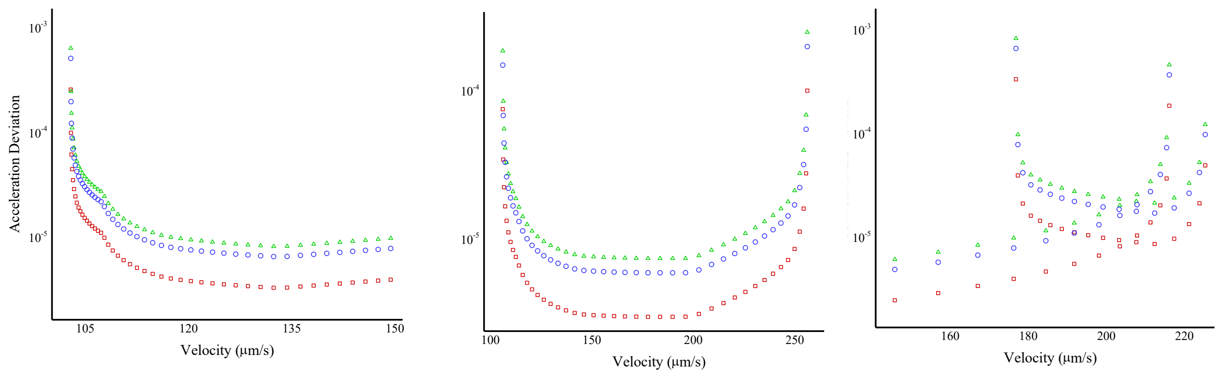

Fig. 4. Percentage of Deviation of the Acceleration of the Sperm Swimming in a Nanofluid versus Base Velocity Values. Blue Symbols: $\varphi = 0.1$,Green Symbols: $\varphi = 0.2$.

4- Discussions

We have developed an ODE framework based on an experimental time evolution of parallel velocity values of a sperm swimming in water. The ODE can also be used for the extrapolation purpose in the short time ranges. The low-dimensional ODE is then used to simulate external forcing and the optimal excitation frequencies of the forcing is then discussed. It is shown that only in a limited number of certain frequency numbers the forcing could be effective for the fertilization purposes.

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