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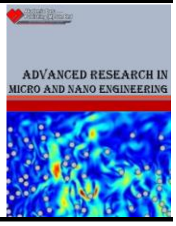

MHD Stagnation-Point Flow and Heat Transfer Over an Exponentially Stretching/Shrinking Vertical Sheet in a Micropolar Fluid with a Buoyancy Effect

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ABSTRACT

The MHD stagnation point flow and heat transfer in a micropolar fluid over an exponentially vertical stretching/shrinking sheet are investigated in the presence of convective boundary conditions in the current work. The buoyancy effect, also known as mixed convection, is also taken into account. Several researchers are conducting research on the fluid in the current state of mixed convection. To convert the governing equations from a system of partial differential equations to ordinary differential equat ions, similarity variables were used. The modified equations are then numerically solved in MATLAB using BVP4c. There is a lot of agreement when compared to previous findings. Contradictory phenomena are observed between Micropolar and mixed convection for fluid velocity, angular velocity and temperature distribution profiles. The significant variables' characteristics are graphically presented, and the numerical results are tabulated.

Keywords:
Micropolar fluid; MHD stagnation point flow; heat transfer; exponentially stretching/shrinking; buoyancy effect

1. Introduction

Stagnation point flow with heat transfer across a stretching/shrinking sheet has several industrial uses. Some applications include boundary layer along material handling conveyors, blood flow difficulties, aerodynamics, plastic sheet extrusion, cooling of metallic plates in a bath, textile and paper industries, and so forth [1]. According to Crane [2], Hiemenz was the first to investigate it, demonstrating that using similarity transformation, the Navier-Stokes equations regulating the flow can be reduced to an ordinary differential equation of third order. Because of the nonlinearities in the reduced differential equation, no analytical solution is accessible, and the nonlinear equation is normally solved numerically with two-point boundary conditions, one of which is set to infinity. Extrusion of polymer fluids, solidification of liquid crystals, cooling of a metallic plate in a bath, animal blood, exotic lubricants, and colloidal and suspension solutions are a few of the uses for micropolar fluids. These fluids resemble rigid molecules, magnetic fluids, dusty clouds, muddy fluids, and some biological fluids [3]. The problem of stagnation point flow in a micropolar fluid has been extended in numerous ways to include various physical effects such as Nazar *et al.,* [4] Ishak *et al.,* [5], Borrelli *et al.,* [6], Dash *et al.,* [1], Soid *et al.,* [7], Attia [8], and Mishra *et al.,* [9]. Many academics have examined

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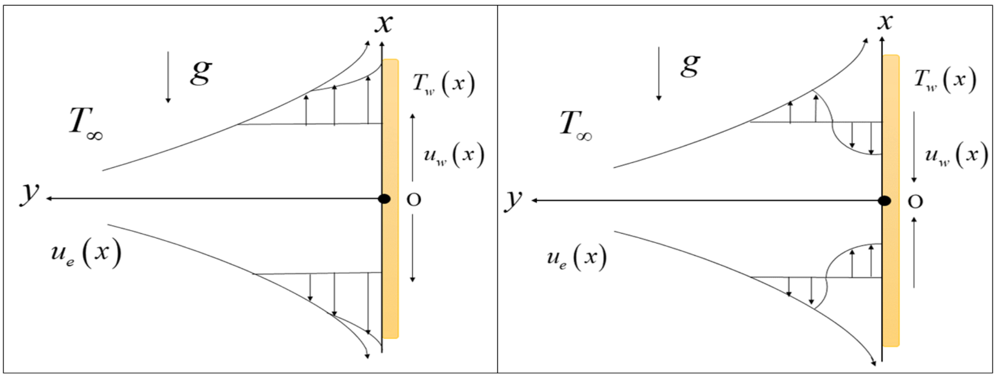
the non-uniqueness of solutions to fluid flow issues on moving, shrinking, and stretching sheets in the presence and absence of a buoyancy effect in the last few years [10] such as Lakshmi Devi *et al.,* [11] and Song *et al.,* [12].

The purpose of this study is to investigate the MHD stagnation point flow and heat transfer over an exponentially stretching/shrinking vertical sheet submerged in a micropolar fluid with a buoyancy effect. Similarity variables were employed to turn the governing partial differential equations (PDEs) into ordinary differential equations (ODEs). The amended equations were then numerically solved in MATLAB using BVP4c. We expect that the results gained will be beneficial for applications and as a supplement to prior research.

2. Methodology

Consider a micropolar fluid flow towards a stagnation point on a vertical sheet as illustrated in Figure 1. The designed was exactly based on the original profile of Waini *et al.,* [13]. The x and y axes are Cartesian coordinates where x is assigned vertically along the surface and y is orthogonal to it with the origin o. The symbol $u_e(x)=ae^{x/L}$ is free stream velocity where $a>0$ (constant) and L is reference length. Next, $u_w(x)=be^{x/L}$ is an exponential velocity when the surface is stretched $(b>0)$, shrunk $(b<0)$ or $b=0$ is for the static surface. The surface temperature is given as $T_w(x)=T_\infty+T_0e^{2x/L}$ where T_0 is a constant and T_∞ is the ambient temperature. Hence, g is a symbol of acceleration due to gravity.



(a) Stretching (b) Shrinking

Fig. 1. The geometry of the flow problem of (a) Stretching and (b) Shrinking sheet

The governing boundary layer parabolic partial differential equations (PDEs) are written as continuity, linear momentum, angular momentum and energy equations [14, 10, 13]:

$$\partial u/\partial x+\partial v/\partial y=0 \tag{1}$$

$$u(\partial u/\partial x)+v(\partial u/\partial y)= u_e(\partial u_e/\partial x)+(\upsilon+\kappa/\rho)(\partial^2 u/\partial y^2)+\kappa/\rho(\partial N/\partial y)+g\beta_T(T-T_\infty)-\sigma B^2/\rho(u-u_e) \tag{2}$$

$$u(\partial N/\partial x)+v(\partial N/\partial y)=\chi/\rho j(\partial^2 N/\partial y^2)-\kappa/\rho j(2N+\partial u/\partial y) \tag{3}$$

$$u(\partial T/\partial x)+v(\partial T/\partial y)=k/\rho c_p(\partial^2 T/\partial y^2)-1/\rho c_p(\partial q_r/\partial y) \tag{4}$$

Subject to the boundary conditions:

$$u = u_w,\ v = 0,\ T = T_w,\ N = -m\left(\partial u / \partial y\right) \text{ at } y = 0$$
$$u \to u_e,\ T \to T_\infty,\ N \to 0 \text{ as } y \to \infty \tag{5}$$

where u and v are component of velocity along x and y direction respectively. The symbol $\upsilon = \mu / \rho$ is the kinematic viscosity, ρ is the fluid density, μ is the coefficient of fluid viscosity, N is microrotation or angular velocity, $j = 2L\upsilon e^{-x/L} / a$ is micro-inertia per unit mass, $\chi = \mu\left(1 + K/2\right) j$ is spin gradient, κ is the vortex viscosity. The electrical conductivity of the fluid assigned as σ, $B\left(x\right) = B_0 e^{x/2L}$ is the variable magnetic field where B_0 is a constant and c_p is the specific heat. The symbol β_T is the thermal expansion coefficient. Eq. (1) to Eq. (4) along with the boundary condition Eq. (5) can be expressed in a simpler form by introducing the following similarity transformation [13]:

$$\eta = \sqrt{\frac{a}{2\upsilon L}} e^{\frac{x}{2L}} y, \qquad u = a e^{\frac{x}{L}} f'\left(\eta\right), \qquad v = -\sqrt{\frac{a\upsilon}{2L}} e^{\frac{x}{2L}} \left(f\left(\eta\right) + \eta f'\left(\eta\right)\right), \qquad \theta\left(\eta\right) = \frac{T - T_\infty}{T_w - T_\infty},$$
$$N = a\sqrt{\frac{a}{2\upsilon L}} e^{\frac{3x}{2L}} h\left(\eta\right) \tag{6}$$

where η is similarity variable, while u and v denotes the stream function that the continuity Eq. (1) is identically fulfilled. Thus, the transformed linear momentum Eq. (2), angular momentum Eq. (3) and energy Eq. (4) become:

$$f'''\left(1 + K\right) + f'' f - 2\left(f'\right)^2 - M\left(f' - 1\right) + K h' + 2\lambda\theta + 2 = 0$$
$$\left(1 + K/2\right) h'' - K\left(2h + f''\right) - 3f' h + f h' = 0 \tag{7}$$
$$\Pr\left(4f'\theta - f\theta'\right) - \theta''\left(1 + 4R/3\right) = 0$$

The corresponding boundary conditions:

$$f\left(\eta\right) = 0,\ f'\left(\eta\right) = \varepsilon,\ \theta\left(\eta\right) = 1,\ h\left(\eta\right) = -m f''\left(\eta\right) \text{ at } \eta = 0$$
$$f'\left(\eta\right) \to 1,\ \theta\left(\eta\right) \to 0,\ h\left(\eta\right) \to 0 \text{ as } \eta \to \infty \tag{8}$$

where the prime indicates differentiation with respect to η and $M = 2\sigma B_0^2 L / \rho a$ is Hartmann number or magnetic parameter. $\lambda = g\beta_T T_0 L / a^2$ is buoyancy parameter, $\varepsilon = b/a$ is the stretching/shrinking parameter where $a > 0$ (constant) and the surface is stretched ($b > 0$) or shrunk ($b < 0$). Therefore, the value of ε is directly proportional to b. Next, $K = \kappa / \mu$, $\Pr = \mu c_p / k$ and $R = 4\sigma^* T_\infty^3 / k^*$ are the micropolar parameter, Prandtl number and the radiation parameter respectively. The involved physical quantities are the skin friction coefficient C_f, the local Nusselt number Nu_x and the local couple stress M_x [8]:

$$C_f = \frac{\tau_w}{\rho u_e^2},\ M_x = \frac{\chi\left(\partial N / \partial y\right)_{y=0}}{\rho x u_e^2},\ Nu_x = -\frac{x}{\left(T_w - T_\infty\right)}\left(\partial T / \partial y\right)_{y=0} \tag{9}$$

where the surface shear stress $\tau_w = \left[\left(\mu + \kappa\right)\partial u / \partial y + \kappa N\right]_{y=0}$. Then, the reduced skin friction coefficient, the local couple stress, and the reduced local Nusselt number:

$$C_f\left(\mathrm{Re}_x\right)^{1/2}\sqrt{2L/x}=\left[1+\left(1-m\right)K\right]f''\left(0\right),\; M_x\,\mathrm{Re}_x=\left(1+K/2\right)h'\left(0\right)$$
$$Nu_x\left(\mathrm{Re}_x\right)^{1/2}\sqrt{2L/x}=-\theta'\left(0\right) \tag{10}$$

$\mathrm{Re}_x = xu_e/\upsilon$ is the local Reynolds number.

3. Results and Discussion

An analysis of the behaviors of the velocity, angular, and temperature profiles is carried out. The numerical solutions are obtained using BVP4c in MATLAB. The numerical values of $C_f\left(\mathrm{Re}_x\right)^{1/2}\sqrt{2L/x}$ and $Nu_x\left(\mathrm{Re}_x\right)^{1/2}\sqrt{2L/x}$ are obtained for various values of stretching/shrinking parameter ε, when $\mathrm{Pr}=6.2$ and non-buoyant case $\lambda=0$ with other parameter was set to be constant at $K=M=m=R=0$. Tables 1 and 2 show the comparison of the skin friction coefficient and the local Nusselt number between the results found by Ur Rehman *et al.,* [15] and Waini *et al.,* [13] with the present study, respectively.

The current results were in great agreement with the earlier study when the numbers in Tables 1 and 2 were compared. As a result, the approach employed for this study was valid and accurate to verified. The values of the skin friction coefficient decrease while the values of the local Nusselt number increase when the parameter stretches/shrinking increases.

Table 1

Comparison for Numerical Values $C_f\left(\mathrm{Re}_x\right)^{1/2}\sqrt{2L/x}$ for $\varepsilon=-0.5,0,0.5$

ε	Ur Rehman *et al.,* [15]	Waini *et al.,* [13]	Present Study
-0.5		2.1182	2.11816867
0	1.68720	1.6872	1.68721817
0.5	0.96040	0.9604	0.96041608

Table 2

Comparison for Numerical Values $Nu_x\left(\mathrm{Re}_x\right)^{1/2}\sqrt{2L/x}$ for $\varepsilon=-0.5,0,0.5$

ε	Waini *et al.,* [13]	Present Study
-0.5	0.0588	0.05878644
0	2.5066	2.50662545
0.5	4.8016	4.08157327

Micropolar parameter K and buoyancy or mixed convection parameter λ will be examine. The other parameters such as material parameters m, Prandtl number Pr, magnetic M and radiation R are fixed to $m=0.5$, $\mathrm{Pr}=6.2$, $\varepsilon=-1$ and $M=R=2$ are considered. This result is focusing for shrinking plate where the ratio is 1. The Figures 2, 3 and 4 illustrate the effects of the velocity $f'\left(\eta\right)$, the angular velocity $h\left(\eta\right)$, and the temperature $\theta\left(\eta\right)$ profiles on the values of micropolar parameter K respectively. The value for micropolar parameter is $K=0,1,2,5$ and buoyancy parameter is $\lambda=2$. These profiles asymptotically satisfy the boundary condition in Eq. 8, giving us confidence in the solutions' accuracy. The decreasing behavior of $f'\left(\eta\right)$ is observed with the increase of K as shown in Figure 2. The behavior of $h\left(\eta\right)$ is given in Figure 3. It is illustrated that $h\left(\eta\right)$

increases to $h(\eta) \approx -0.5$ and decreases to the boundary condition with increasing of K. Meanwhile, Figure 4 shows $\theta(\eta)$ increases due to increasing of K.

The Figures 5, 6 and 7 show the effects of $f'(\eta)$, $h(\eta)$, and $\theta(\eta)$ on the buoyancy parameter λ values, respectively. There is an opposite phenomenon with micropolar effect. The value of the buoyancy parameters is $\lambda = 0,1,2,3$ and micropolar parameter $K = 2$. the fluid velocity increases while the angular velocity decreases to $h(\eta) \approx -0.4$ and then increases to zero when λ increases as depicted in Figures 5 and 6 respectively. Meanwhile, the temperature behavior decreases due to the presence of λ.

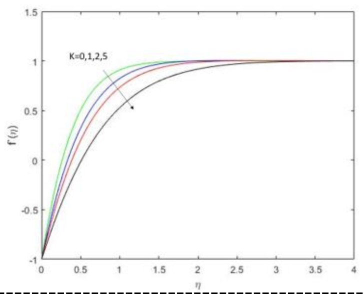


Fig. 2. Velocity profile $f'(\eta)$ for varies of K

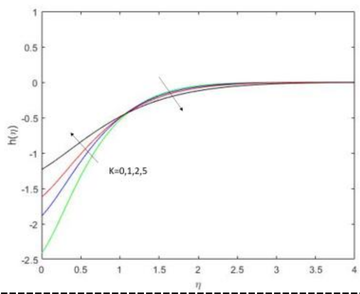


Fig. 3. Angular velocity profile $h(\eta)$ for varies of K

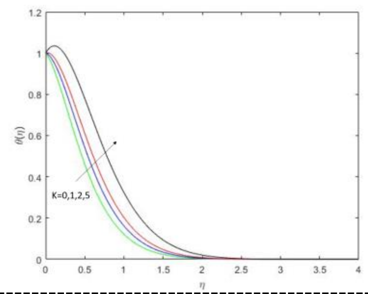


Fig. 4. Temperature profile $\theta(\eta)$ for varies of K

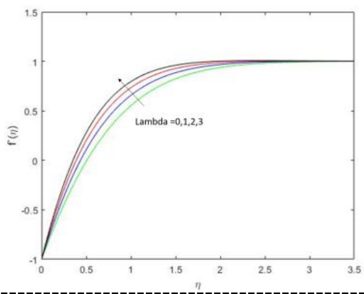


Fig. 5. Velocity profile $f''(\eta)$ for varies of λ

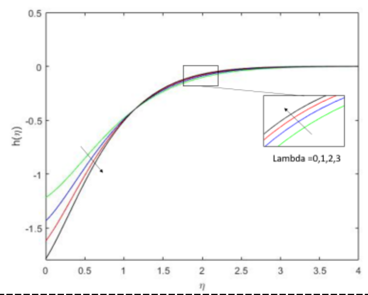


Fig. 6. Angular velocity profile $h'(\eta)$ for varies of λ

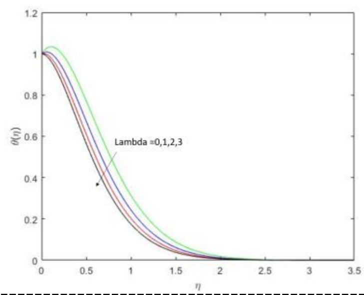


Fig. 7. Temperature profile $\theta(\eta)$ for varies of λ

4. Conclusions

From this present paper, we detect the following:

i. The velocity profile at the vertical plate increases on increasing the buoyancy force but reduce with micropolar.

ii. The angular profile for the particle rotation occurs in two phenomena which are decrease and increase for buoyancy force reversible for micropolar.

iii. The temperature of the fluid is drop with buoyancy force and rises for micropolar.

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