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Exploring Compression-Only Structures and Innovations in Earthen Floor Slab Systems

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ABSTRACT

This study presents a systematic literature review tracing the evolution of compression-only, earthen-based floor slab systems from early empirical design to modern computational methods. Earthen construction, renowned for its sustainability and natural aesthetic, historically relied on rule-of-thumb approaches for structural stability. Our review documents the transition from ancient masonry techniques and proportional rules to the adoption of static analysis and computational simulations, such as thrust line and force network analyses. From an initial pool of 300 sources, 99 were selected based on rigorous inclusion criteria. Data visualization with VOSviewer—via timeline density, cross-citation, and keyword maps revealed a surge in research interest since 2020 and underscored the lasting influence of foundational studies. Key themes include computational innovations, material trade-offs, and recent integration of materials science with structural analysis.

1. Introduction

The Built Environment's detrimental impact on Earth's ecological systems has been a topic of significant contention particularly over the past two decades. Numerous Scholars including [1-3] provide extensive insight into these adverse effects, notably building activities account for 39% of energy-related global CO2 emissions according to the United Nations 2021 survey, and are responsible for 35% of the waste disposed in landfills not to mention the significantly high amounts of greenhouse gases (GHG) that the Built Environment releases into the atmosphere [1], Which, according to the Climate Watch Report as of 2023, was measured at more than 6 billion tons or 12% of the overall GHG emissions.

These environmental impacts, combined with the rapid population-growth and the increasing rural-urban migration underscore the urgent need for decisive action to mitigate the crisis. The awareness towards the Built Environment's effects has resulted in a rising demand for sustainable, low-carbon construction materials to substitute modern stables such as Steel and Concrete; both of which are notorious for their negative impact on the environment, as early as the 1960s to mitigate

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the damage construction activities pose on the environment and has culminated in an array of potential green construction materials such as Cross laminated Timber (CLT), Recycled Plastic, and Foamed Light weight Concrete.

These newly developed materials however suffer greatly in terms of global adoption mainly due to practical and economical limitations associated with using them, this in turn has shifted Academic and Practitioners' efforts to investigate a class of materials known as Earthen construction materials due to the minimal impact these materials pose to the environment, not to mention their abundant availability in nature and relatively easy manufacturing as highlighted by [2], and indeed there's a noticeable rise in popularity of earthen building components such as Rammed Earth.

These efforts, however, have been stunned by one major and critical building component, Floor slabs, as horizontal elements in buildings Floor slabs present unique challenges in earthen systems due to their susceptibility to bending and shear forces which are not allowed by Earthen materials, Earthen materials Afterall are notorious for their low tensile strength and thus require vertical support at very short intervals. Yet, historical works of Architecture provide countless precedents of instances where Earthen materials were used to span lengths as great as 24 meters and that wasn't accomplished through supplementing or manipulating the material properties in any form but simply through modifying the geometry of these structures such that the geometry allows its constituents to be placed under constant compression which is favored by Earthen materials which are nearly indestructible under compression and axial loading [6].

These geometric forms combined with Earthen materials result in what's coined as Compression-only structures which are academically geometric and engineered assemblies of earthen or masonry materials that transfer all the structure's loads and forces through compressive paths which allows the overall structure to overcome the material's limited tensile strength and span long distances not possible with an earthen-based post and lintel system while also substantially reducing the amount of material used [3,5,6,13].

Compression-only structures can be viewed within many contexts, be it from a Historical, Aesthetic or Technical point of view. Historically the building technology played a significant role at propagating civilization and establishing monuments that endured centuries with the first of such monuments being attributed to the Mesopotamians who built small Barrel Vaults over important Burial sites, detailed Analysis of the Historical Evolution of the Building Technology can be found in [4,6,9] who have offered great insights regarding the Historical evolution of the technology.

Aesthetically the building technology marks and even plays a paramount role in identifying several Architectural periods spanning over 21 Centuries from the Roman republic (ca 500 BC) through the Medieval period and reaching the peak of Aesthetic and Artistic expression in the Gothic period (ca 16th Century AD) for detailed Accounts of the Aesthetic and Architectural qualities of the technology readers are directed to [10,11] as well as the works of Nicholus Pevsner. The efficiency of compression-only structures in comparison to post and lintel systems is attributed to the mode of load transfer in post and lintel systems which is characterized by bending resistance, this mode requires materials with higher tensile strength which isn't available in earthen materials while compression only structures don't resist the forces, rather re-directs them through the Structure which makes use of the high compressive strength of the material (Figure 1).

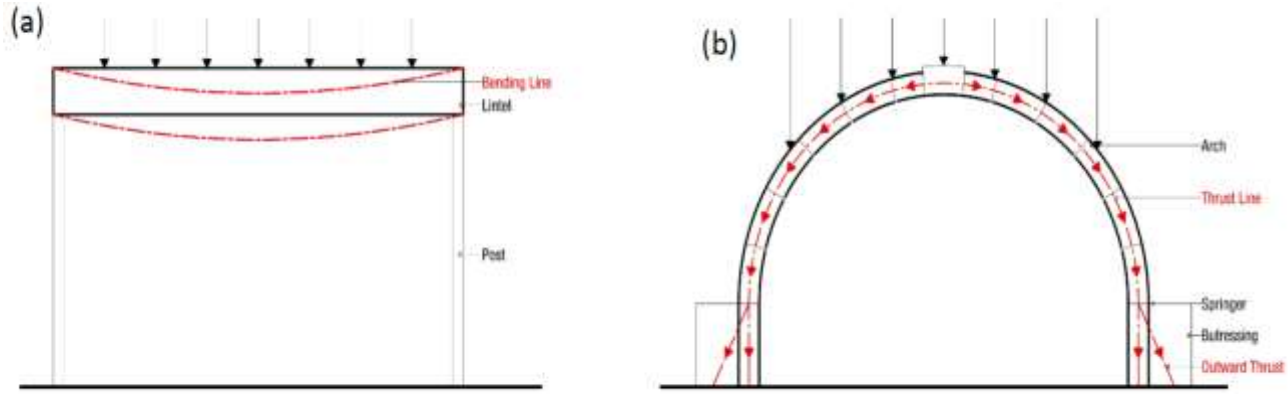

Fig. 1. Comparison between Post and Lintel system and Compression-only structural system: (a) Post and Lintel System and (b) Compression-only structure

The statement regarding the superior efficiency of compression-only structures can be intuitively verified through observing any ancient and historical building that utilized the technology, Taq-Kisra for instance spans 26 meters at the Iwan hall with a shell thickness of 1 meter throughout, compared to the Parthenon which spans 31 meters with columns at 3.8-meter intervals while the lintels (composed of the architrave and the frieze) have a total thickness of about 2.15 meters Later instances of compression-only structures consistently showcased a gradual optimization of the span to shell thickness ratio which arguably peaked in the Gothic period, for instance, King's College Chapel in London spans 12.5 meters with a shell thickness of 12-17 centimeters wherein the shell is about 1% of the vault's span, these figures are on par with an eggshell which has a ratio of 0.5-1% [6].

There is no doubt that the technology is very efficient at transferring high loads with as little material as possible, indeed the first instances of the technology are humble 1-meter wide barrel vaults, the technology has continued evolving over the following 6600 years out of sheer necessity and within those years a myriad of forms have been developed, ranging from simple arches to complex vaults, fan vaults, and even free-form shells the forms are diverse in geometry, materials, and even construction processes with some requiring centering while others are self-centering [7].

Given the long history of Compression-only structures, numerous forms have been developed ranging from two-dimensional arches and parabolas to three-dimensional vaults, domes, and shells. In an effort to aid the identification and categorization of these structural forms as well as the undertaking of research efforts surrounding the topic, several authors have proposed different systems to categorize these structures; [8] proposed categorizing the structural forms under three categories based on a geometric point of view, these are arches, vaults, and domes. He suggests that arches are the basis of all the other forms and that the three-dimensional forms are achieved through the rotation of or the translation of the basic forms, vaults for instance are achieved by translating arches while domes are achieved by rotating them around a central axis. Another equally influential categorization system was proposed by [9], who proposed a categorization based on two primary factors, the need for centering in construction and the geometrical shapes involved in designing the vaults. This categorization appears more suitable as it solves a myriad of discontinuities that Duarte's creates.

Ochsendorf proposed three main categories of vaults; specifically Nubian Vaults, which are self-centering and derived from arches or parabolas; Classical or Roman Vaults, which include Barrel, Groin, Ribbed, and Gothic Vaults, all of which require centering and are based on translations or rotations of circular, semi-circular, pointed, or parabolic arches; and Catalan Vaults, which also do not require centering and are constructed using a thin-tiling technique. However, Ochsendorf's classification focuses solely on three-dimensional spatial forms, and does not account for variations of arches and parabolas, which are not included in this system. Moreover, categorizing structures based on whether they require centering leads to complications, as there are numerous instances where a structure that required centering in one case was built without it in another, and *vice versa*. Figure 2 illustrates the most common forms of vaulted compression-only structures.

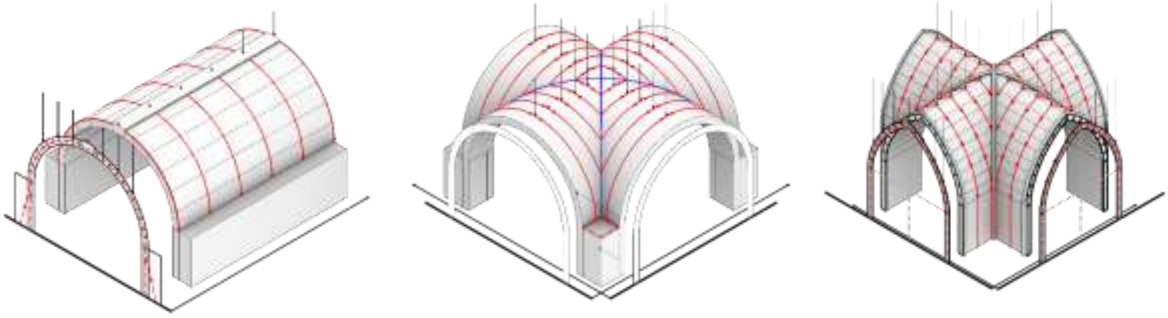

Fig. 2. Compression-only structural forms and their categorization systems

The earliest known instances of compression-only structures are attributed to the Mesopotamians, with examples dating back to 6000 B.C. Over the course of several millennia, this foundational building technology has evolved continuously, shaped by the innovations of successive civilizations. Notably, the Persians and the Romans made significant contributions—refining techniques such as vaulting, arch construction, and other geometric strategies that exploit the inherent strengths of earthen materials under compression (see Figure 3 for an overview of the historical evolution of this building technology).

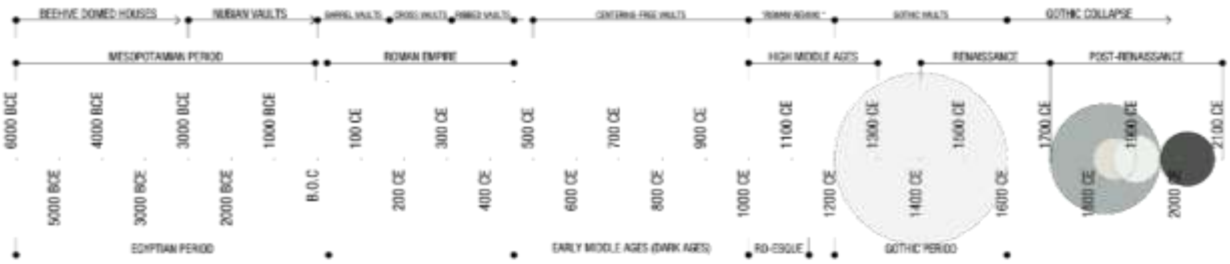

Fig. 3. Historical timeline of compression-only structures

One of the earliest examples of compression-only structures that have survived to the present day are Beehive domed houses. Despite the cleverness of their construction methods, these structures are often under-documented, although some research has been conducted on this fascinating subject. Civilizations such as the Egyptians, Mesopotamians, and those in the Mediterranean and Aegean regions, all of which were characterized by arid climates, employed these forms. It is widely believed that the need for such structures arose due to the limited availability of construction-grade timber in these areas [10]. Figure 4 presents a broad sample of compression-only structures, potential categorization systems, and the primary categories.

Fig. 4. Compression-only structural forms and their categorization systems

The structures mimic nature with their beehive-like form, and it's believed that the form was used partially as a response to the hot climate as the height of the dome's upper hole serves as a chimney which improves ventilation, and partially due to the very nature of stone-laying process. While no Systematic thermal Analysis for the Performance of these Structures exist, Özdeniz *et al.,* (1998) suggested an 8 degree difference between the internal and external environment of the Beehive domed houses in Harran, Turkey. From the Mesopotamians who used similar forms of Housing for the general Populous, the technology spread to multiple areas that the Mesopotamians ruled over such as Turkey[19]. Some of the latest remains are the Tholos tombs at Mycenae, Greece, extensively studied by British archaeologist Alan Wace (1879-1957). Some of the buildings in this area are subterranean, (Figure 5a), while others are built on-ground such in Figure 5b.

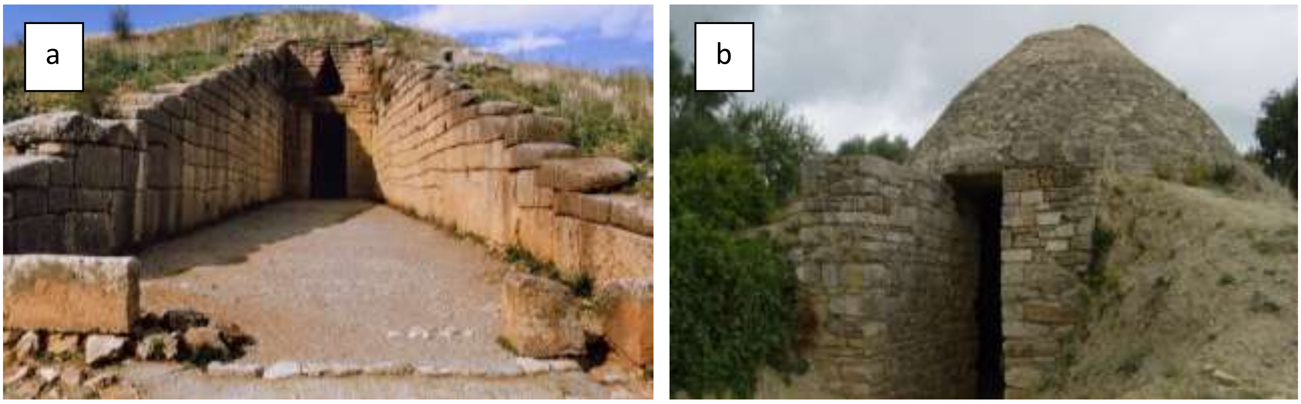

Fig. 5. Buildings in subterranean area (a) Mycenaean Tholos Cyclopean Tomb and (b) Above-ground Beehive Domed Tomb in Tholos [11]

The second oldest form of Compression-only structures Nubian Vaults, which were used throughout the region of Nubia [12]. While Beehive Domed Houses are built upwards; Nubian Vaults are more resembling of a classical Barrel Vault in their form and may be precursors to the Barrel Vault purely from a topological point of view, alas no evidence exists to suggest that. These Vaults were built using mud bricks and clay mortar in regions with tree scarcity as stated by [12]. Nubian Vaults (Figure 1.6) have a parabolic cross section earning them the compression-only status, the cross-section is a byproduct of the initial outline of the vault's design, using strings to proportionate the Vault from a singular point in the load bearing wall. The use of proportions in design and with the innovative building process characterized by slanted loops of arches combined with the Scarcity of wood in the Regions these Vaults were built resulted in a yet another Centering-free Structural form.

Following the Architectural developments in Egypt and Mesopotamia, the second earliest instances of compression-only structures can be found in Macedonia. much like the Mesopotamians, the Macedonians, had used Barrel Vaults in their tombs and Burial Chambers Albeit at larger and more monumental scales such as the Tombs found in Derveni near Thessaloniki. Following the Macedonians, the Greek had left no evidence of Compression-only structural forms making the Persians and Romans the next civilizations to have advanced the building technology. In this regard one of the most magnificent edifices to have reached us from the Persian civilization includes *Taq-Kasra* or *Taq-Kisra "the Crown of Kisra"* was Built at Ctesiphon, modern day Iraq to the South of Baghdad, The Barrel Vault making the *Iwan* Hall spans a monumental 26 meters and rises to a Height of 30 meters with a Shell thickness of 1 meter making it the Largest Barrel Vault even by modern standards. Not much is known about the Vault aside from the Centering-free Construction evidenced by the Bricks-laying technique that's identical to Nubian Vaults, the Vault is Buttressed by two monumental structures Flanking Both sides. Likewise, the Roman Empire showcased diverse compression-only structures with varying spans and applications [8] Many of these buildings have become cultural landmarks attracting millions of visitors annually, Noteable Roman precednets include the Basilica of Maxentius and the Baths of Diocletian. Barrel Vaults that were made by the Romans and the Persians are testament to the extent these Civilization had utilized the Structural Form in their Construction works which culminated into the Groin Vault, Geometrically the Groin Vault is the result of intersecting two Barrel Vaults at their centres, at right angles to each other [3], and [13]. Groin Vaults are a critical Innovation in the History of Classical Architecture and to understand why Groin Vaults were such a pivotal innovation in the history of Compression only structures we must first look at the Limitations of Barrel Vaults, which are the consequence of the Structural Behaviour and Geometrical topology of these Structures.

Several Architectural and civilizational periods such as the Dark Ages, the Roman Revival and Gothic then ensued, each adding several innovations to the building technology such as Ribbed Vaults and Fan Vaults, however due to several significant developments over the following 200 years the technology has ultimately become obscure, these developments started with the transition into the Renaissance; this period witnessed a plurality of transitions and changes, from the decline of the medieval structural theory and the rise of scientific structural analysis as the predominant design method, to the revival of Vitruvian rules. subsequent time Periods brought further changes that rendered the Gothic Structural theory obsolete, the Industrial Revolution (ca 1700 AD – 1800 AD) for instance provided new, modern construction materials such as steel and iron, both of which possessed incredible tensile Strengths and allowed for great spans without the complexity of a Fan Vault and its time-consuming scaffolding construction. Following the invention of steel, concrete which was invented in the 18th century promised even faster and easier construction processes while its environmental effects weren't clear at the time. The combination of steel and concrete culminated in Reinforced Concrete, today's environmental antagonist; this combination of materials could span distances up to 12 meters in the case of waffle slabs, this combined with the post-World War 2 conditions of Europe, characterized by lack of housing and scarcity of resources; has boosted the adoption of reinforced concrete and steel construction at the expense of Earthen and masonry materials. As mentioned earlier, due to the trajectory the Built environment has been following and the challenges faced by the industry, several authors have revisited Compression-only structures in effort to investigate the possibility of adapting the building technology and adapting Earthen materials into modern construction practices. The extended period during which this technology remained obscure has resulted in significant loss of the expertise once possessed by medieval master builders. Much of this knowledge has either been lost to history or is preserved only in fragmented literature across various academic disciplines. This dispersion of sources presents considerable challenges for studying compression-only structures, as no comprehensive compendium exists that outlines the major technological developments. Consequently, the present study aims to address this gap by conducting a systematic literature review that traces the historical evolution of compression-only structures and earthen floor slabs—from their modest origins to the advanced computational methods employed today. In doing so, the study contributes to the field by lowering the barriers to future research and providing a holistic reference with an emphasis on equilibrium analysis.

2. Methodology

This paper's research methodology is structured as a Systematic Literature Review in which the history and evolution of the topic of Compression-only structures and Earthen based floor slabs are traced to gain insight into the evolution of Masonry Analysis methods and how they've historically reflected on the practical applications of Compression-only structures including the construction of Earthen floor slabs.

The Study begins with a Desktop search aimed at identifying and extracting relevant papers on the topic, the search was carried out via major scientific databases with emphasis placed on English academic Literature published between 2000 and 2024, this is then followed by a Quality assessment ensuring that the studies included in the review are credible and uphold scientific integrity to ensure a critical coverage of the topic. Duplicate and Irrelevant studies will be filtered manually through manual inspection of papers' Titles, Abstracts and Introductions with studies that fit the scope of this research being shortlisted and explored through Content Analysis.

2.1 Research Design

The SLR process follows a sequence of well-defined steps which ensures that all relevant academic literature is identified and synthesized systematically, the overall workflow aligns with the Preferred Reporting Items for Systematic Reviews and Meta-Analyses Extension for Scoping Reviews (PRISMA-ScR) and is structured into four main stages outlined in Table 1.

Table 1
Overview of SLR process

No.	Stage	Description	Tools/Methods
1	Identification	Comprehensive search across multiple databases using targeted keywords and Boolean operators	Scopus, Web of Science, ResearchGate, Google Scholar
2	Screening	Removal of duplicate records and preliminary assessment based on titles and abstracts	PRISMA guidelines, Zotero reference management software
3	Eligibility	Detailed full-text review applying inclusion/exclusion criteria; quality assessment using a CASP checklist	Data extraction forms
4	Inclusion	Final selection of studies, thematic categorization, and synthesis of findings	Thematic Analysis, Qualitative synthesis, narrative construction

The SLR begins with a Scoping stage to identify studies relevant to compression-only, earthen-based floor slab systems. Data is gathered from peer-reviewed sources such as Scopus and Web of Science, both renowned for extensive, multidisciplinary coverage, with Web of Science focusing on papers published since 1990 and Scopus covering an even broader range. Given the obscurity of compression-only structures, which leads to a scarcity of recent literature, conference proceedings and grey literature (e.g., technical reports and dissertations from ResearchGate and Google Scholar) are also included. Emphasis is placed on academic literature published between 2000 and 2024, while seminal works by key authorities from before the 21st century are also considered. Search terms encapsulating the topic are used in this stage and include terms aimed at Earthen Materials, such as "Rammed Earth", "Adobe", "Compressed earth block", "Bio-Stabilized earth". Terms aimed at Structural systems such as "Compression-only structures", "Thrust networks", "masonry vaults", "floor slabs". and finally, terms aimed at recent innovations such as "3D-printed earth", "Fiber Reinforcement", "Topology Optimization" and "Building codes". Table 2 highlights the search keywords and strategies used in this stage:

Inclusion and exclusion criteria are then applied to ensure that only the most relevant, high-quality studies are analysed. Included studies focus on the historical evolution of compression-only or earthen-based floor slabs in the context of equilibrium or computational analysis or recent innovations; they are peer-reviewed articles, conference papers, academic books, or reputable technical reports written in English and providing clear historical context and technical insights. Conversely, studies were excluded if they primarily compared earthen structures to modern concrete or steel systems, originated from non-academic sources such as blogs, or lacked a comprehensive historical perspective or detailed technical discussion.

Table 2
Search keywords and strategies used in the literature identification

No.	Keyword Group	Boolean Operators	Example Search String
1	Core Concepts	AND	“Compression-only structures” and “Earthen Floor slabs”
2	Historical Perspective	AND/OR	“Historical Evolution” or “Mesopotamian” OR “Roman Vaults”
3	Equilibrium Methods	AND	“Computational analysis” and “Equilibrium Analysis”
4	Material Trade-offs	AND	“Earthen Construction materials” and “Material trade-offs”
5	Recent Innovations	AND	“Recent Advances” and “Compression-only floor slabs”

Key information from each study is collected using a standardized form that captures publication details, historical context, analysis methods, material trade-offs, and recent innovations. The extracted data are then organized and synthesized into a coherent narrative through thematic coding, which assigns labels to themes such as early innovations, computational methods, material trade-offs, and recent advancements.

A Qualitative thematic analysis is employed to integrate and interpret the data through reviewing the extracted information and assigning codes to text segments that represent specific themes. Similar codes are then grouped to identify overarching themes that capture major trends in the evolution of compression-only structures, a coherent narrative is then made to explain the evolution of these systems emphasizing the impact of modern computational methods, material trade-offs and recent innovations.

3. Results

The systematic literature review (SLR) identified 300 potential sources, including books, peer-reviewed academic papers, grey literature, and conference proceedings. After applying the inclusion and exclusion criteria detailed in the methodology, 30 sources were deemed relevant for this study. The collected data was encoded into an RIS file and imported into VOSviewer, a data visualization software. VOSviewer was used to generate several analytical maps:

1- Timeline Density Map (Figure. 6): This map visualizes the evolution of research interest over the past two decades, revealing a significant increase in scholarly attention since 2020.
2- Cross-Citation Map (Figure. 7): This map highlights influential studies by tracking cross-citations among authors, indicating that although more studies have been published after 2020, earlier works remain highly influential.
3- Keyword Frequency Map (Figure. 8): This analysis shows that keywords such as "thrust line analysis," "masonry," and "sustainability" occur most frequently, underscoring the primary research foci within the literature.

These visualizations confirm that while the topic of compression-only, earthen-based floor slab systems has garnered increased attention in recent years, foundational research from earlier periods continues to shape current scholarly discourse.

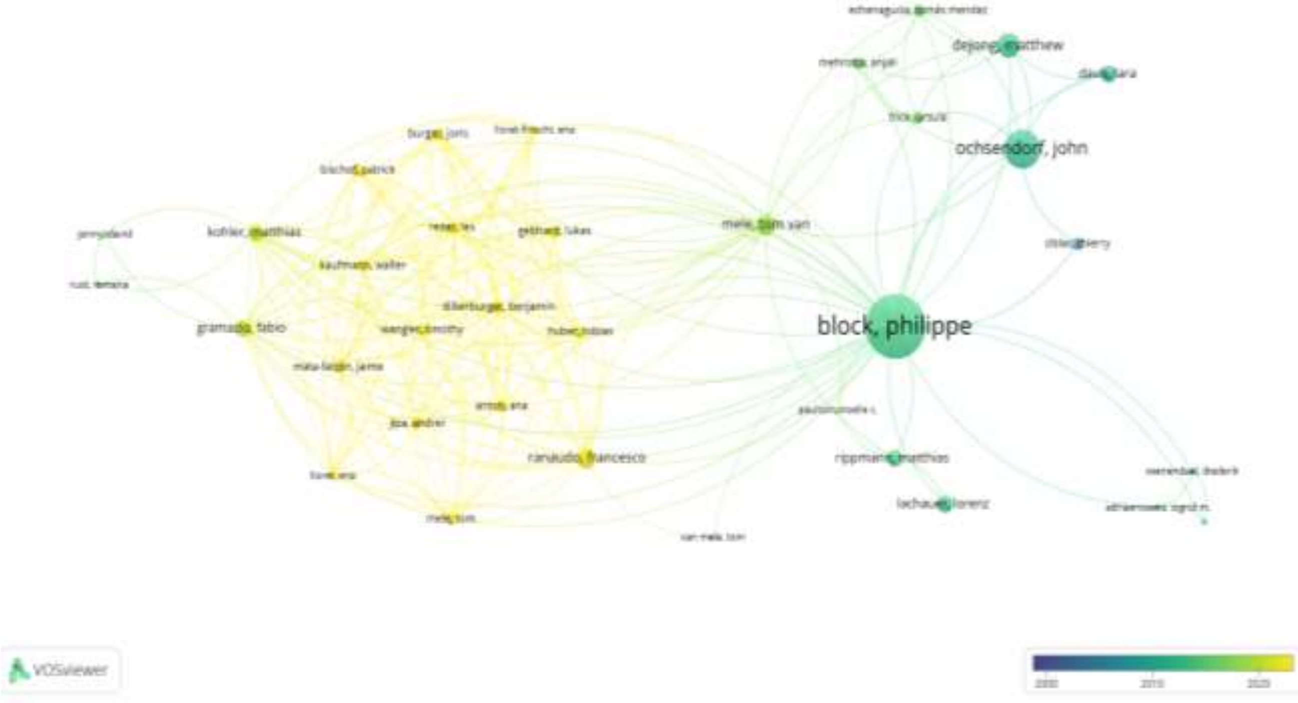

Fig. 6. Timeline density map

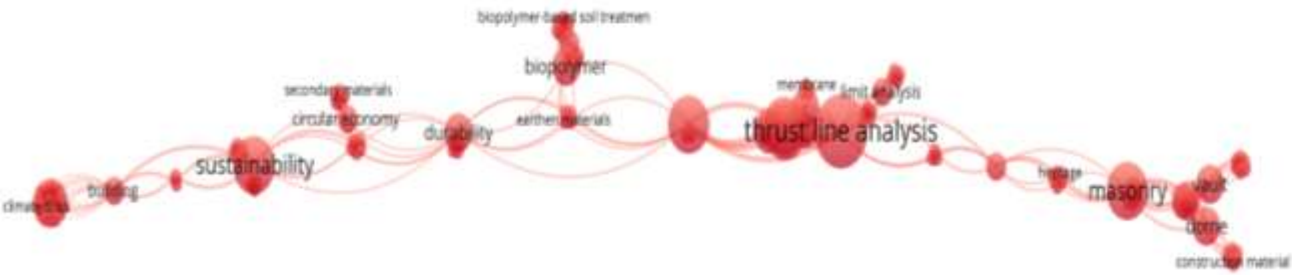

Fig. 7. Cross-citation map

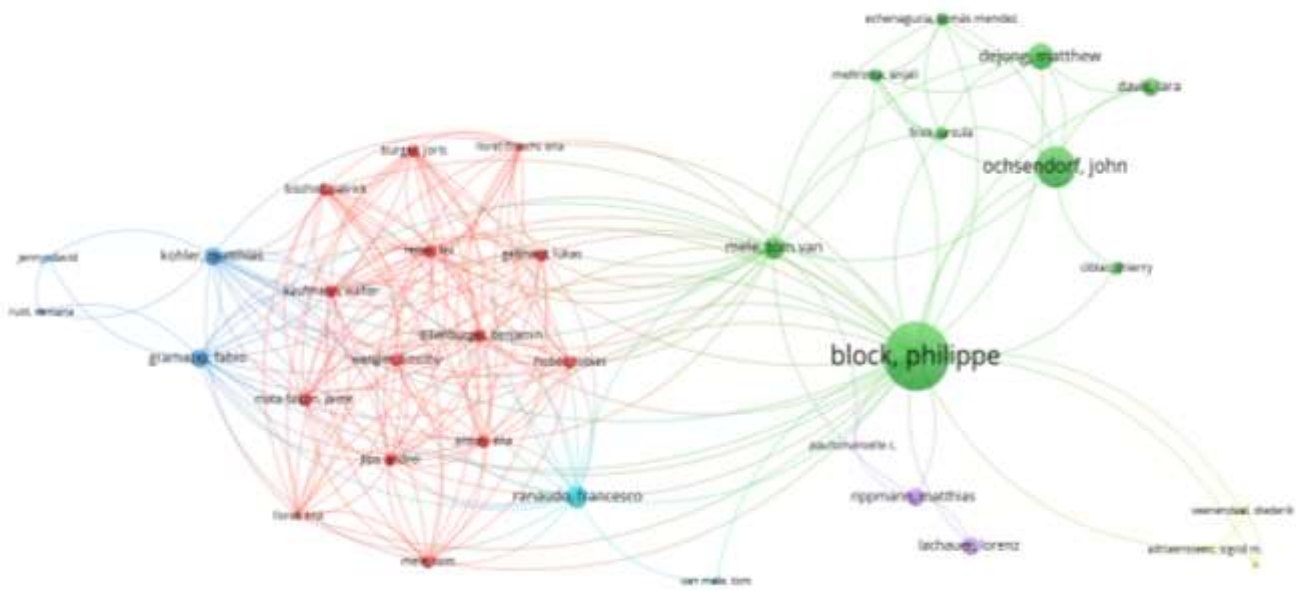

Fig. 8. Keywords frequency map

3.1 Theory of Masonry and Equilibrium Analysis
3.1.1 Rules of thumb

The first and earliest understanding master-builders had of masonry equilibrium was confined to Rules of thumb and Empirical rules which were used throughout the design process, these rules were used to obtain all the dimensions needed to ensure and maximize the structural stability of the building. There were for instance rules to obtain the size of Buttresses, the spans of Choirs, the breadth and height of walls, or the cross sections of the Ribs to name a few [14].

The SLR has identified a massive scarcity of literary records and manuscripts on rules of proportion and especially those predating the Gothic era. Gaetani *et al.,* [15] note that only Heron of Alexandria (ca. 60 AD) wrote about Vault construction in antiquity, later cited by Anthemius of Tralles; both documents are lost. Subsequently, the most reliable evidence for classical builders' use of proportion rules comes from Vitruvius and sections of his ten books. Vitruvius's documentation, along with Villard de Honnecourt's work, formed the basis for Heyman's assertion that builders from the Classical period through the Middle Ages used these rules. However, Vitruvius did not provide any practical design rules as noted by [15], and Villard was silent on structural matters, as reported by [16].

O'Dwyer [27] noted that in his 10 books on Architecture, Vitruvius synthesized what he believed to be key to Architectural activity. Most notably he described the human figure as the principal source for proportions, he also provided rules for the design of various objects from temple fronts to Cranes and water works and even weapons to name a few. In his fifth book for instance, Vitruvius explains the rules to determining the dimensions of the Roman Forum's Basilicas, he states that the width must be equal or less than half of the length but never smaller than third, the width of the Aisles should be a third of the central Nave width, and the height of the columns should be equal to the aisle width. In his third book he describes rules for intercolumniation spacing for species of temples, which he called *'eustyle'* as reported by [28].

Following Vitruvius's rules, the legalization of Christianity in the Roman empire by Constantine through the decline and subsequent collapse of the Roman empire (ca 500 CE – 600 CE) was the period commonly termed as the Dark Ages, in this period many of the traditional skills and constructive processes had disappeared, and most of what remained of the Roman imperial Architecture was Ransacked, used and repurposed in what's known as *Spolia*, it wasn't until the 10th Century that Vaulted Structures started to appear again in Europe[22].

During this period there were no building conventions for the construction of the early Christian buildings, it was believed to be a period of apparent Randomness, until [29] introduced the possibility that the builders of this period may have consciously or unconsciously imitated Vitruvius's rules of proportions using the spoliated elements from various Roman Basilicas. The Authors have analyzed the relationship between various dimensions of the columns including the diameter, intercolumniation and height of the shaft of Churches from the 5th and 6th centuries CE. Their findings supported the claim that there was indeed an underlying building convention that was an extension of the Vitruvian rules. it's noteworthy however that their study was limited to churches from 2 centuries only while the Dark ages have spanned 5-6 Centuries, expanding the Research scope to include more churches throughout the period is likely to yield better results and provide a conclusive statement on the use of building conventions during this period.

From the 10th to 15th century AD, architectural treatises and manuscripts rarely covered technical matters like vault design. This knowledge was exchanged in secrecy during the Gothic period and documented only in the Renaissance and Baroque eras, about four Centuries later [15]. Notable examples include Derand's and Blondel's treatises. Although some of the rules presented in the treatises have been in use a century prior to the treatises. These early efforts to collect the Gothic rules have finally culminated in the Gothic treatises which are widely regarded as the first complete manuscripts governing the technical information of Gothic Church design, for a more detailed analysis the reader is directed to [14,16,18]. Around the late Gothic and Renaissance majority of our knowledge of the period's building rules of proportions comes from Gil de Hontañón, one of the most important Spanish Architects of the past, and the Polymath Leon Battista Alberti; who is the most prominent person to write about Architecture after Vitruvius, joined by various other authors of this period such as Blondel, Derand & Hernandez Ruiz de Joven. These Authors' remaining manuscripts are dated to the 15th and 16th centuries, after which Galileo's structural analysis brought about the dawn of the scientific engineering theory which signalled the end of the Medieval masonry theory. Both [15,16] have provided detailed accounts for some of these rules and contrasted them in the results they gave to common issues. One such issue, perhaps the most interesting; was Gil de Hontañó's trouble with finding the needed Buttresses to account for the outward thrust of an Arch, he's reported his attempts to find a universal rule for the problem but had failed to do so, and according to Hontañó no Architect he's spoken to in that period had known of such Rule. Huerta reports on Hontañó's four proposed rules and verifies that his first rule, the simplest one, agrees with the Renaissance structures. The Rule, as reported in [16].

> 'The elevation of the intrados of the arch or barrel vault with its piers is drawn. Then a line is traced joining the keystone of the Arch with base of the pier. The intersection of this line with the horizontal impost line gives a point. The distance of this point to the vertical axis of symmetry is the thickness of the buttress' (p.179).

Huerta [16] and earlier [18] devised an Algebraic interpretation presented in equation Eq. (1), Where c is the buttress thickness, s is the span, h is the height of the impost line and f is the height of the Arch. with this expression Huerta confirms the validity of Hontañó's geometric rule.

$$\frac{c}{s} = \frac{\frac{h}{s}}{2\,(\frac{h}{s} + \frac{h}{f})} \tag{1}$$

While Huerta validated the rules provided by Gil de Hontañó, [19] disputed his Rules and noted inconsistencies contained in the rules themselves, Heyman went as far as to say that it seems that Gil de Hontañó was copying mechanically rules he learned by rote. Another Prominent Rule of this period was Blondel or Duran's Rule, named after the Authors, respectively and pertained determining the Butress breadth of a simple Arch or Barrel Vault. Retrieved from [3] the Rule states:

> 'The semicircular arc AD is divided into three equal parts by points B and C. The line CD is then prolonged so that CD = DF. Point F defines the outer edge of the buttress FG' (p.419).

The Rule was used time and time again throughout the 18th Century and reappeared in numerous Building Treatises of the 19th Century all the way until the second half of the 20th Century. Heyman notes that Blondel's Rule accords with intuition for single span bridges, irregularities and exceptional cases were Pointed, Shallow and Circular Arches which required depth than the rule would suggest. Following these Authors, very few treatises and manuscripts reached us from the 18th through 20th century relating to compression-only structures, notable instances include de La Hire and Belidor in the early 1700s who rigorously tried to study an Arch's stability according to wedge theory but ended up relying on geometrical basis. While it is impossible to pinpoint the exact moment in history to spell the doom for Rules of Proportions, it is generally accepted that the invention of steel during the

Industrial Revolution and then concrete and finally Reinforced Concrete were the final nail in the Coffin for this era of Geometries and proportion rules. These *"Modern"* materials allowed for faster construction and better control over Stresses and Limit states, dropping masonry compression-only structures irrelevant in the process. As previously mentioned in the Literature Review, compression only structures were first designed in accordance with various design rules, these rules were based on rules of proportions and culminated in a Body of science that was based on empirical rule of thumb that were transferred from Master builders to their apprentices in secrecy throughout the majority of the history of the building technology until they were compiled into Treatises in the Renaissance and Bourque eras (15th Century AD and onwards), examples include [17,30].

3.1.2 Emergence of static analysis

The first evidence for structural analysis and static analysis started to appear at the turn of the 15th Century AD; in the works of Leon Battista Alberti (1404-1472), Palladio (1508-1580) and Leonardo Davinci (1452-1519), the former of which was the first to propose the Basis of Scientific Analysis [20]. these early concepts and ideas weren't developed enough to be used for designing compression only Structures alas many significant authors and Scientists of this time offered their contributions to this new Science, one of the most significant of these figures is Galileo Galilei who in his Book *Two New Sciences* in 1638 investigated how to measure the transverse stress on a Beam as function of its Breadth and Width. Galileo died 4 years after his book was Published and several Authors have continued his work and went on to define the modern theory of Structural Analysis which gradually rendered the rules of proportion redundant until it was fully developed in the 17th Century AD then used thoroughly in the 18th century AD [16]. After Galileo, the most significant development in this area is attributed to Robert Hooke who made the association between the Behaviour of compression-only structures which are characterized by high compressive strength and low tensile strength, and robes or chains which are characterized by high tensile strength and low compressive strength [20], the most significant part of his work was his famous Anagram in Figure 9.

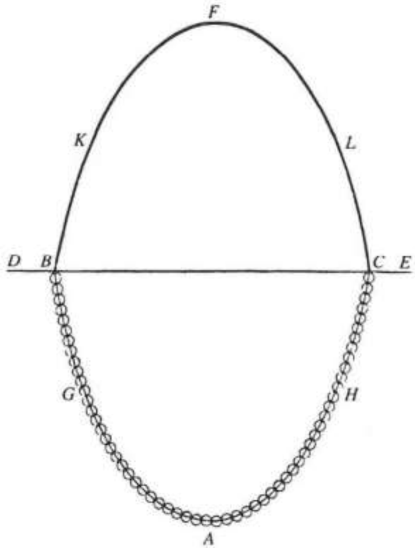

Fig. 9. Poleni's Interpretation of Hooke's Hanging Chain [21]

The Anagram in Figure 9 demonstrates how we may get the shape of a compression-only rigid Arch as a function of the Catenary, since the latter can be described Mathematically then the Arch could be drawn as an inverted Catenary resulting in a line or path of force which, when enveloped by the Arch' Voussoirs, creates a compression only structure, Hooke mentioned: as hangs the Flexible line, so but Inverted will stand the Rigid Arch (p.12).

Obtaining the geometry of an Arch through the inversion of a hanging cable or chain had its advantages and dis-Advantages, for starters the integrated approach was highly visceral, rather intuitively describing the principles compression only structures are built upon, additionally the geometry of the chain was independent of its dimensions and therefore an infinite number of equilibrium configurations could be drawn around the chain as long as the resultant form contains the shape generated by the hanging chain [22].

This approach was adopted at multiple instances with the earliest being Poleni's implementation of the Hanging chain model during his Restoration work for the Dome of San Peter in Rome [21]. Hooke in this regard is considered to be the first Author to describe a Mathematical, scientific design and Structural Analysis process for compression-only Structures, However; it must be mentioned that the discovery of the Catenary shape by David Gregory (1659-1708) according to [23] is what permitted Hooke's discovery, Gregory formulated the equation of the Catenary shape and concluded that the Catenary is the real shape of a Load Bearing Arch. This early form of compression-only Structural Analysis had significant limitations which permitted the Gothic Theory of Masonry to stay in use for a while, these new methods were highly limited to planar structures such as Arches or even Barrel Vaults since these were mere repetitions of Arches and no alternative solutions for Fan Vaults or Flying Buttresses had been discovered, it wasn't until the 21st Century that some reliable methods to calculate the Stresses for complex compression-only Structures were made alas these too aren't fully developed yet. On this, as mentioned by [24]; despite the advances in newly developed Analysis methods; no single method has been able to consistently give definitive solutions to problems Previously conquered by Masons and Master Builders Centuries ago.

Other significant Authors on the topic include James Stirling (1692-1770) which described how a Catenary can be built with hanging spheres which then inspired Poleni to adopt the technique. Philippe de La Hire, around the same time, formulated the principles of Equilibrium of Arches and Vaults, which was later Simplified. In 1773, The Engineer Charles-Augustin de Coulomb (1736-1806) determined the maximum and minimum strengths of an Arch. While in Spain the works of Bernard Forest Belidor (1698-1761) which was Partially Translated by John Muller (1699-1784) has Enabled the upcoming Authors to Complete Elastic theory of Masonry, a more Comprehensive Review of the Evolution of Scientific Structural Analysis can be found in [6].

3.1.3 Interactive design approaches

As mentioned earlier, the Hanging Chain model was Deemed an effective form Generation method for simple Structural forms such as Arches and simple Shell Structures such as Barrel Vaults but wasn't an effective method of Complex forms, this however didn't stop Practitioners from adopting the method in several ways due to its Appealing Characteristics, the method was interactive and required no time to make conclusions, not to mention that it Represents Structural Behavior independent of Scale. Various Architects and Engineers found interest in applications of the Technique and implemented it in works as early as Poleni's undertaking of the Restoration of St Peter's Dome in Rome [21]. The LR found that the most Significant Contributions to the topic are found in the works of A. Gaudi], H. Isler and F. Otto. The SLR has found that Gaudi is likely to be one of the most famous Architects who's implemented the Hanging Chain model in his works, although Previous, less sophisticated examples exist in the works of Giovvani Poleni and Heirich Huebsch (1795-1863). Gaudi had used the technique to determine the optimal Geometry that enables the structure to carry all the loads in Compression only and he did so by Drawing the Floor plans in scale on a base Board to then Hang Chains and Robes with Weights simulating the final Loads imposed on the Structure, the model was then inverted, and Gaudi would draw Forms that Contained the Force diagram. The Process sounds simple enough though it Requires an expert to make it work. Chilton *et al.,* [25] mentioned that starting 1950s, Swiss engineer Heinz Isler pioneered shell structures with minimal thickness. He used scale models of bent metal rods as boundaries for tensile materials, determining the ideal curvature by inverting these shells. However, manually translating and scaling these shapes to full size was a tedious process that sometimes resulted in errors. Otto developed the soap film solution, where he'd use bent metal rods to create the boundaries for what's to become a tensile Membrane structure, since soap film has a negligible weight Otto didn't have to invert the model and could keep it upright.

3.1.4 Recent re-interpretation efforts

While interest in compression only structures and their mechanical properties has been limited in comparison with modern materials, it is evident through the literature review that the topic has recently started to engage some authors and researchers driven by how these structures offer a highly sustainable construction model due to the intrinsic sustainable qualities of Earthen materials as well as the high material efficiency of these structures [26]. The first problem faced when attempting to harness the mechanics of compression-only structures to be adapted into contemporary building components, as mentioned previously; is the absence of a concise masonry structural theory since the original structures were made following empirical design rules.

The advent of computational simulations and robotic assembly processes have incited some authors to investigate further potentials of these Structural forms, these computational processes played a pivotal role in shaping this period as authors engaged in interpreting previously empirical and proportion-based Structural forms into Algorithmic processes suitable for contemporary applications, most notably Professor Philippe Block and his lab has provided great advances in the modernization and adaptation of the structural forms into contemporary building elements.

While the first instances of adapting these Building technologies came from Architects & Practitioners such as Gaudi, Frei Otto, and Hans Izler, all of whom relied on Graphics analysis, a tedious and time consuming process the interpretations remained in the realm of Avant-garde construction projects and styles, while Antoni Gaudi's hanging chain models produced magnificent structure with unmatched structural performance; the technique isn't a viable approach for Developers and thus effectively died with Gaudi, the popularization of these structural forms and by extension of Earthen construction requires the formulation of streamlined processes.

As mentioned earlier, one of the first interpretations of compression-only structures into mathematical expressions came from Robert Hooke's Anagram which was further developed by Poleni, using this interpretation to determine the equilibrium conditions of a compression-only structure is referred to as "Thrust Line Analysis" wherein the Thrust Line is an imaginary line that runs through the structure and represents the compressive force path the loads and stresses follow, [6] Heyman mentions that for as long as the thrust line lies within the cross section of the Arch; the structure is stable and there is no question of material failure such as crushing. This statement regarding the safety of the Arch as a function of the Thrust Line is Plastic in nature most notably, Heyman has pointed out that it can be applied to Masonry structures only if three assumptions are true about the structure, these assumptions were that the stone has no tensile strength, stone has an infinite compressive strength, and sliding failure can't occur assuming the materials has sufficient friction between the Voussoirs. This in turn entails that the material experiences stresses only in the parts where the thrust line flows, if the thrust line flows on the surface of the Voussoirs, then only the surface of the Arch experience stresses while the rest of the Voussoirs are stress-free. Furthermore, the *Safe Theorem* of Plastic theory means that for any given Arch with whatever loading conditions it is experiencing an infinite number of viable Thrust Lines may form, the Arch however doesn't need to contain all those imaginary lines and if it contains only one such line the Arch will be stable and safe. Up until the 21st Century, the Line of Thrust was calculated manually, with great contributions from Yvon Villarceau and Charles Inglis, see [19] for detailed analysis of their contributions, it is imperative to note that while the Shape of the Thrust Line of an Arch supporting its own weight only is expressed as the inverse of the Catenary spanning the same distance, a more complex loading condition requires solving for the loads imposed on the structure. Figure 10 showcases a reproduction of an example presented by Heyman, wherein and Arch of height $a + h$ is supporting a road of Length l and it is assumed that the infill is of uniform distribution and density.

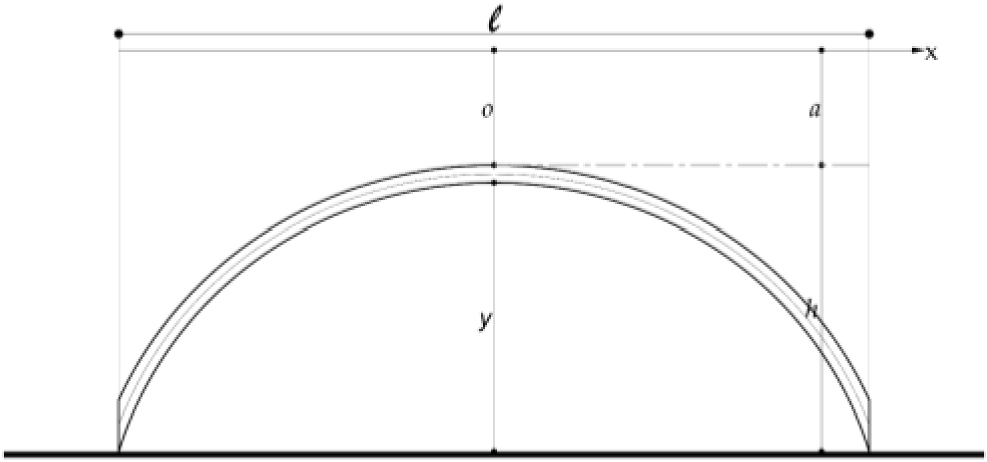

Fig. 10. Simple Arch bridge illustration, reproduction from [6]

To find a possible position of the Line of Thrust Inglis proposed the following Eq. (2), which represents a family of curves with α being equal to the rise of the infill from the keystone level to the top of the Arched wall:

$$\gamma = \alpha\, cosh\, [\frac{2x}{l}\, cosh^{-1}(\frac{a+h}{h})] \quad (2)$$

Eq. (2) is used to determine the centreline of the Arch which coincides with the Thrust Line and in it the parameters l and $a + h$ are variables controlled by the designer, Heyman affirms that the depth of the Arch is a significant contributor to the equation as shallow Arches result in higher thrusts than deep ones (Pointed Arches for instance). Yvon Villarceau developed a similar equation to that of Inglis, However Yvon allows for much more complex loading conditions and provides tables for the numerical step by step solution for any given problem, confined in the realm of 2D Structures. Through the advances in computational simulations, it has become possible to automate the calculation of the Line of Thrust in a Graphic Statics procedure as proposed by [3], in this process the Arch is divided into segments based on the Voussoirs and forces are solved at each stone.

The key difference is that Finite Element Analysis (FEA) incorporates a material's elastic properties—which are minimal in masonry making it useful for predicting initial deformations and cracking but ineffective at forecasting collapse mechanisms [2]. Thus, alternatives like the Line of Thrust method, which better predicts collapse, are generally recommended [3]. However, Thrust Line analysis is limited to structures that can be represented in a single cross-section (such as arches, flying buttresses, and barrel vaults) and cannot directly handle three-dimensional vaults like cross and fan vaults [24]. To extend this method to 3D problems, the Slicing technique first explored by Oxman divides a vault into several 2D cross-sectional slices, effectively reducing a 3D problem to multiple 2D analyses. While this approach works for historic spatial masonry compression-only structures, it does not capture their full 3D behaviour and is a manual process [24].

Another prominent method for computing the thrusts of 3D structural forms was recently proposed by Jacques Heyman, in [6] a rigorous framework for 3D spatial Vaults was introduced based on the membrane theory proposed, Professor Block mentions that the limitation of this approach lies in that it takes a safe Lower bound approach which limits the material use efficiency. The most promising of 3D analysis methods for spatial compression-only structures is found in the works of Dermot O'Dwyer, in [27] an equilibrium approach based on force networks was proposed, O'Dwyer implemented optimization methods based on finding compression-only force networks that are entirely contained within the boundaries of the shell, the method allowed for different choices of force patterns to be used in representing the infinite internal force equilibrium states of masonry structures not to mention that the method incorporated loading and structural discontinuities and Vault pathologies such as cracks, the main drawback of this approach was the inability to deal with the high degree of indeterminacy of 3D forms which limited the method to applications where assumptions could be made on the force distribution [24].

The most recent, and perhaps promising, contribution on the topic of Equilibrium Analysis of three-dimensional Vaults is found in the works of Professor Philippe Block who proposed using Funicular Network Analysis (FNA), in a way an extension of the Line of Thrust onto 3D structures, see [3] and [24] for a comprehensive analysis of the method and process.

Several authors explored the structural analysis of medieval vaults using Thrust Line Analysis (TLA) which was used to calculate the equilibrium of compression-only structures (unreinforced masonry) mainly in 2D format, while useful for Arches and Barrel Vaults it wasn't effective for more complex 3D forms. Thrust Network Analysis (TNA) was first proposed in [27] as an extension of TLA onto 3D forms, the technique (TNA) was later applied through using reciprocal figures and linear optimization, enabling better understanding and visualization of vaulted structures' stability. The Technique is implemented in Matlab and Rhinoceros 3D and offers an intuitive design and analysis tool for compression-only structures, providing rapid solutions and insightful force distribution visualizations. The technique functions through dividing the surface of the shell into nodes (connected with force paths) and the Equilibrium is calculated at each node using the Eq. (3),

$$Vi = \sum Fij \tag{3}$$

Where, *Vi* = Vertical force at node i.

- *Fij* = Force in the branch between nodes *i* and *j*.

However, while the method is advanced, it has certain limitations reported by the authors. Mainly its implementation in Matlab and Rhino Script restricts the number of elements in the node network due to inherent software constraints, this in turn impacts its scalability and overall accuracy. Block and Lachauer [28] further advances the Thrust Network Analysis method for 3D equilibrium analysis of Masonry Vaults through combining the algorithm with structural Matrix Analysis and various efficient optimization strategies which culminate in the new algorithm offering faster and more robust solutions and processes than previous efforts. The paper does so through identifying and visualizing the network's nodes' force dependencies or degrees of freedom, this additional data is then used to obtain an algorithm to find a solution, mapping a given height field best which is then used to obtain a 3D form's equivalent of the Line of Thrust based on Wrinkler's Elastic theory. The method, as reported by the authors, represents an admissible stress state that provides a stable lower bound on the geometric safety factor which is expressed as the ratio between the thinnest possible shell enveloping the Thrust Network and the Vault's actual geometry.

An important aspect of analyzing the equilibrium solutions for a given masonry vault with complex geometry, that is to find a compressive solution in equilibrium with the given loads that stays within the vault's section [25-30] whereas in planar structures such as Arches the analyst has to consider and solve for only one horizontal thrust in thrust networks there are highly dependent combinations of thrusts in their elements which requires the algorithm to control all of the Parameters to perform an effective search, in that regard the paper extends the force density method (FDA) using the insights provided by the TNA method and Matrix Analysis of structures.

In this context, the problem is formulated as finding the closest fit network to the mid-section of the shell, between the intrados and extrados (Figure 11), obtained as the height deviation $f(z_i) = \|z_i - z_i^M\|^2$with $z_i^M = (z_i^I + z_i^E)/2$ in which $z_i^I \& z_i^E$ represents the vault's Intrados and Extrados height fields for the free nodes.

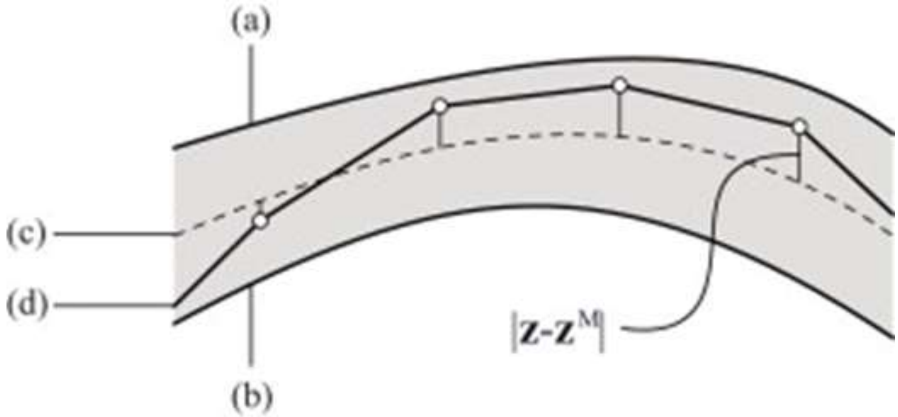

Fig. 11. Height Deviation of the thrust network from the middle surface. (a) Vault's intrados, (b) Vault's extrados, (c) middle surface and (d) thrust network [2]

This method was later used by Professor Block and his team to develop RhinoVAULT, a software plugin integrated into Rhinoceros 3D to solve for Free-form masonry Shells of discrete elements, it used Thrust Network Analysis and reciprocal diagrams to provide an intuitive approach to form-finding and geometry optimization of Masonry shells, the software was in fact used in several projects, the most important of which was the Armadillo Vault and Nest Hilo Vault. For the complete research in this regard, Readers are directed to [29].

Prominent aspects of the Research that must be highlighted include the Computation method which is a combination of Form Diagram and Force, Cremona, or Maxwell Diagram, the latter represents the equilibrium of Internal and External forces of the structure while the former represents the Geometry of a Pin-Jointed structure. According to Rippmann, the relation between Form and Force diagrams is coined as *Reciprocal Form*, in short this is from made of Polygonal Edges in which the edges correspond to and are equal in both the Form Diagram and Force Diagram, the Form is a Freestanding and Free-form compression-only structure. When the Reciprocal Diagram is combined with Linear Optimization and Funicular Force Networks, Thrust Network Analysis is formed.

Furthermore, the biggest advantage Thrust Network Analysis offers in the computation and analysis of three-dimensional compression-only structures under vertical loads is the interactive and intuitive process where the designer can control the multiple degrees of freedom in statically indeterminate structures due to visual nature of the Form and Force Diagrams. The method mainly targets the early stages of the Design Process where the user can directly control the edges of the Form Diagram and subsequently changing the Force Diagram Edges in accordance with the Form Diagram, this ultimately arrives at a compression-only structure designed to support its own weight at a minimal shell thickness, notwithstanding, the user can change the gravitational Force Index to increase the shell's strength under higher Loads than the Shell's Blocks. This Process However is mainly limited to Pin-Jointed Structures, these are Reciprocal Shell Structures, supported at Discrete Boundaries, at this stage of Development one couldn't use the method for Analyzing Slab-Like Compression only Structures to Design Doubly Curved Rib Stiffened Shells and it wasn't until [30] that such building technology was developed alas with major issues in regards to global adoption. Ranaudo *et al.,* [30] mentions that a combination of Thrust network Analysis followed by a compliance based topology optimization by evaluating the material distribution for several volume fractions which made it possible to identify regions where the material was and wasn't necessary which then lowered the weight of the Slab, the final step is to compute for and place the funicular ribs which were simple vertical extrusions along the network edges. The slab is 350 mm deep at its thickest points and tapers down to 50 mm at the midsection, the Ribs are 50mm thick for an easy concrete pour.

The technology is very sophisticated and while it has several limitations, it nevertheless is a great contribution to the topic of compression-only structures as a while and compression-only floor slabs specifically. Perhaps the biggest limitation found in the technology proposed in [30] is the complexity of the fabrication process, each floor slab is formed by pouring a specially wet concrete mix into a CNC wire-cut polyurethane formwork, the formwork consists of two components both of which are very complicated and require large space and long time to accurately cut, in developing regions where this technology is limited or non-existent it is very difficult to replicate such floor slab system, even if it were available, mass adoption by Developers would still lag behind and face a lot of resistance due to the costs involved. Secondly, the slab is still using concrete when it is ideal to use Earthen Materials from a sustainability standpoint, a slab made out of Compressed Earth Blocks (CEB) or a special configuration of Rammed Earth is better in this regard compared to concrete alas the challenges of using such material are obvious [31-35]. The final limitation is the applicability of incorporating Building Services, the Slab in Figure 12 showcases the Hilo floor slab with an edge thickness of 350 mm, while the midsection at 50 mm and the region surrounding the midsection may house some Building Services it is still difficult to fit all the Ducting and wiring a Typical Floor Slab allows to house.

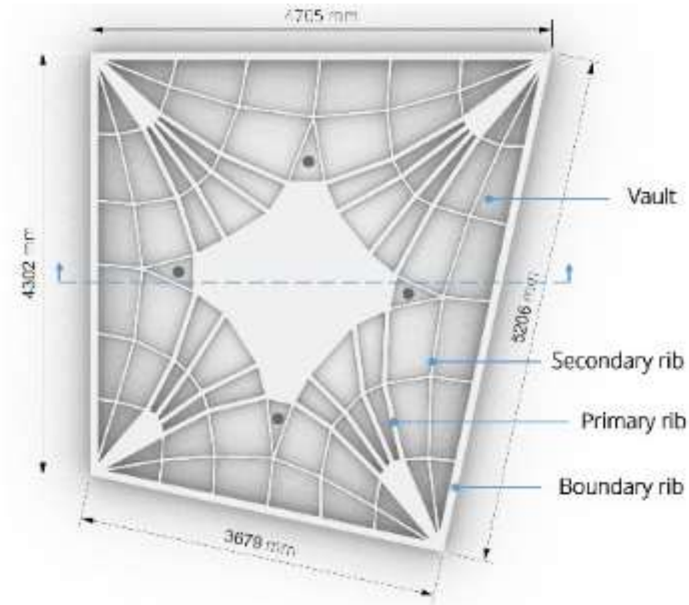

Fig. 12. Anatomy of the Hilo Funicular Slab [30]

No option for an Earthen based floor slab system with the potential for mass-adoption has been developed yet, as mentioned earlier in the research, one possible approach to integrate compression-only structures into the Larger Built Environment is through the Industrialized Building System (IBS), a Two-Way or a One-Way, Modular Floor Slab designed under uniform load distribution and built through Fabrication processes as simple as existing floor slabs is of paramount importance [36-49].

In regards to Fabrication processes, there has been a surge in interest in Robotic assembly, in [26] an alternative for manual labor in the construction of these structural forms is presented in the form of a Robotic assembly unit which minimizes the need for Labor and scaffolding, it's suggested by the Authors that the scaffolding needed to support the out-of-balance forces which offsets the material efficiency, in This method robotic arms hold the Arch ends in place and later in the research multi-branched Arches are built using a two-arm Robotic setup which reduces the required number of Arms [50]. The form finding approach in this paper is based on a Particle-spring system which generates compression-only Arches using catenary curves by defining supports as anchors and the components as chains [51]. The forms were generated using "Kangaroo" in the computer software "Grasshopper". The reduction of the number of Robotic arms was of paramount importance in the research. Various similar efforts with varying degrees of success, however, and once again while these methods and fabrication processes hold great promise for the future of compression-only structures, they do not meet current needs for popularizing the technology.

4. Conclusions

This review traces the evolution of compression-only, earthen-based floor slab systems—from the empirical, rule-of-thumb methods of early master-builders (evident in the works of Heron, Vitruvius, and medieval practitioners) to modern computational and experimental approaches. It synthesizes findings across three areas: historical design rules and empirical methods, the progression of static and computational analyses, and the material and fabrication aspects of traditional earthen construction. Early builders relied on intuitive guidelines to ensure structural stability, even though direct records from the pre-Gothic period are sparse. Over time, these empirical methods evolved into systematic approaches, with the transition beginning in the 15th century through contributions from Alberti, Palladio, and Leonardo da Vinci, and later refined by Galileo, Hooke and Gregory using mathematical models such as the inverted catenary. Pioneers like Gaudi, Heinz Isler, and Frei Otto further advanced these techniques, leading to modern computational methods like thrust network analysis (TNA) and funicular network analysis (FNA) that simulate traditional designs despite challenges in modeling complex 3D behaviors. In summary, this review not only charts the historical evolution of compression-only, earthen-based floor slab systems but also lays the foundation for future innovations that merge traditional construction principles with modern engineering precision, paving the way for sustainable and structurally sound building practices. Future research should focus on, Integrating Historical and Modern Approaches, Bridging the gap between empirical, rule-based methods and contemporary computational models to enhance predictive accuracy and design efficiency. Furthermore, it's necessary to advance 3D Analysis Techniques and develop more robust tools that capture the behavior of complex 3D compression-only structures, potentially through enhanced force network and optimization algorithms.

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