

## Mixed Convection in a Lid-Driven Horizontal Rectangular Cavity Filled with Hybrid Nanofluid By Finite Volume Method

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### ABSTRACT

In the present work, a new type of nanofluid called the hybrid nanofluid ( $\text{Al}_2\text{O}_3\text{-Cu-water}$ ) is used to enhance the heat transfer. The Finite-Volume-Method (FVM) along with the SIMPLE-algorithm has been utilized to study the heat-transfer and, mixed convection fluid-flow of the hybrid nanofluid ( $\text{Al}_2\text{O}_3\text{-Cu-water}$ ), placed within the lid-driven rectangular cavity. The bottom and top walls are subjected to constant high temperature ( $T_h$ ) and low temperature ( $T_c$ ) respectively. The side walls are treated as adiabatic. The top wall moves in the positive x-direction. The effects of Reynolds number and hybrid nanoparticle volume fraction on the flow field have been investigated. It is found that the mean Nusselt number increases with respect to Reynolds numbers and hybrid nanoparticle volume fraction.

### Keywords:

FVM; Mixed convection; Hybrid nanofluid; Rectangular cavity

Received: 19 May 2020

Revised: 25 Jun. 2020

Accepted: 17 Aug. 2020

Published: 30 Sep. 2020

## 1. Introduction

Tremendous amount of heat is generated from various engineering applications such as those encountered in manufacturing, thermal power plants, microelectronics, transportation, etc. Efficient coolers have been designed to adequately dissipate heat. In general, cooling method can be categorized into passive and active methods [1-4]. Traditional coolers such as propylene glycol, water, oils and, ethylene glycol have low thermal conductivities. In order to enhance the heat transfer, the thermal conductivity of the fluid should be enhanced. One of the methods is mixing the base fluid with nano-sized particles (hence the term nano-fluid [5]). The frequently used nanoparticles are metals (Ni, Ag, Au and, Cu), metal-oxides ( $\text{CuO}$ ,  $\text{Al}_2\text{O}_3$ ,  $\text{ZnO}$ ,  $\text{MgO}$ ,  $\text{Fe}_2\text{O}_3$ ,  $\text{TiO}_2$  and,  $\text{SiO}_2$ ), metal-nitride ( $\text{AlN}$ ), metal-carbide ( $\text{SiC}$ ) and carbon materials (MWCNTs, CNT, Diamond and, Graphite). In general, the thermal conductivity of nano-fluid is dependent on parameters such as shape, size, stability of scattered nano-particles, type of base-fluid, mass concentration of nano-particles and fluid temperature [6-7]. As compared to traditional fluids, nano-fluids have been shown to exhibit better thermal performance in a wide range of engineering applications [8-11]. Also, the thermal conductivity of nano-particle can be further enhanced by mixing (hybridization) two (or

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more) types of nano-particles (or hybrid-nano-particles). Turcu *et al.*, [12] were probably the pioneers in synthesizing the hybrid nano-particles (i.e. MWCNTs - Fe<sub>2</sub>O<sub>3</sub>); nevertheless, their hybrid nano-particles are simply the extension of mono-nano-particles. Undoubtedly, hybrid nano-fluids exhibit higher thermal-conductivity than conventional nano-fluids consisting of only one type of nano-particles [13-16].

Internal flow in enclosures could be driven by natural or mixed convections. Mixed convection is commonly found in manufacturing of float glass, solar collectors, solar ponds, food-processing and lubrication. Cimpean *et al.*, [17] studied mixed convection in a trapezoidal porous cavity filled with the hybrid nano-fluid (Al<sub>2</sub>O<sub>3</sub>, Cu -water). They found that higher Reynolds number would increase the heat transfer. Ismael *et al.*, [18] investigated the mixed convection in a lid-driven cavity filled with Al<sub>2</sub>O<sub>3</sub>-Cu-water hybrid nanofluid. The nanofluid was heated by a triangular heater and cooled isothermally from the right vertical wall. They found that the hybrid nano-fluid is a cost-effective solution as the amount of nano-particles can be reduced. Some advanced experimental studies using hybrid nano-fluids have been reported [19-23]. The aim of this study is to investigate the effects of hybrid nano-fluid volume fraction and Reynolds number on the flow field in the rectangular cavity.

## 2. The Mathematical Modelling (Formulation)

The two dimensional (2D) mixed-convection problem in the rectangular cavity of length (L) is shown in Figure 1. The upper and lower walls are isothermal, where the lower wall temperature  $T_h$  is higher than the upper wall temperature  $T_c$ . Both side walls are adiabatic. The fluid inside the rectangular cavity is water-based hybrid nano-fluid (Al<sub>2</sub>O<sub>3</sub>, Cu). The dimensional governing equations are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho_{hnf}} \frac{\partial p}{\partial x} + \nu_{hnf} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \quad (2)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho_{hnf}} \frac{\partial p}{\partial y} + \nu_{hnf} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + g\beta_{hnf}(T - T_c) \quad (3)$$

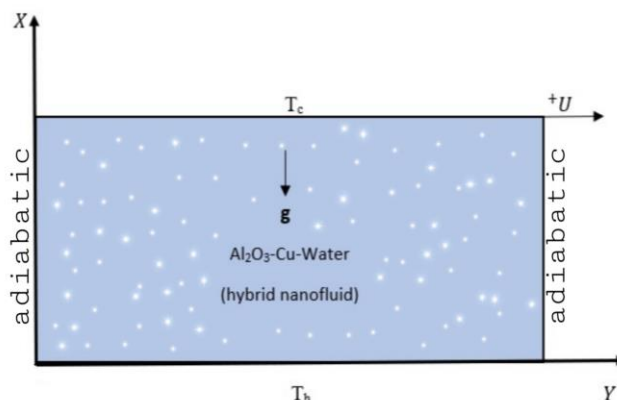
$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{hnf} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \quad (4)$$

The definition of the boundary-conditions as follows:

$$\text{The top wall-side: } u = u_0; v = 0; T = T_c \quad (5)$$

$$\text{The bottom wall-side: } u = 0; v = 0; T = T_h \quad (6)$$

$$\text{The left and, right wall-sides: } u = v = 0; \frac{\partial T}{\partial x} = 0 \quad (7)$$



**Fig. 1.** Physical model

where the  $x$  and  $y$  indicate the Cartesian coordinates in the horizontal and vertical directions, respectively,  $g$  is the gravity acceleration,  $\rho_{\text{hnf}}$  is the density of the hybrid nano-fluid,  $\beta_{\text{hnf}}$  is the thermal expansion coefficient of the hybrid nano-fluid,  $\varphi$  is the solid volume fraction,  $\alpha$  is the thermal diffusivity of the hybrid nano-fluid and,  $\nu_{\text{hnf}}$  is the kinematic viscosity of the hybrid nano-fluid. The physical properties of the hybrid nano-fluid [24] are given below.

The hybrid nanofluid density  $\rho_{\text{hnf}}$  is given as:

$$\rho_{\text{hnf}} = \varphi_{\text{Cu}}\rho_{\text{Cu}} + \varphi_{\text{Al}_2\text{O}_3}\rho_{\text{Al}_2\text{O}_3} + (1 - \varphi_{\text{Cu}} - \varphi_{\text{Al}_2\text{O}_3})\rho_f \quad (8)$$

The hybrid nanofluid heat capacitance  $(\rho c_p)_{\text{hnf}}$  is given as:

$$(\rho c_p)_{\text{hnf}} = \varphi_{\text{Cu}}(\rho c_p)_{\text{Cu}} + \varphi_{\text{Al}_2\text{O}_3}(\rho c_p)_{\text{Al}_2\text{O}_3} + (1 - \varphi_{\text{Cu}} - \varphi_{\text{Al}_2\text{O}_3})(\rho c_p)_f \quad (9)$$

The hybrid nanofluid buoyancy coefficient  $(\rho\beta)_{\text{hnf}}$  can be calculated via:

$$(\rho\beta)_{\text{hnf}} = \varphi_{\text{Cu}}(\rho\beta)_{\text{Cu}} + \varphi_{\text{Al}_2\text{O}_3}(\rho\beta)_{\text{Al}_2\text{O}_3} + (1 - \varphi_{\text{Cu}} - \varphi_{\text{Al}_2\text{O}_3})(\rho\beta)_f \quad (10)$$

The dynamic viscosity ratio of a nano-fluid can be determined using the method developed by Corcione *et al.*, [25]:

$$\frac{\mu_{\text{nf}}}{\mu_f} = 1/1 - 34.87 \left(\frac{d_p}{d_f}\right)^{-0.3} \varphi^{1.03} \quad (11)$$

Finally, the thermal conductivity ratio of the nanofluid is determined as [25]:

$$\frac{k_{\text{nf}}}{k_f} = 1 + 4.4 \text{Re}_B^{0.4} \text{Pr}^{0.66} \left(\frac{T}{T_{fr}}\right)^{10} \left(\frac{k_p}{k_f}\right)^{0.03} \varphi^{0.66} \quad (12)$$

where  $T_{fr}$  is the freezing point of the base fluid (273.15K).

Based on these mathematical models, the dynamic viscosity ratio and the thermal-conductivity ratio of the hybrid nano-fluids (Al<sub>2</sub>O<sub>3</sub>-Cu)-water of particle sizes 33 nm and 29 nm in the ambient condition can be calculated as:

$$\frac{\mu_{hnf}}{\mu_f} = 1/1 - 34.87 (d_f)^{0.3} [(d_{Cu})^{-0.3} (\varphi_{Cu})^{1.03} + (d_{Al_2O_3})^{-0.3} (\varphi_{Al_2O_3})^{1.03}] \quad (13)$$

$$\frac{k_{hnf}}{k_f} = 1 + 4.4 Re_B^{0.4} Pr^{0.66} \left(\frac{T}{T_{fr}}\right)^{10} (k_f)^{-0.03} [(k_{Cu})^{0.03} (\varphi_{Cu})^{0.66} + (k_{Al_2O_3})^{0.03} (\varphi_{Al_2O_3})^{0.66}] \quad (14)$$

where  $Re_B$  defined for hybrid nano-fluid is:

$$Re_B = \frac{\rho_f u_B (d_{Cu} + d_{Al_2O_3})}{\mu_f} \quad (15)$$

$$u_B = \frac{2 k_b T}{\pi \mu_f (d_{Cu} + d_{Al_2O_3})^2} \quad (16)$$

Here;  $k_b = 1.380648 \times 10^{-23}$  (J/K) is the Boltzmann-constant,  $l_f = 0.17$  nm is the mean-path of fluid particles and,  $d_f$  is the molecular-diameter of water [25]:

$$d_f = \frac{6 M}{N^* \pi \rho_f} \quad (17)$$

where;  $M$  denotes the molecular-mass of the working-fluid,  $N^*$  define is the Avogadro-number and,  $\rho_f$  is the working fluid-density at normal temperature (310K). In the present work, the non-dimensional variables are presented as:

$$X = \frac{x}{L}, Y = \frac{y}{L}, U = \frac{u}{U_0}, V = \frac{v}{U_0}, \theta = \frac{T-T_c}{T_h-T_c} = \frac{T-T_c}{\Delta T}, Pr = \frac{\nu_f}{\alpha_f}, P = \frac{\rho L^2}{\rho_f \alpha_f^2}, Ri = \frac{Gr}{Re^2} \quad (18)$$

Then, the non-dimensional governing equations:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (19)$$

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{Re} \frac{\mu_{hnf}}{\mu_f} \frac{\rho_f}{\rho_{hnf}} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) \quad (20)$$

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{Re} \frac{\mu_{hnf}}{\mu_f} \frac{\rho_f}{\rho_{hnf}} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + \left( \frac{(\rho\beta)_{hnf}}{\rho_{hnf}\beta_f} \right) Ri \theta \quad (21)$$

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{\alpha_{hnf}}{\alpha_f} \frac{1}{Pr Re} \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) \quad (22)$$

As well as the dimensionless of boundary conditions are:

$$\text{The top wall-side: } U = 1; V = 0; \theta = 0 \quad (23)$$

$$\text{The bottom wall-side: } U = 0; V = 0; \theta = 1 \text{ and} \quad (24)$$

$$\text{The left and, right wall-sides: } U = V = 0; \frac{\partial \theta}{\partial X} = 0 \quad (25)$$

The local Nusselt numbers for the cold wall, hot wall and the average Nusselt number are given in Eqs. (26-28), respectively:

$$Nu_x = -\frac{k_{hnf}}{k_f} \left(\frac{\partial\theta}{\partial Y}\right)_{Y=0} \quad (26)$$

$$Nu_x = -\frac{k_{hnf}}{k_f} \left(\frac{\partial\theta}{\partial Y}\right)_{Y=1} \quad (27)$$

$$\overline{Nu} = \int_0^D Nu_x dx \quad (28)$$

### 3. Numerical Technique

The governing equations are solved numerically using FVM [26]. The convection term is approximated using the power-law scheme. The SIMPLE algorithm is used for pressure-velocity coupling. Then, the algebraic system of equations is solved using the TDMA algorithm written in FORTRAN 90 programming language. The relaxation factor is set below 0.5 for momentum and energy equations in order to obtain convergence. The convergence criterion is calculated as:

$$\text{error} = \frac{\sum_{j=1}^m \sum_{i=1}^n |\eta_{i,j}^{k+1} - \eta_{i,j}^k|}{\sum_{j=1}^m \sum_{i=1}^n |\eta_{i,j}^{k+1}|} \leq 10^{-7} \quad (29)$$

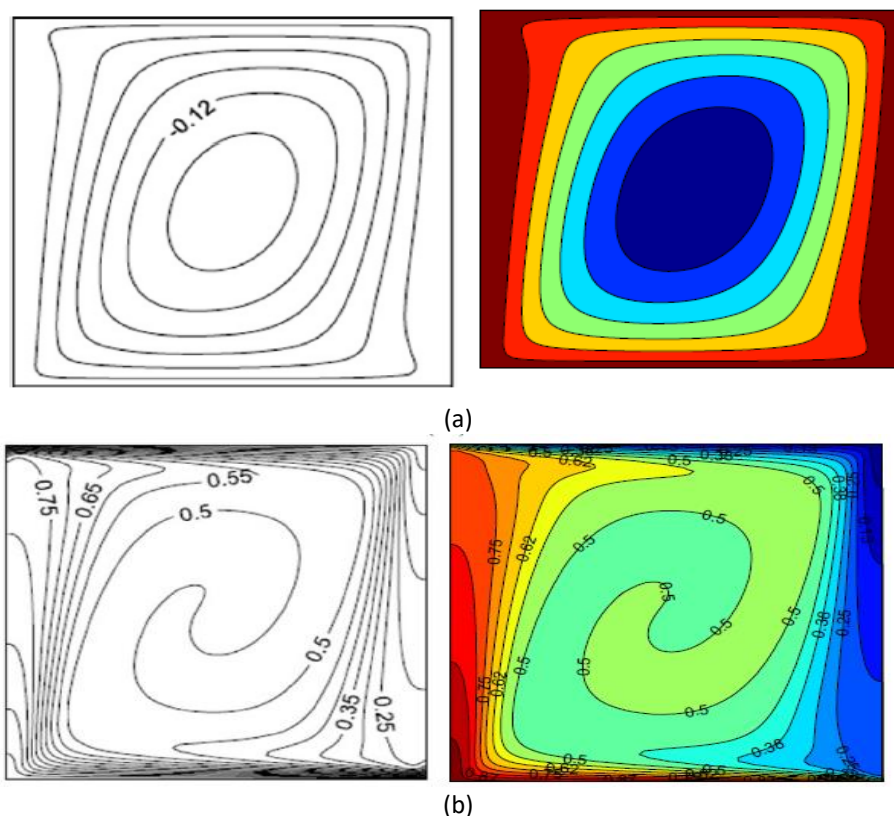
where m and n are denoted as the grid-point numbers in the x and, y directions, respectively,  $\eta$  is any transport quantity and k is the iteration number.

### 4. Mesh Independent and Validation

To check for grid-independence, simulations using seven different grid sizes, i.e.  $80 \times 40$ ,  $100 \times 50$ ,  $120 \times 60$ ,  $140 \times 70$  are executed. The numerical setting can be found in Table. 1. Based on the table, the result obtained on the  $120 \times 60$  grid is already grid - independent. The result has been compared to that of Ismael *et al.*, [27] and good agreement has been found as shown in Figure 2.

**Table 1**  
 Mesh convergence

Size	Average Nusselt number $\overline{Nu}$
80x40	4.869276
100x50	4.889277
120x60	4.897964
140x70	4.900624



**Fig. 2.** Streamlines patterns (a) in left Ismael *et al.*, [27]; in right present study and, validation of isotherms (lines) (b) in left Ismael *et al.*, [27], in right present study

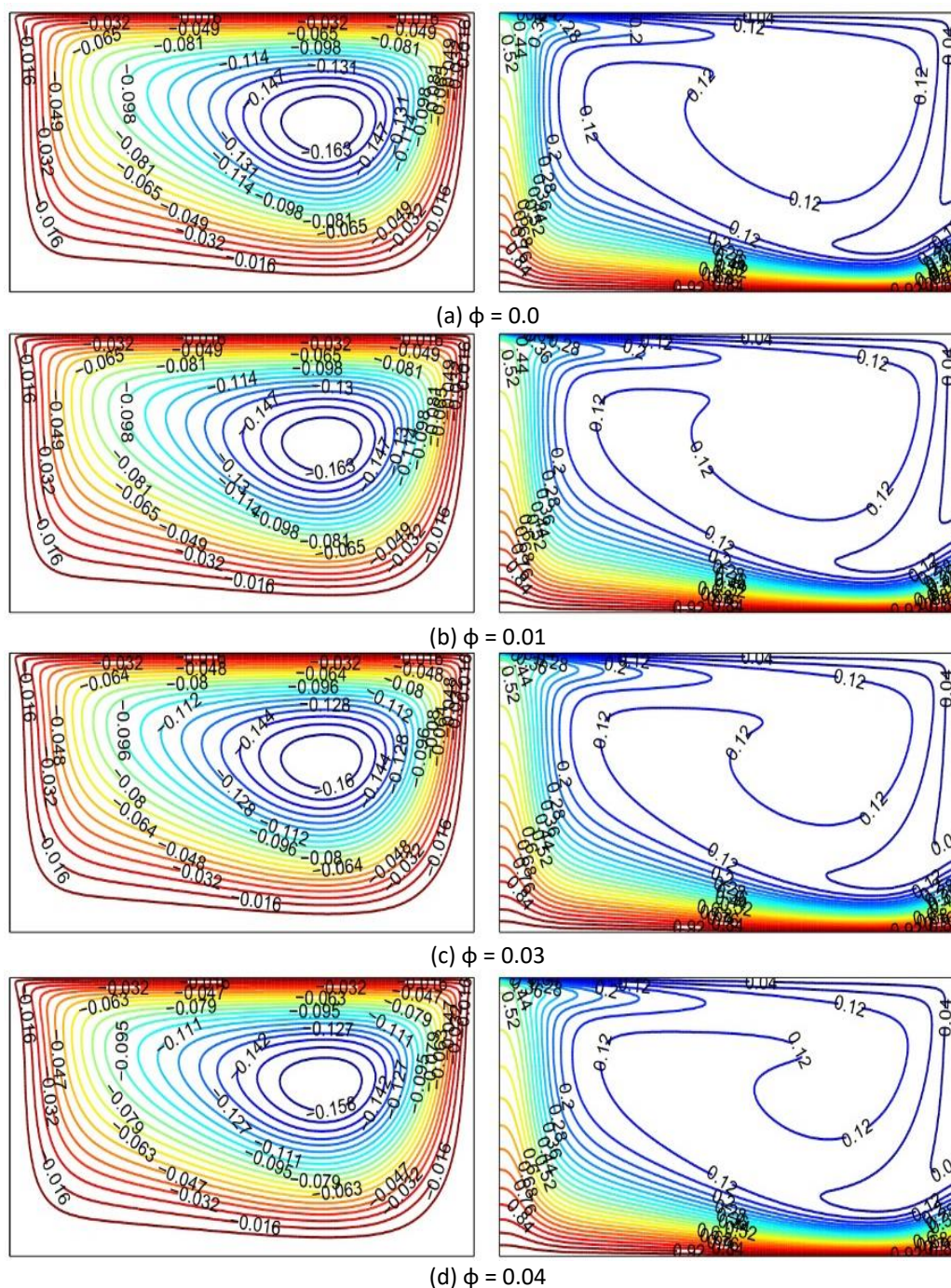
## 5. Results and Discussion

The heat transfer properties within the rectangular cavity filled with hybrid nano-fluid ( $\text{Al}_2\text{O}_3\text{-Cu-H}_2\text{O}$ ) have been studied. The effects of parameters such as volume fraction and Reynolds number on the heat transfer with  $\text{Pr} = 6.2$  have been investigated. Streamlines, isotherms and averaged Nusselt number have been investigated. The thermo-physical characteristics of the basic fluid ( $\text{H}_2\text{O}$ ) and the  $\text{Al}_2\text{O}_3 + \text{Cu}$  nanoparticles are reported in Table 2. Figure 3 shows the streamlines and isotherms at  $\text{Re} = 10$ ,  $L = 2.5$ ,  $\text{Ri} = 1$ . A primary recirculation cell rotating in the clockwise direction can be observed. As the volume fraction of the hybrid nano-particle increases, the intensity of the vortex increases, and the streamlines move closer to each other. The isothermal lines are clustered near the isothermal walls, indicating that the temperature gradients are relatively high in these regions. As the volume fraction of nano-particle increases, the isothermal lines become less congested.

**Table 2**

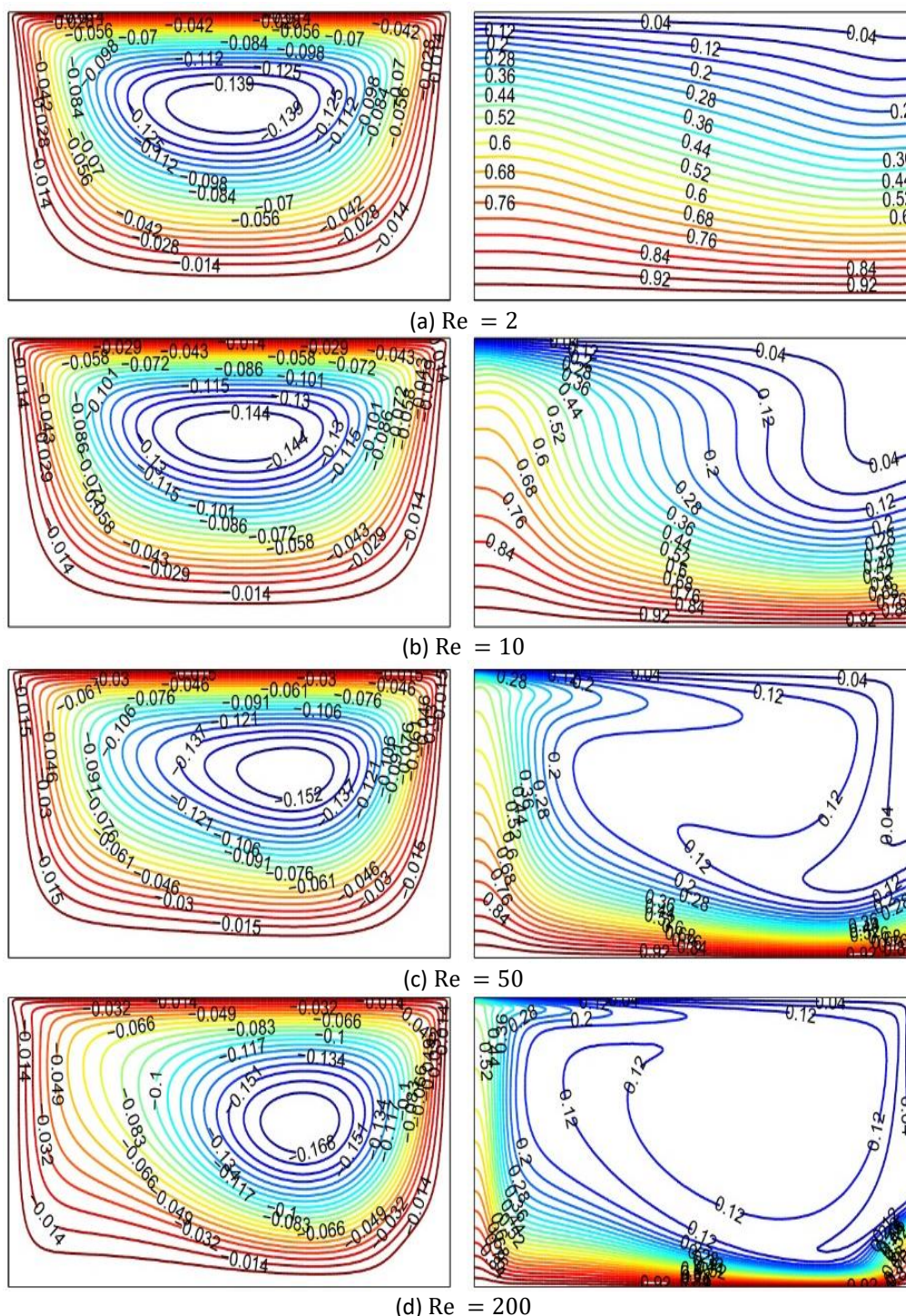
Thermo-physical characteristics of  $\text{H}_2\text{O}$ , Cu,  $\text{Al}_2\text{O}_3$  nanoparticles ( $T = 310 \text{ K}$ ) [28]

Physical properties	Base fluid	Cu	$\text{Al}_2\text{O}_3$
$k (\text{Wm}^{-1}\text{K}^{-1})$	0.628	400	40
$\mu \times 10^6 (\text{kg/ms})$	695	-	-
$\rho (\text{kg/m}^3)$	993	8933	3970
$C_p (\text{J/kgK})$	4178	385	765
$\beta \times 10^{-5} (1/\text{K})$	36.2	1.67	0.85
$d_p (\text{nm})$	0.385	29	33



**Fig. 3.** Variations of streamlines in left and, isotherms in right, when (a)  $\phi = 0.0$ , (b)  $\phi = 0.01$ , (c)  $\phi = 0.03$ , (d)  $\phi = 0.04$

Figure 4 shows the effect of  $Re$  on the streamlines and isotherms at  $Ri = 10, L = 2.5, \varphi = 0.02$ . There is a central vortex rotating in the clockwise direction. The isotherms at  $Re = 1$  follow closely to those observed in the pure conduction case. Pure conduction occurs due to weak convection as shown in Figure 4(a) and (b). Stratification tends to be more apparent as  $Re$  is increased up to 10. By further increasing the  $Re$  as shown in Figure 4(c) and (d), the cavity center tends to become isothermal.



**Fig. 4.** Variations for streamlines in left and, isotherms in right, when (a)  $Re = 2$ , (b)  $Re = 10$ , (c)  $Re = 50$ , (d)  $Re = 200$

Figure 5 shows the distribution of local (area) Nusselt number along the hot wall for different  $Re$  values ( $Ri = 10$ ,  $\varphi = 0.02$  and  $L = 2$ ). As shown in Figure 6, the local Nusselt number increases with respect to  $Re$  due to more intense mixing. Figure 7 shows the effect of volume fraction on the local Nusselt number ( $Re = 10$ ,  $Ri = 10$ ,  $L = 2$ ). As expected, the increase in volume fraction can enhance the local Nusselt number in the warm regions.



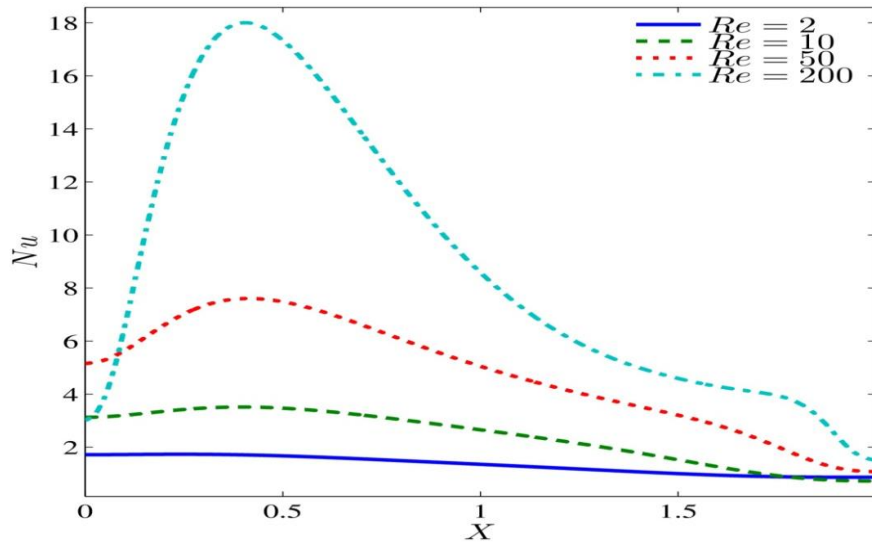


Fig. 5. Variations of the local Nusselt-number against X for different Re

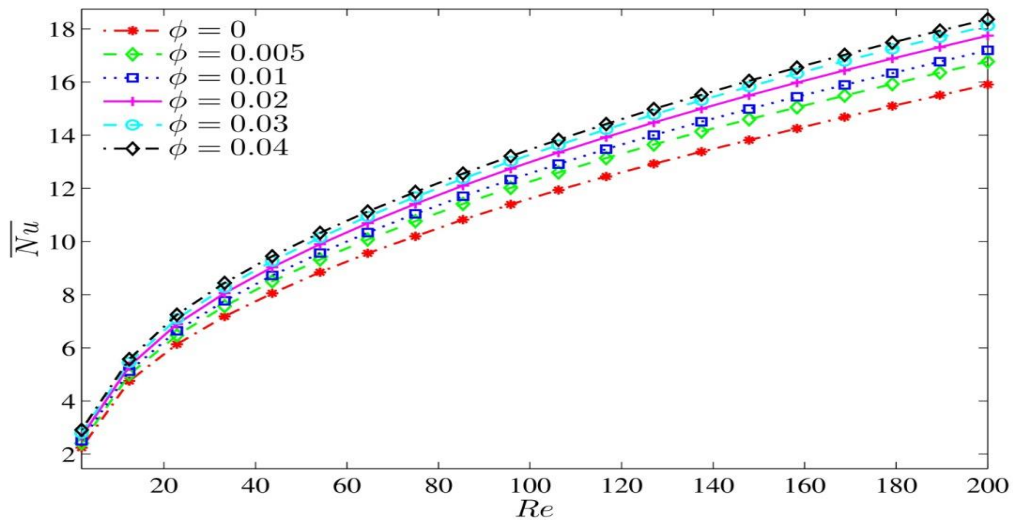


Fig. 6. Variations of the average Nusselt-number against Re for different  $\phi$

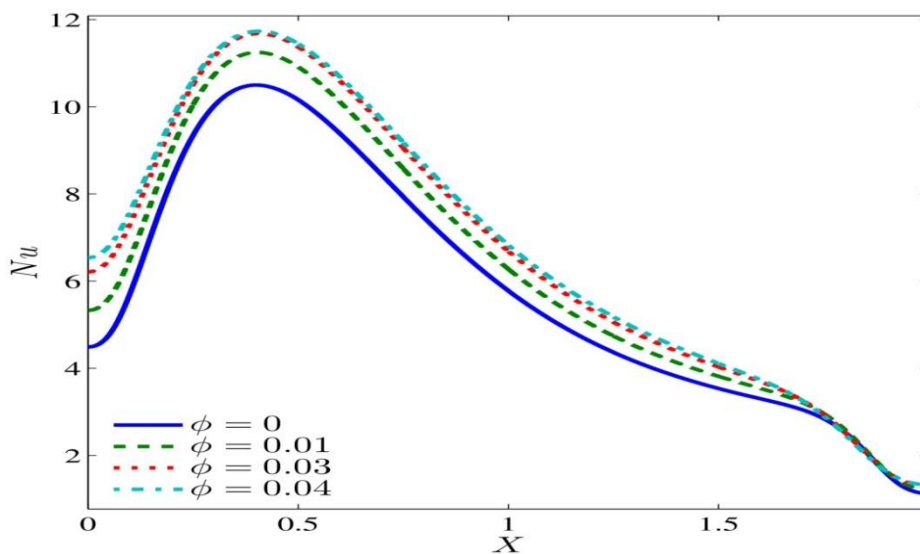


Fig. 7. Variations of the average Nusselt-number against X for different  $\phi$

## 6. Conclusions

The simulation model of a rectangular cavity filled with hybrid nano-fluid ( $\text{Al}_2\text{O}_3\text{-Cu-water}$ ) has been solved using the SIMPLE algorithm. The effect of hybrid nano-particle volume fraction on the heat transfer for different Reynolds numbers (2, 10, 50, 200) has been investigated. Various patterns of streamline have been found. The effect of Reynolds number on the local and average Nusselt numbers is quite significant. The increase in hybrid nano-particle volume fraction ( $\text{Al}_2\text{O}_3\text{-Cu}$ ) can enhance the Nusselt number.

## Acknowledgement

The authors would like to acknowledge the financial support received from the Ministry of Higher Education Malaysia (Grants no FRGS/UTHM/K172).

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