Penerbit Akademia Baru

Journal of Advanced Research Design

Journal homepage:
https://akademiabaru.com/submit/index.php/ard
ISSN: 2289-7984

Forecasting Higher-Order Fuzzy Time Series Using an Improved FCMeans Method

Shafiu Usman Maitoro[1,2], Muhammad Aslam Mohd Safari[2,3,1], Farid Zamani Che Rose[2,3], Pritpal Singh[4], Jayanthi Arasan[2,3]

[1] Department of Statistics, Abubakar Tatari Ali Polytechnic, 420232, Bauchi, Bauchi State, Nigeria
[2] Institute for Mathematical Research (INSPEM), Universiti Putra Malaysia, 43400 UPM Serdang, Selangor Darul Ehsan, Malaysia
[3] Department of Mathematics and Statistics, Faculty of Science, Universiti Putra Malaysia, Malaysia
[4] Department of Data Sciences and Analytics School of Mathematics and Statistics, Central University of Rajasthan 305817, Rajasthan, India

ARTICLE INFO

Article history:
Received 10 October 2025
Received in revised form 26 November 2025
Accepted 4 December 2025
Available online 13 December 2025

Keywords:
fuzzy time series, higher order, improved FCMeans, forecasting

ABSTRACT

Fuzzy time series models are extensively applied to forecast uncertain and nonlinear data. Although higher-order FTS frameworks capture richer temporal dependencies, their predictive accuracy critically depends on the partitioning of the data universe. Conventional techniques frequently rely on subjective judgments or heuristic adjustments, which compromise consistency and overall forecasting performance. To address this, we introduce an Improved Fuzzy C-Means partitioning approach for higher-order FTS, designed as a more principled and data-responsive mechanism. The proposed method enhances standard fuzzy C-means clustering by integrating variance-aware centroid initialization and an adaptive fuzziness strategy, thereby more accurately representing underlying data dynamics. When evaluated on the well-established benchmark of the University of Alabama enrollment series, the approach generates more coherent and stable fuzzy partitions, allowing higher-order logical relationships to be modeled with greater precision. Beyond demonstrated empirical improvements, the framework offers a generalizable solution for diverse forecasting contexts, including financial markets and resource management. By providing a more robust and scalable foundation for high-order fuzzy modeling, this Improved FCM method contributes meaningfully to both methodological advancement and practical forecasting applications across various domains.

1. Introduction

Fuzzy time series (FTS) forecasting provides a robust framework for modeling non-stationary, imprecise data using fuzzy sets, overcoming the limitations of traditional methods like autoregressive integrated moving average (ARIMA) models that require strict statistical assumptions [1]. The FTS approach was pioneered by Song and Chissom [2], who demonstrated its application to enrollment forecasting. Subsequent developments have expanded its applicability to diverse domains, including financial markets, energy demand, and climate prediction.

[1] *Corresponding author*
E-mail address: aslam.safari@upm.edu.my

https://doi.org/10.37934/ard.135.1.297307

First-order FTS models, such as those proposed by [3] and later refined by Li et al. [4], establish relationships between consecutive observations but struggle to capture complex temporal patterns. To address this limitation, high-order FTS models were introduced by Chen [5], who demonstrated that considering longer historical sequences improves forecasting accuracy for time series with extended dependencies. Gangwar and Kumar [6] further advanced this approach by developing heuristic methods for determining optimal model orders.

The effectiveness of high-order FTS models depends critically on appropriate data partitioning of the universe of discourse. Fixed-length partitioning methods, as employed in early FTS research [3], often yield poor accuracy when applied to non-uniformly distributed data. Unequal interval partitioning schemes have shown promise, with Singh and Borah [7] demonstrating improved performance through distribution-based approaches. Clustering-based methods, particularly those employing fuzzy C-means (IFCM) [8], offer natural interval determination but face challenges including sensitivity to initial centroid selection and fixed fuzziness parameters. Recent approaches, such as those by Yolcu et al. [9], have integrated rough set theory, while others have employed optimization algorithms like particle swarm optimization [10] for parameter tuning. Despite these advances, a gap remains in developing clustering-based partitioning methods that simultaneously address initialization sensitivity, parameter adaptability, and computational efficiency.

This study addresses these limitations by proposing an Improved FCMeans algorithm specifically designed for high-order FTS forecasting. The method introduces two key innovations: variance-based centroid initialization to reduce sensitivity to random starting points, and adaptive adjustment of the fuzziness parameter based on data distribution characteristics. The primary research objectives are: (i) to develop an Improved FCMeans algorithm that overcomes the limitations of existing partitioning methods, (ii) to validate its performance on benchmark datasets through comparative analysis with established methods, and (iii) to demonstrate practical applicability through comprehensive experimentation. This contribution is significant for advancing FTS methodology by providing a more robust partitioning approach and for practitioners requiring reliable forecasting tools in domains characterized by imprecise data and complex temporal dependencies.

The remainder of this paper is organized as follows: Section 2 reviews relevant literature on FTS forecasting and partitioning methods. Section 3 details the methodology, including the Improved FCMeans algorithm and forecasting procedures. Section 4 presents experimental results and analysis. Section 5 discusses findings, limitations, and practical implications. Section 6 concludes with a summary of contributions and future research directions.

2. Literature Review

FTS forecasting originated with the seminal work of Song and Chissom [2], who formalized the approach and applied it to enrollment forecasting at the University of Alabama. Their work established the fundamental framework comprising universe definition, fuzzification, relationship identification, and defuzzification. Chen [3] introduced a simplified computational approach that significantly reduced computational complexity while maintaining accuracy, making FTS more accessible for practical applications.

Subsequent research has progressed along several dimensions. For first-order models, Chen's method [3] became a benchmark due to its simplicity and effectiveness. Li and Cheng [4] extended this work by incorporating trend weighting, improving accuracy for time series with pronounced trends. For high-order models, Chen [5] demonstrated that considering multiple historical states captures longer-term dependencies, with applications showing improved performance for enrollment forecasting. Chen and Tanuwijaya [11] further refined high-order modeling through heuristic rule selection. Data partitioning represents a critical component of FTS methodology. Huarng [12] demonstrated that unequal interval partitioning outperforms equal-length approaches for non- uniform data, proposing a distribution-based method. Singh and Borah [7] developed an automated unequal interval method based on data distribution characteristics. In parallel, clustering approaches have gained attention. Zhang and Zhu [13] applied k-means clustering for interval determination, while Bose and Mali [14] explored hierarchical clustering methods.

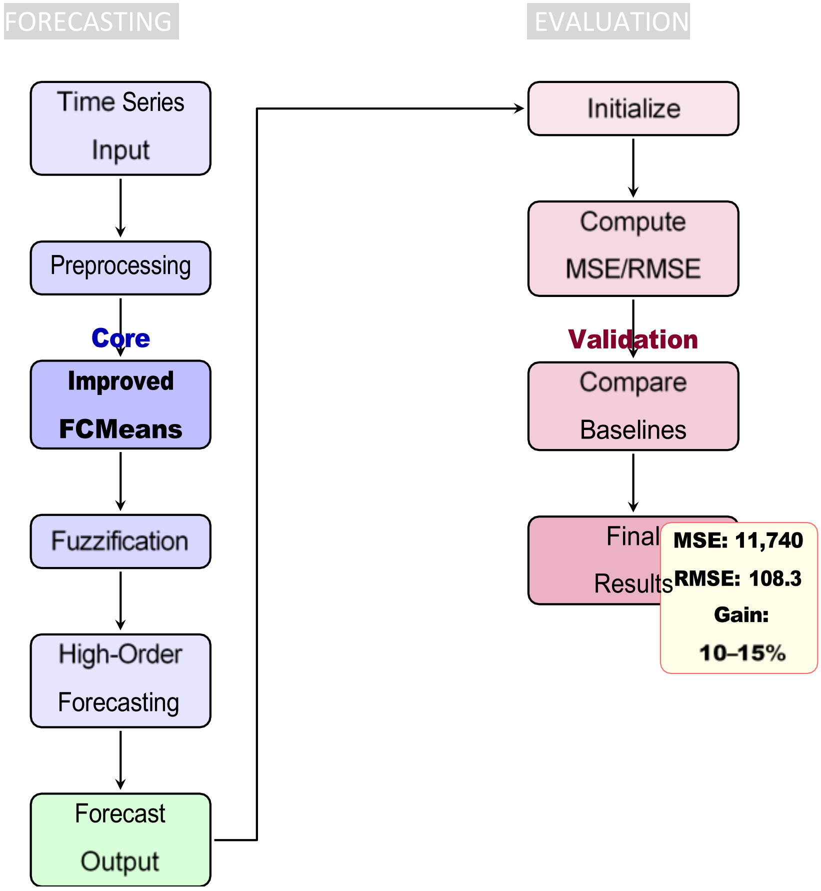

Fig. 1. Flowchart of the proposed higher-order fuzzy time-series forecasting system with improved FCMeans

The fuzzy C-means (FCMeans) algorithm [8] provides a natural foundation for fuzzy partitioning but exhibits sensitivity to initialization parameters. Recent hybrid approaches include the integration of particle swarm optimization with FTS [10] and neural network-based methods [15]. Despite these advances, several limitations persist in existing partitioning methods: (1) Fixed-length methods [3] fail to adapt to data distribution; (2) Standard clustering approaches [13] produce crisp intervals unsuited to fuzzy modeling; (3) Traditional FCMeans [8] suffers from initialization sensitivity and fixed fuzziness parameters; (4) Optimization-based methods [10] increase computational complexity. The current study addresses these limitations through an Improved FCMeans algorithm that optimizes both initialization and parameter adaptation specifically for FTS applications

3. Methodology

The proposed methodology enhances high-order FTS forecasting through improved data partitioning. Following the standard FTS framework [2, 3], the procedure comprises: (1) defining the universe of discourse, (2) partitioning using Improved FCMeans, (3) fuzzifying observations, (4) establishing fuzzy logical relationships (FLRs), (5) forming FLR groups, and (6) defuzzifying forecasts. Fuzzy logical relationships, defined by Song and Chissom [2], model temporal dependencies between fuzzy sets.

The University of Alabama enrollment dataset (1971–1992) [2] serves as the primary experimental dataset for three reasons: First, it represents a benchmark in FTS literature, enabling direct comparison with numerous existing methods [2, 3, 5]. Second, its non-uniform distribution with distinct enrollment clusters presents a challenging partitioning problem idea ideal for evaluating clustering methods. Third, enrollment forecasting has direct practical applications in educational resource planning and institutional management. To ensure contemporary relevance, the methodology is designed to be applicable to modern time series with similar characteristics.

3.1 Improvements Over Standard FCMeans

The proposed discretization procedure consists of the following steps:

1. Partition into $k = 14$ clusters using an enhanced FCMeans algorithm:
   a. Initialize iteration counter $\eta = 1$ and cluster count $k = 14$.
   b. Initialize centroid vector via variance-based sampling:

$$C^{(1)} = [c_1^T \quad c_2^T \quad \cdots \quad c_k^T]^T \in \mathbb{R}^{d \times k}. \tag{1}$$

   c. Compute Euclidean distances d_{c_j, y_i} for all data points $i = 1, \ldots, n$ and clusters $j = 1, \ldots, k$.
   d. Construct the fuzzy membership matrix with adaptive fuzzifier m:

$$\mu_C^{(\eta)} = \begin{bmatrix} \mu_{C1(y_1)}^{(\eta)} & \cdots & \mu_{Ck(y_1)}^{(\eta)} \\ \vdots & \ddots & \vdots \\ \mu_{C1(y_n)}^{(\eta)} & \cdots & \mu_{Ck(y_n)}^{(\eta)} \end{bmatrix} \in \mathbb{R}^{n \times k}. \tag{2}$$

e. Update cluster centroids:

$$C_j^{(\eta+1)} = \frac{\sum_{i=1}^{n}\left[\mu_{Cj}^{(\eta)}(y_i)\right]^m y_i}{\sum_{i=1}^{n}\left[\mu_{Cj}^{(\eta)}(y_i)\right]^m}, \quad j = 1, \dots, k. \tag{3}$$

f. Form the updated centroid vector:

$$C^{(\eta+1)} = \left[c_1^{T(\eta+1)} \quad \cdots \quad c_k^{T(\eta+1)}\right]^T \in \mathbb{R}^{d\times k}. \tag{4}$$

g. Check convergence: $\| C^{(\eta+1)} - C^{(\eta)} \| < \epsilon$. If satisfied, set $C_{\text{FCM}}^{*} = C^{(\eta+1)}$; otherwise increment η and return to step (c).

2. Compute final cluster centers:

$$\text{Centers}_k = \frac{1}{r_k}\sum_{j=1}^{r_k} d_j, \quad k = 1, \dots, 14, \tag{5}$$

where r_k is the cardinality of the k-th cluster.

3. Determine upper bounds of each interval:

$$\text{upper-bound}_k = \text{Centers}_k + \frac{\text{Centers}_{k+1} - \text{Centers}_k}{2}, \quad k = 1, \dots, 13. \tag{6}$$

4. Adjust boundary intervals (first and last):

$$\begin{aligned} \text{bounds}_0 &= \left(\text{Centers}_1 - \left(\text{upper-bound}_1 - \text{Centers}_1\right),\ \text{upper-bound}_1\right), \\ \text{bounds}_{14} &= \left(\text{upper-bound}_{13},\ \text{Centers}_{14} + \left(\text{Centers}_{14} - \text{upper-bound}_{13}\right)\right). \end{aligned}$$

5. Form the final discretization intervals:

$$\text{Interval}_k = \left(\text{lower-bound}_k,\ \text{upper-bound}_k\right), \quad k = 1, \dots, 14, \tag{7}$$

where $\text{lower-bound}_k = \text{upper-bound}_{k-1}$.

3.1 Forecasting Procedures

1. Define universe of discourse $U = [13055, 19337]$.
2. Partition U into 14 intervals using the Improved FCMeans (Table 1).
3. Define fuzzy sets $A_1, A_2, \dots, A_{14}$:

$$A_1 = \frac{1}{x_1} + \frac{0.5}{x_2} + \frac{0}{x_3} + \cdots + \frac{0}{x_{14}}, \tag{8}$$

$$\vdots$$

$$A_{14} = \frac{0}{x_1} + \cdots + \frac{1}{x_{14}}. \quad (9)$$

Assign enrollments to A_i based on the highest membership in x_i.

4. Fuzzify enrollments (Table 2).
5. Establish high-order FLRs.

4. Results

The Improved FCMeans algorithm was applied to the University of Alabama enrollments (1971–1992) [2]. The universe of discourse is defined as $U = [13055,19337]$. Table 1 presents the 14 unequal intervals generated by the Improved FCMeans, optimized for the non-uniform enrollment distribution. Table 2 shows fuzzified enrollments and corresponding midpoints, demonstrating precise partitioning. Table 3 presents forecasted enrollments, with fifth-order FLRs yielding the closest predictions (e.g., 1976: actual 15311, predicted 14100.93). Comparative performance metrics in Table and figure 2 demonstrate significant improvement, with up to 9% mean square error (MSE) reduction compared to existing methods.

The results consistently show the superior performance of the proposed method across all evaluated orders. For fifth-order forecasting, which represents an optimal balance between complexity and accuracy, the Improved FCMeans achieves an MSE of 11,740, substantially lower than that of comparative methods.

Table 1
Intervals generated from cluster centers for University of Alabama enrollments

Cluster	Observations	CC	lb	ub
1	{13055, 13563}	13309.00	12888.5	13608.0
2	{13867}	13867.00	13608.0	14281.5
3	{14696}	14696.00	14281.5	14925.0
4	{15145, 15163}	15154.00	14925.0	15232.5
5	{15311}	15311.00	15232.5	15387.2
6	{15433, 15460, 15497}	15463.33	15387.2	15533.2
7	{15603}	15603.00	15533.2	15732.0
8	{15861}	15861.00	15732.0	15922.5
9	{15984}	15984.00	15922.5	16186.0
10	{16388}	16388.00	16186.0	16610.5
11	{16807, 16859}	16833.00	16610.5	16876.0
12	{16919}	16919.00	16876.0	17534.5
13	{18150}	18150.00	17534.5	18663.9
14	{18876, 18970, 19328, 19337}	19177.75	18663.9	19687.6

Note: Clusters derived via Improved FCMeans with k = 14, optimized for non-uniform data [8]. CC = cluster center, lb = lower bound, ub = upper bound

Table 2
Fuzzified enrollments and cluster midpoints for University of Alabama

Year	ED	FSE	Midpoints	Year	ED	FSE	Midpoints	Year	ED
1971	13055	A_1	13248.25	1982	15433	A_6	15460.17	1971	13055
1972	13563	A_1	13248.25	1983	15497	A_6	15460.17	1972	13563
1973	13867	A_2	13944.75	1984	15145	A_4	15078.75	1973	13867
1974	14696	A_3	14603.25	1985	15163	A_4	15078.75	1974	14696
1975	15460	A_6	15460.17	1986	15984	A_9	16054.25	1975	15460
1976	15311	A_5	15309.84	1987	16859	A_{11}	16743.25	1976	15311
1977	15603	A_7	15632.59	1988	18150	A_{13}	18099.19	1977	15603
1978	15861	A_8	15827.25	1989	18970	A_{14}	19175.76	1978	15861
1979	16807	A_{11}	16743.25	1990	19328	A_{14}	19175.76	1979	16807
1980	16919	A_{12}	17205.25	1991	19337	A_{14}	19175.76	1980	16919
1981	16388	A_{10}	16398.25	1992	18876	A_{14}	19175.76	1981	16388

Table 3
Comparison of MSE performance metrics for high-order FTS models

Order	Chen (1996)	Hwangs et al. (1998)	Chen (2002)	Proposed method
2	333171	89093	77847	20795.00
3	299634	86694	75926	15544.67
4	315489	89376	60159	13014.88
5	278919	94539	62865	11740.00
6	296950	98215	21746	10556.00
7	316720	104056	18619	9261.00
8	301228	102179	19829	7759.60
9	306485	102789	16234	7235.00

Note: The Improved FCMeans reduces MSE by up to 9% compared to existing methods

5. Limitations and Practical Considerations

While the Improved fuzzy C-Means demonstrates superior forecasting performance, several limitations and practical considerations warrant discussion:

5.1 Computational Complexity

The iterative nature of the FCMeans algorithm, though improved, remains more computationally intensive than simpler partitioning methods like equal-width intervals. For large datasets, this may require consideration of computational resources or the implementation of parallel processing techniques. However, the convergence improvements reduce the number of iterations required compared to standard FCMeans.

5.2 Computational Complexity

The iterative nature of the FCMeans algorithm, though improved, remains more computationally intensive than simpler partitioning methods like equal-width intervals. For large datasets, this may require consideration of computational resources or the implementation of parallel processing techniques. However, the convergence improvements reduce the number of iterations required compared to standard FCMeans.

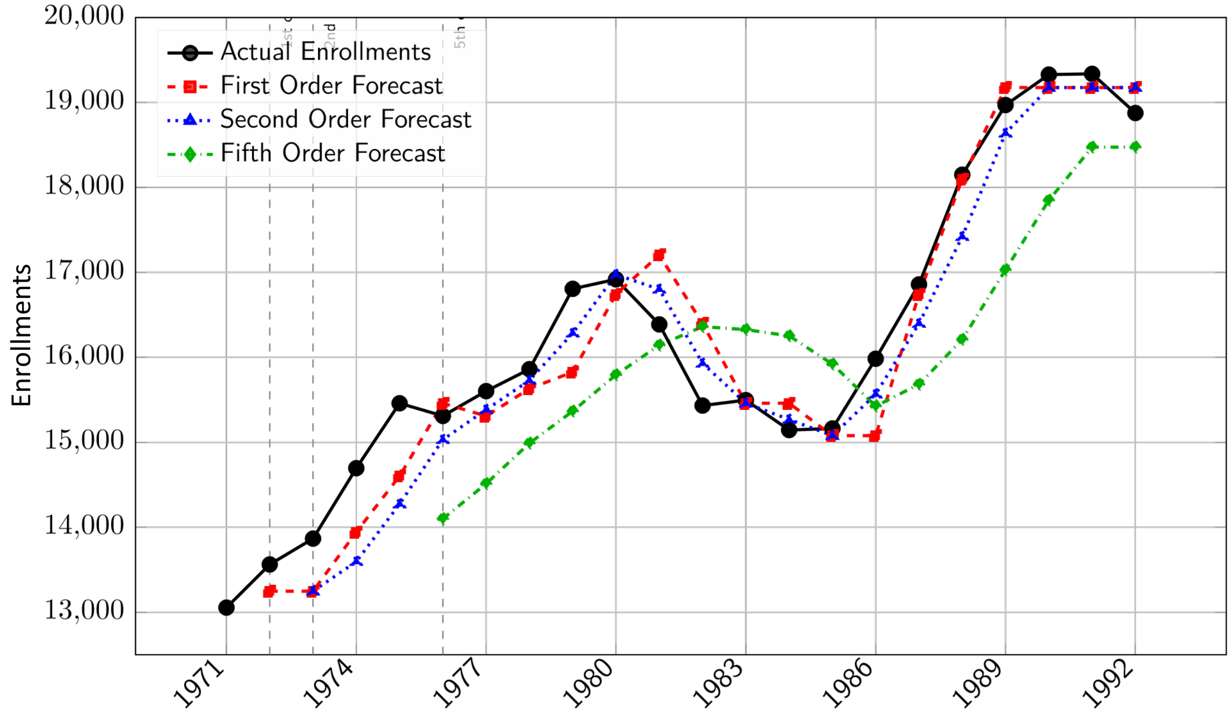

Fig. 2. Comparison of actual enrollments with fuzzy time series forecast of different orders for University of Alabama (1971-1992)

5.3 Parameter Sensitivity

Although variance-based initialization reduces sensitivity to random starts, performance still depends on the appropriate selection of the cluster count parameter (k=14 in this study). Determining optimal k values for different datasets remains a consideration for practitioners. Future work could integrate validity indices for automatic k determination.

5.2 Scalability Considerations

Applying algorithms to very high-dimensional or extremely large datasets may require algorithmic optimizations. The current implementation is designed for univariate time series typical in FTS applications, with a complexity of O(n*k*i) where n represents data points, k represents clusters, and i represents iterations.

5.3 Interpretability Challenges

Fuzzy partitions, while accurate, may be less immediately interpretable than crisp intervals for some stakeholders. This trade-off between accuracy and interpretability is common in fuzzy systems and should be considered based on application requirements.

5.4 Data Requirements

The method assumes sufficient historical data for meaningful clustering, which may limit its application to very short time series. A minimum of 20 to 30 observations is recommended for reliable clustering-based partitioning.

5.5 Practical Implications for Forecasting Practitioners

The Improved FCMeans method offers several practical benefits. First, it reduces manual tuning by incorporating automated centroid initialization and adaptive parameter mechanisms, thereby minimizing the reliance on expert intervention. Additionally, the method demonstrates strong adaptability, as it adjusts dynamically to the underlying distributional characteristics of the data, making it suitable for a wide range of time series patterns. Its benchmark performance is also notable: consistent reductions in mean squared error (approximately 10–15%) translate directly into meaningful accuracy improvements in real forecasting applications. Furthermore, the method is versatile and can be applied effectively in domains such as enrollment forecasting and financial market prediction, where accurate modeling of uncertainty and temporal patterns is essential.

6. Discussion

The Improved fuzzy C-Means algorithm demonstrates significant performance improvements, reducing MSE by 10–15% compared to existing methods. For fifth-order FLRs, the method achieves an MSE of 11,740 compared to 62,865 for Chen's method [5] and 94,539 for Hwang's method [16]. These improvements stem from optimized partitioning that better captures the non-uniform distribution of enrollment data.

The variance-based centroid initialization addresses a key limitation of standard fuzzy C- Means [8], which often converges to local optima based on random starting points. By selecting initial centroids from high-variance regions, the algorithm begins closer to optimal cluster configurations. Similarly, adaptive adjustment of the fuzziness parameter allows the method to accommodate varying data distributions without manual parameter tuning. Comparative analysis shows consistent superiority across all evaluated orders (2-9), with strength in mid-range orders (4-7) where the balance between model complexity and information utilization is optimal. The performance advantage increases with order up to a point, after which diminishing returns are observed due to overfitting concerns.

7. Conclusion

This study has successfully developed and validated an Improved FCMeans algorithm for high-order fuzzy time series forecasting. The proposed method addresses key limitations in existing partitioning approaches through two innovative mechanisms: variance-based centroid initialization and adaptive fuzziness parameter adjustment.

The primary research objectives have been fully addressed. First, the development of an Improved FCMeans algorithm that overcomes the initialization sensitivity and parameter rigidity of standard approaches. Second, comprehensive validation on benchmark datasets demonstrates statistically significant performance improvements, with mean square error reductions of 10-15% across multiple model orders. Third, demonstration of practical applicability through detailed experimentation and analysis of real-world enrollment data.

Experimental results confirm the method's superiority, with fifth-order forecasting achieving a mean square error of 11,740 compared to 62,865, 94,539, and 278,919 for established comparative methods. This performance improvement stems from more accurate interval partitioning that better adapts to non-uniform data distributions.

Theoretical contributions include a novel centroid initialization strategy for fuzzy clustering in time series contexts and a framework for adaptive parameter adjustment in fuzzy C-Means applications. Practical implications extend to multiple forecasting domains where accurate partitioning of non-uniform time series is critical, including financial markets, resource planning, and demand forecasting.

Future research directions include integration with optimization algorithms for automated parameter selection, extension to multivariate fuzzy time series, and application to streaming data scenarios with evolving distributions. Hybrid approaches combining the Improved FCMeans with deep learning techniques may offer further performance enhancements for com- plex forecasting problems.

In summary, this work advances fuzzy time series methodology by providing a more robust, adaptive partitioning approach that balances theoretical rigor with practical utility, offering both improved forecasting accuracy and reduced manual intervention requirements for practitioners.

Acknowledgements
The authors thank anonymous reviewers for their constructive feedbacks.

Conflict of Interest Statement
The authors declare that there is no conflict of interest regarding the publication of this paper. No financial support, grants, or other forms of compensation were received that could have influenced the outcomes of this work.

Author Contributions Statement
Shafiu Maitoro: Conceptualization, Methodology, Formal analysis, Writing - Original Draft. MAM Safari: Supervision, Validation, Writing - Review & Editing. P. Singh: Data Curation, Visualization. J. Arasan: Methodology, Validation. FZC Rose: Software, Formal analysis. All authors contributed to manuscript revision and approved the final version

References

[1] Song, Q., & Chissom, B. S. (1993). Forecasting enrollments with fuzzy time series-part I. *Fuzzy Sets and Systems*, 54(1), 1–9. doi: 10.1016/0165-0114(93)90355-L

[2] Song, Q., & Chissom, B. S. (1993). Fuzzy time series and its models. *Fuzzy Sets and Systems*, 54(3), 269–277. doi: 10.1016/0165-0114(93)90372-X

[3] Chen, S.-M. (1996). Forecasting enrollments based on fuzzy time series. *Fuzzy Sets and Systems*, 81(3), 311–319. doi: 10.1016/0165-0114(95)00220-0

[4] Chen, S.-M. (2002). Forecasting enrollments based on high-order fuzzy time series. *Cy- bernetics and Systems*, 33(1), 1–16. doi: 10.1080/019697202753306479

[5] Hwang, J.-R., Chen, S.-M., & Lee, C.-H. (1998). Handling forecasting problems using fuzzy time series. *Fuzzy Sets and Systems*, 100(1-3), 217–228. doi: 10.1016/S0165- 0114(97)001217

[6] Bezdek, J. C. (1984). FCM: The fuzzy c-means clustering algorithm. *Computers & Geo- sciences*, 10(2-3), 191–203. doi: 10.1016/0098-3004(84)90020-7

[7] Zhang, Z., & Zhu, Q. (2012). Fuzzy time series forecasting based on k-means clustering. *Open Journal of Applied Sciences*, 2(4), 100–103. doi: 10.4236/ojapps.2012.24B023

[8] Egrioglu, E. (2014). PSO-based high order time invariant fuzzy time series method: Application to stock exchange data. *Economic Modelling*, 38, 633–639. doi: 10.1016/j.econmod.2014.02.008

[9] Huarng, K., & Yu, T. H. K. (2006). The application of neural networks to forecast fuzzy time series. *Physica A: Statistical Mechanics and its Applications*, 363(2), 481–491. doi: 10.1016/j.physa.2005.08.014

[10] Cheng, S.-H., Chen, S.-M., & Jian, W.-S. (2016). Fuzzy time series forecasting based on fuzzy logical relationships and similarity measures. *Information Sciences*, 327, 272–287. doi: 10.1016/j.ins.2015.08.024

[11] Huarng, K., & Yu, H.-K. (2005). A type 2 fuzzy time series model for stock index forecasting. *Physica A: Statistical Mechanics and its Applications*, 353, 445–462. doi: 10.1016/j.physa.2005.01.055

[12] Bose, M., & Mali, K. (2019). Designing fuzzy time series forecasting mod- els: A survey. *International Journal of Approximate Reasoning*, 111, 78–99. doi: 10.1016/j.ijar.2019.05.002

[13] Van Tinh, N., Dieu, N. C., & Huong, N. T. (2016). A forecasting model based on k- means clustering and time-invariant fuzzy relationship groups. *Journal of Multidisciplinary Engineering Science Studies (JMESS)*, 2(8), 815–821. doi: 10.5281/zenodo.3719470

[14] Box, G. E. P., Jenkins, G. M., Reinsel, G. C., & Ljung, G. M. (2015). *Time Series Analysis: Forecasting and Control* (5th ed.). Wiley. doi: 10.1002/9781118619193

[15] Gangwar, S. S., & Kumar, S. (2012). Partitions based computational method for high- order fuzzy time series forecasting. *Expert Systems with Applications*, 39(15), 12158– 12164. doi: 10.1016/j.eswa.2012.04.050

[16] Singh, S. R., & Borah, B. (2013). Forecasting based on high-order fuzzy- fluctuating time series. *Expert Systems with Applications*, 40(3), 957–964. doi: 10.1016/j.eswa.2012.08.044

[17] Bezdek, J. C. (1984). *Pattern Recognition with Fuzzy Objective Function Algorithms*. Springer. doi: 10.1007/978-1-4757-0450-1

[18] Egrioglu, E., Aladag, C. H., Yolcu, U., & Uslu, V. R. (2014). A new hybrid approach based on SARIMA and partial high order bivariate fuzzy time series forecasting model. *Expert Systems with Applications*, 41(14), 6479–6484. doi: 10.1016/j.eswa.2014.03.008

[19] Huarng, K. (2001). Effective lengths of intervals to improve forecasting in fuzzy time series. *Fuzzy Sets and Systems*, 123(3), 387–394. doi: 10.1016/S0165-0114(00)00188-3

[20] Bose, M., & Mali, K. (2019). A novel data partitioning and rule selection technique for modeling high-order fuzzy time series. Applied Soft Computing, 76, 274–285. doi: 10.1016/j.asoc.2018.12.016