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# Chemical Reaction on Unsteady MHD Flow Over a Vertical Stretching Plate Embedded in Porous Medium with Richardson Number

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**ABSTRACT**

The chemical reaction of heat and mass transfer in an unsteady magnetohydrodynamics (MHD) mixed convection stagnation point flow towards a vertical stretching plate embedded in porous medium was studied. Richardson number and concentration were taken into consideration in this study. By selecting suitable similarity variables, the non-linear partial differential equations (PDEs) are transformed into a set of ordinary differential equations (ODEs), and then solved numerically using the boundary value problem solver (bvp4c) in MATLAB software. The solution is dependent on the governing parameters including Prandtl number, reaction rate parameter, Schmidt number, thermal Richardson number and buoyancy ratio parameter. The numerical results are depicted graphically in velocity, temperature and concentration profiles. The findings have shown that as the chemical reaction parameter increases, the temperature profile and Sherwood number also increases.

## 1. Introduction

The boundary layer flow is the region adjacent to a solid surface where viscous stresses are present. Ludwig Prandtl, a German aerodynamicist in 1904, was the first person who introduced the concept of a boundary layer. A boundary layer extends over a long, essentially flat surface in many natural flow situations, such as the flow over the ship and submarine hulls, aircraft wings and atmospheric motions over flat terrain [1].

The chemical reaction occurs due to foreign mass in air or water involves rearrangement of the molecular or ionic structure of a substance.  The chemical reaction involves in the conversion process from raw material into desired product. For example, agricultural product needs to undergo pre-processing and purification which involves many disciplines especially chemistry and biochemistry in

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order to produce supplementary foods. There are two types of response: homogeneous reaction which occurs uniformly throughout a given phase of a flow and heterogeneous reaction in a particular region or within the boundary of a phase [2]. For example, smog is formed as a by-product of a first order homogeneous chemical reaction [3]. The study of convective flow under the influence of chemical reaction in porous medium has various applications such as inducing the mixing drugs and transportation, food preservatives technology, plastic technology and petrochemicals. Joseph *et al.* [4] investigated the effect of chemical reaction on unsteady Magnetohydrodynamic (MHD) free convective two immiscible fluids flow. They found that the increase in chemical reaction parameters suppresses both velocity and concentration profiles.

MHD refers to the dynamics of electrically conducting fluids and plasmas in a magnetic field [5]. The application of magnetic field is used widely in many industries such as to heat, pump, stir and levitate liquid metals and in the earth's core, such as the solar magnetic field, which generates sunspots and solar flare [6]. The MHD liquid metal could give high efficiency and good fuel utilization, enhance energy conversion application and be a high-power propulsion application. Despite the opinion that steady flow is an ideal flow environment in a boundary layer problem, unsteady flow has also gained attention as it reflects real-world applications due to its relationship with time as insinuated by [7].

Stagnation point flow is the fluid motion near the stagnation region at the surface of objects in the flow. Nowadays, the stagnation flow of an incompressible viscous fluid over a stretching or shrinking sheet is a popular research theme. It is often studied due to its effective uses in the engineering industry, such as aerodynamics, extrusion of plastic sheets and cooling of the cooling metallic plate [8]. Azmi *et al.* [9] analysed the unsteady MHD flow about a stagnation point on a stretching plate embedded in porous medium. They solved the problem by using Runge-Kutta Fehlberg fourth-fifth order method with shooting technique. They found that the velocity increases with the increase of unsteadiness parameter for case velocity ratio parameter $\lambda = 0.5$, meanwhile it decreases for case $\lambda = 1.5$.

Both heat transfer and skin friction coefficient are essential in determining a product's quality. Heat transfer occurs when there is a difference in the temperature between the surface and the fluid, leading to a thermal boundary layer. There are three different mechanisms to transfer heat, namely heat conduction, thermal radiation and convection [10]. Convective heat transfer is when heat is transferred from the hotter particles away from the heat source by the fluid motion. Resulting in more significant buoyancy produced, as the hotter fluid is less dense and cooler fluid will be heated. Forced convection occurs when the mechanism in which an external source generates fluid motion. In contrast, free convection is a mechanism in which fluid motion is caused by buoyancy forces alone. The combined forced and free convection mechanism will produce mixed convection, and the buoyancy effect drives the flows due to gravitational acceleration and density variations from one fluid layer to another.

Research on mixed convection flow near stagnation point on vertical plate in a porous medium is a popular topic. Merkin [11] studied mixed convection flow on vertical plate in a porous medium and found dual solutions that exist for certain conditions. He then extended the research 6 years later by determining the solutions' stability. Moreover, [12] studied MHD free convection flows of a viscous fluid moving along a vertical plate in a porous medium. Mixed convection in stagnation flows is important when the buoyancy forces are high since it will significantly affect the flow and thermal fields. Recent studies about dual solutions on mixed convection flow using vertical plate are also done by [13-16].

A porous medium consists of a solid matrix with an interconnected void [17]. Fluid flow in porous media can be treated as one-, two- or three-dimensional depending on the complexities involved under a given circumstance and can be categorized as unsteady state, steady-state and pseudo-steady state. The flow through a porous medium has become the core of some applications in the engineering field as it is essential in technical problems like a stream through blood rheology, packed beds, environmental pollution and sedimentation. Makinde *et al.* [18] discussed porous medium on an unsteady MHD flow in a channel with heat source/sink effects. They found that when the Grashoff number increases, the temperature difference of the fluid intensifies which then increases buoyancy force.

The research in convective boundary layer flow of an electrically conducting fluid in the presence of a magnetic field has been the subject of many investigations due to its fundamental importance in industrial and technological applications. However, the chemical reaction formed from these processes could be an issue, especially for industries because without knowing what sort of chemical reaction can occur, industries could be in deep waters, drowning in lawsuits due to mishandling of the possibly dangerous chemicals which causes water pollution, air pollution as well as soil pollution. This then leads to more complications such as the death of aquatic creatures, cases of food poisoning occurring among nearby residents due to impure water and the reduction of the air quality in the surrounding area. Besides that, the chemical reaction could also significantly affect the quality and quantity of the product. Therefore, it is significant to investigate chemical reactions on an unsteady MHD to avoid these issues and prepare strategies to counter the predicament.

This study involved chemical reaction on unsteady 2D laminar flow in the vicinity of the stagnation point over a stretching plate with time dependent free stream embedded in a porous medium. The stretching plate is placed on the vertical surface where the vertical deals with mixed convection and we consider the buoyancy effects and the concentration. The combined buoyancy effects of thermal and concentration in a fluid-saturated porous medium could be beneficial in many applications such as soil pollution, fibrous insulation and nuclear waste disposal. This behaviour can be analysed from a dimensionless parameter which is Richardson number. Richardson number is a ratio between buoyancy term and the flow shear term. It also has a relation with the gravitational force. Richardson number enables to predict the occurrence the fluid turbulence and measure the index of flow stability. The higher value of Richardson number indicates the buoyancy is significant that lead to unstable flow. Meanwhile, the lower value leads to a stable flow.

Despite the numerous studies on unsteady MHD mixed convection flow with vertical plate, there is not much research done on this topic that involves viscous fluid considering the buoyancy effect and concentration as well as the byproducts from the chemical reaction. Thus, the aim of this study is to extend the research done by [9] to include the chemical reaction and buoyancy effects with mixed convection flow over a vertical stretching plate. The solutions obtained from this study can be used as a reference for future research in analysing and comparing the effect of chemical reaction on MHD with mixed convection in stagnation flow of a viscous fluid.

## 2. Methodology

Consider an unsteady 2D laminar flow in the vicinity of the stagnation point over a stretching plate with time dependent free stream embedded in a porous medium. This topic discusses on the formulation of the governing boundary layer equations which are the continuity, momentum, energy and concentration equations.

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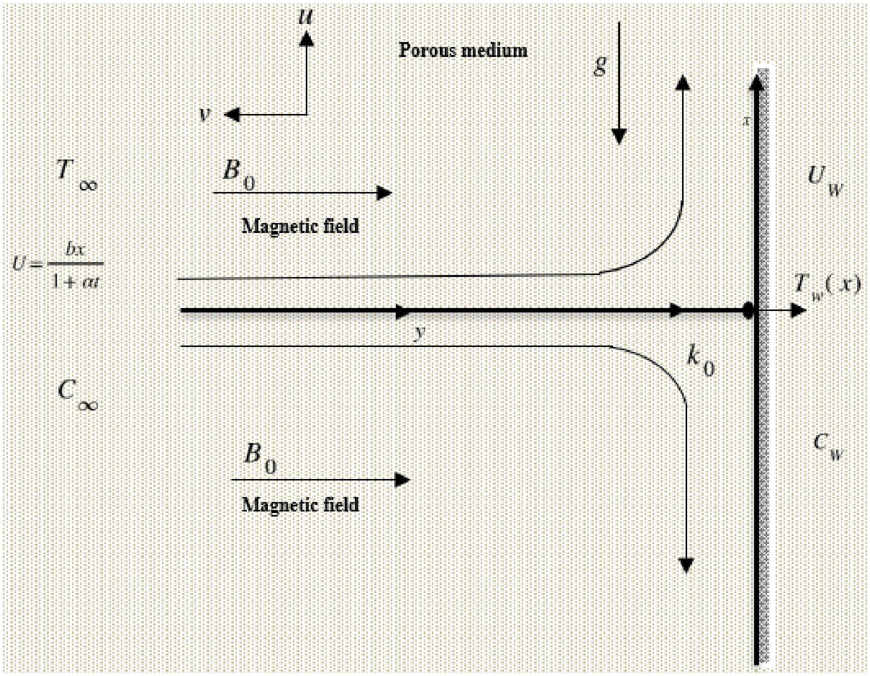



**Fig. 1.** Physical model of boundary layer flow on a vertical stretching plate

The governing equations for continuity, momentum, energy and concentration can be expressed as,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{\partial U}{\partial x}+\frac{\partial U}{\partial t}+v\frac{\partial^2 u}{\partial y^2}-\frac{\upsilon(u-U)}{k}-\frac{\sigma B_0{}^2(u-U)}{\rho}+ \left(\beta_T\right)\left(T-T_\infty\right)g+(\beta_C)(C-C_\infty)g \tag{2}$$

$$\rho C_p\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\kappa\frac{\partial^2 T}{\partial t^2} \tag{3}$$

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial x^2}-k_1\left(C-C_\infty\right) \tag{4}$$

where $u$ is the velocity component along the $x$-direction, $v$ is the velocity component along the $y$-direction, $\upsilon=\mu/\rho$ is the kinematic viscosity of the fluid, $\beta_T$ is the thermal expansion coefficient of temperature of the fluid, $\beta_C$ is the thermal expansion coefficient of fluid concentration, $g$ is the buoyancy parameter, $\rho$ is the density of fluid, $C_p$ is the fluid specific heat capacity at constant pressure, $T$ is the temperature of fluid, $T_w$ is the surface temperature, $T_\infty$ is the free stream temperature, $\kappa$ is thermal conductivity, $k$ is the permeability of the porous medium, $\sigma$ is electrical conductivity of the fluid, $B_0$ is the constant magnetic field strength applied in the positive $y$-direction to the stretching plate, $D$ is the diffusivity of the concentrations, and $k_1=k_0/\left(1+\alpha t\right)$ is the chemical reaction rate where $k_0$ is the chemical reaction. The boundary conditions are,

$$u = U_w(x,t) = \frac{cx}{1+\alpha t},\ v = 0,\ T = T_w(t),\ C = C_w(t)\ at\ y = 0 \tag{5}$$

$$u = U(x,t) = \frac{bx}{1+\alpha t},\ \ T = T_\infty,\ C = C_\infty\ as\ y \to \infty \tag{6}$$

where $b$ and $c$ are positive constants, $\alpha$ is a constant, $U_w$ is the velocity of the stretching plate, $U$ is the free stream velocity, $C$ is the concentration of the fluid, $C_w$ is the surface concentration of the fluid and $C_\infty$ is the constant ambient concentration. These similarity variables are used to transform the partial differential equations (PDE) of Eq. (1) - (4) into ordinary differential equations (ODEs) where $\psi$ is the stream function defined by $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$ which satisfies Eq. (1) and $\eta$ is the similarity variable, $\theta(\eta)$ is the dimensionless temperature and $\phi(\eta)$ is the dimensionless concentration fraction function,

$$\eta = \sqrt{\frac{c}{\upsilon(1+\alpha t)}}y,\ \ \psi = \sqrt{\frac{c\upsilon}{(1+\alpha t)}}xf(\eta),\ \ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}\ \text{where}\ T_w - T_\infty = \frac{1}{(1+\alpha t)^2},$$
$$\phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}\ \text{where}\ C_w - C_\infty = \frac{1}{(1+\alpha t)^2}, \tag{7}$$

Thus, we attain

$$u = \frac{\partial\psi}{\partial y} = \frac{cx}{(1+\alpha t)}f'(\eta),\ \ v = \frac{\partial\psi}{\partial x} = -\sqrt{\frac{c\upsilon}{(1+\alpha t)}}f(\eta) \tag{8}$$

where $f(\eta)$ is the dimensionless stream function. After applying the similarity variables, the Eq. (2), Eq. (3) and Eq. (4) with boundary conditions Eq. (5) and Eq. (6) are transformed into a set of ordinary differential equations as shown below:

$$\begin{aligned} f'''(\eta) + \left(f(\eta) + \frac{S\eta}{2}\right)f''(\eta) - \left(f'(\eta) - S\right)f'(\eta) + \lambda^2 \\ - \lambda S - \left(\frac{1}{D} + M\right)\left(f'(\eta) - \lambda\right) + \lambda_t(\theta + \varepsilon\phi) = 0 \end{aligned} \tag{9}$$

$$\theta''(\eta) + \Pr\left(f(\eta) + \frac{1}{2}S\eta\right)\theta'(\eta) + 2\Pr S\theta(\eta) = 0 \tag{10}$$

$$\phi''(\eta) + Scf(\eta)\phi'(\eta) + SSc\left(\frac{\eta}{2}\phi'(\eta) + 2\phi(\eta)\right) - Sc\gamma\phi(\eta) = 0 \tag{11}$$

where $\Pr = C_p\mu/\kappa$ is the Prandtl number, $\lambda = b/c$ is the velocity ratio parameter where $\lambda > 0$ is stretching plate while $\lambda < 0$ is shrinking plate. However, this research only focuses on stretching plates. $S = \alpha/c$ is the unsteadiness parameter, $1/D = \upsilon(1+\alpha t)/kc$ is the porosity parameter, $Sc = \upsilon/D$ is the Schmidt number, $\gamma = k_0/c$ is the chemical reaction rate parameter, $M = \sigma B_0^{\,2}(1+\alpha t)/\rho c$ is the magnetic parameter, $\varepsilon = \lambda_c / \lambda_t$ is the buoyancy ratio parameter, and $\lambda_c = Gr_c / \mathrm{Re}_x^{\,2}$ are the solutal Richardson number and $\lambda_t = Gr_t / \mathrm{Re}_x^{\,2}$ the thermal Richardson number, where $\mathrm{Re}_x = U_w x / \upsilon$ is the Reynolds number, $Gr_t = g\beta_t (T_w - T_\infty) x^3 / \upsilon^2$ the local thermal Grashof number and $Gr_c = g\beta_c (C_w - C_\infty) x^3 / \upsilon^2$ is the local solutal Grashof number. The transformed boundary conditions are,

$$f(0)=0, \quad f'(0)=1, \quad \theta(0)=1, \quad \phi(0)=1 \quad at\ \eta=0 \tag{12}$$

$$f'(\infty)\to\lambda, \quad \theta(\infty)\to 0, \quad \phi(\infty)\to 0 \quad as\ \eta\to\infty \tag{13}$$

To obtain the transformation solution, all parameters must be constant. The skin friction coefficient, $C_f$, Nusselt number, $Nu$, and Sherwood number, $Sh$, are defined by the following relations,

$$C_f = \frac{\mu}{\rho U_w^{\,2}} \left( \frac{\partial U}{\partial y} \right)_{y=0}, \quad Nu = \frac{-x}{T_w - T_\infty} \left( \frac{\partial T}{\partial y} \right)_{y=0}, \quad Sh = \frac{-x}{C_w - C_\infty} \left( \frac{\partial C}{\partial y} \right)_{y=0} \tag{14}$$

where $\mu$ is the dynamics viscosity. Using the variables in Eq. (7) and Eq. (8), the parameters are given by,

$$C_f = \mathrm{Re}_x^{-\frac{1}{2}} f''(0), \quad Nu = -\mathrm{Re}_x^{\frac{1}{2}} \theta'(0), \quad Sh = -\mathrm{Re}_x^{\frac{1}{2}} \phi'(0) \tag{15}$$

## 3. Results and Discussion

An analysis of the behaviours of the momentum, thermal and concentration of the chemical reaction on mixed convection flow in porous medium is carried out. Several values are considered which are the chemical reaction rate parameter, $\gamma$ . Prandtl number *Pr*, Schmidt number *Sc*, thermal Richardson number $\lambda_t$ and buoyancy ratio parameter $\varepsilon$ while the other parameters which are unsteadiness *S*, magnetic $M$ , porosity *D* and velocity ratio $\lambda$ remain constant. The validation for the skin friction is in good agreement with [9] and [19], as shown in Table 1.

**Table 1**
Comparison of the values of skin friction coefficient $f''(0)$ for several values of $\lambda$

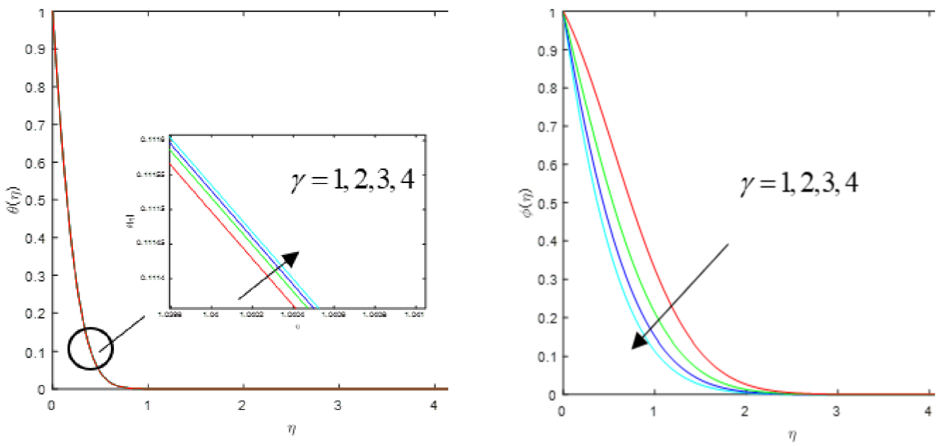
| $\lambda$ | Azmi *et al. [9]* | Pop *et al. [19]* | Present results |
|---|---|---|---|
| 0.1 | -0.969386 | -0.9694 | -0.96938654 |
| 0.2 | -0.918107 | -0.9181 | -0.91810743 |
| 0.5 | -0.667264 | -0.6673 | -0.66726384 |
| 2.0 | 2.017503 | 2.0174 | 2.01750302 |
| 3.0 | 4.729282 | 4.7290 | 4.72928275 |

Table 2 shows $-\phi'(0)$ increases significantly while $f''(0)$ and $-\theta'(0)$ decrease consistently with the increment of chemical reaction $\gamma$. The more the Sherwood number, the higher the mass transfer rate compared to the mass diffusivity on the plate.

**Table 2**
Result of the skin friction, $f''(0)$, Nusselt Number $-\theta'(0)$ and Sherwood Number $-\phi'(0)$ at the various values of the chemical reaction rate parameter $\gamma$

| $\gamma$ | $f''(0)$ | $-\theta'(0)$ | $-\phi'(0)$ |
|---|---|---|---|
| 1 | 0.51572068 | 4.01582577 | 0.46079816 |
| 2 | 0.46377143 | 4.01061867 | 0.99434818 |
| 3 | 0.43212608 | 4.00747791 | 1.38038949 |
| 4 | 0.41000410 | 4.00530592 | 1.69246889 |

There is a minimal increment in fluid temperature in boundary layer as illustrated in Figure 2(a). Meanwhile the concentration profile decreases expressively by the rising of chemical reaction parameter as depicted in Figure 2(b). This implies the reduction of the solute boundary layer thickness significantly and enhances the transport phenomenon.




(a) (b)

**Fig. 2.** Effect of chemical reaction $\gamma$ on (a) Temperature profile $\theta(\eta)$ and (b) Concentration profiles $\phi(\eta)$ for various values of $\gamma = 1, 2, 3, 4$ with $\Pr = 7$, $\lambda = 1$, $M = 1$, $S = 1$, $Sc = 1$, $\lambda_t = 1$ and $\varepsilon = 1$.

In the Tables 3 and 4 show that $f''(0)$, $-\theta'(0)$ and $-\phi'(0)$ increase with the increment of thermal Richardson number $\lambda_t$ for $\lambda$=0.5 and $\lambda$=1. It means that the drag force, the rate of heat transfer and the rate of mass transfer increase on the surface as $\lambda_t$ for both cases. However, the mass transfer rate for $\lambda$=1 is faster than $\lambda$=0.5. It means that the mass transfer rate is more significant when the wall velocity is equal to the fluid velocity, $\lambda$=1, while the mass transfer rate is less significant when the wall velocity is double the fluid velocity, $\lambda$=0.5. There is a negative value of $f''(0)$ for case $\lambda$<1 which means surface exerts the drag force on the fluid, while the other values of $f''(0)$ are positive which show the opposite behaviour. Moreover, all the values of $-\theta'(0)$ are positive which practically means that the heat rate is transferred from the surface to the fluid. This is because the surface is hotter than the fluid.

**Table 3**
Result of the skin friction, $f''(0)$, Nusselt Number, $-\theta'(0)$ and Sherwood Number, $-\phi'(0)$ at the various values of the thermal Richardson Number, $\lambda_t$

| $\lambda_t$ | $f''(0)$ | $-\theta'(0)$ | $-\phi'(0)$ |
|---|---|---|---|
| 1 | -0.29379586 | 3.93825272 | 0.27881377 |
| 3 | 0.75919115 | 4.01665446 | 0.38703486 |
| 5 | 1.72678724 | 4.08181596 | 0.46374447 |
| 6 | 2.18858093 | 4.11110014 | 0.49546206 |

**Table 4**
Result of the Skin Friction, $f''(0)$, Nusselt Number, $-\theta'(0)$ and Sherwood Number, $-\phi'(0)$ at the various values of the thermal Richardson Number, $\lambda_t$

| $\lambda_t$ | $f''(0)$ | $-\theta'(0)$ | $-\phi'(0)$ |
|---|---|---|---|
| 1 | 0.51572068 | 4.01582577 | 0.46079816 |
| 3 | 1.48984903 | 4.08101772 | 0.52559683 |
| 5 | 2.40807072 | 4.13800794 | 0.57822779 |
| 6 | 2.85098409 | 4.16420239 | 0.60138440 |

The velocity profile enhances in the increasing of the thermal Richardson number as shown in Figure 3(a) and 3(b) for cases $\lambda$=0.5 and $\lambda$=1 respectively. The presence of thermal Richardson number over a stretching plate for both cases have a significant impact on velocity profiles as it increases due to the enhancement in buoyancy force that leads to an increase in the values of velocity along with increment values of $\lambda_t$.

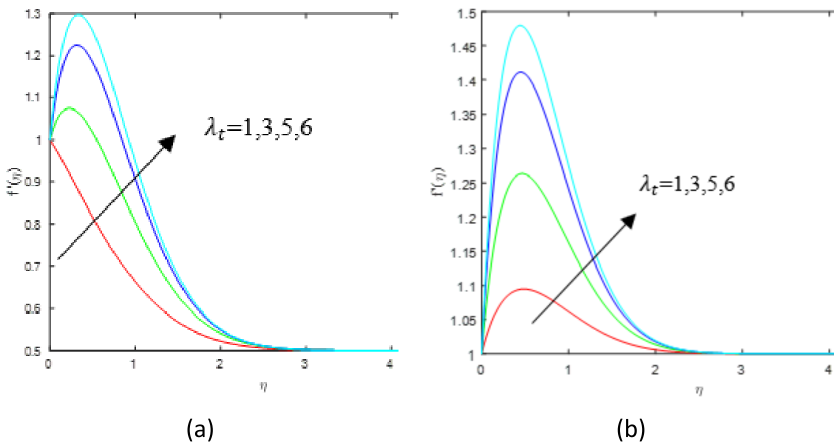
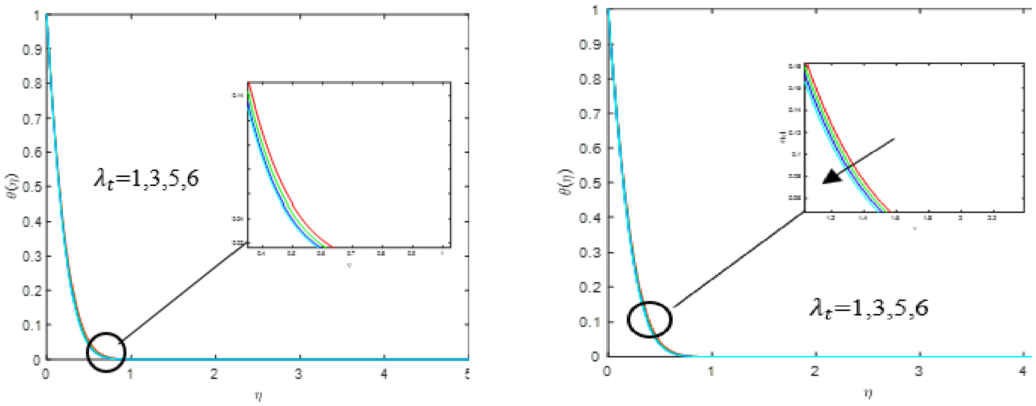


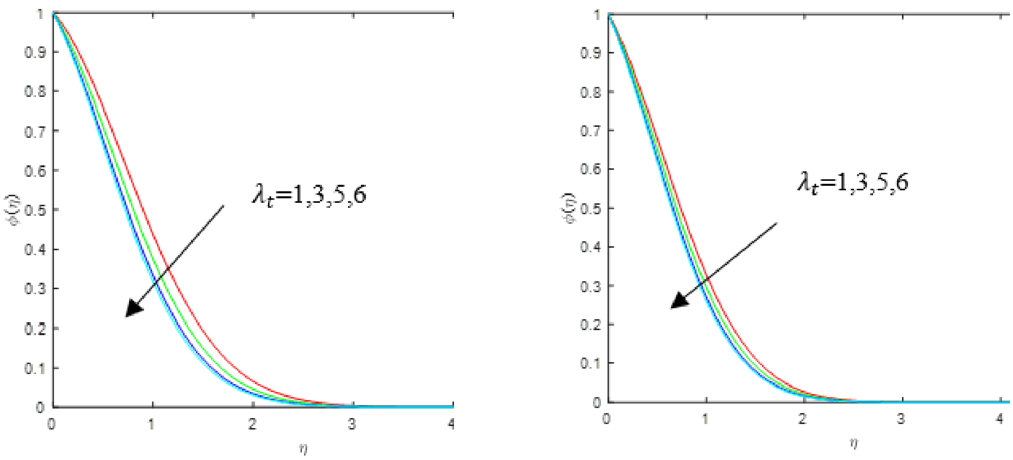

**Fig. 3.** Effect of thermal Richardson Number $\lambda_t$ for values of (a) $\lambda = 0.5$ and (b) $\lambda = 1.0$ =1.0 with $\Pr = 7$, $M = 1$, $S = 1$, $Sc = 1$, $1/D = 1$, $\gamma = 1$ and $\varepsilon = 1$ on velocity profile, $f'(\eta)$

At the same time, in the boundary layer, all the temperature and concentration profiles declined with an increase in $\lambda_t$ as shown in Figures 4 and 5 for cases $\lambda$=0.5 and $\lambda$=1 respectively.




(a) (b)

**Fig. 4.** Effect of thermal Richardson Number $\lambda_t$ for values of (a) $\lambda = 0.5$ and (b) $\lambda = 1.0$ =1.0 with $\Pr = 7$, $M = 1$, $S = 1$, $Sc = 1$, $1/D = 1$, $\gamma = 1$ and $\varepsilon = 1$ on temperature profile, $\theta(\eta)$




(a) (b)

**Fig. 5.** Effect of thermal Richardson Number $\lambda_t$ for values of (a) $\lambda = 0.5$ and (b) $\lambda = 1.0$ with $\Pr = 7$, $M = 1$, $S = 1$, $Sc = 1$, $1/D = 1$, $\gamma = 1$ and $\varepsilon = 1$ on concentration profile, $\phi(\eta)$

Tables 5 and 6 indicate a similar conclusion can be drawn for all the three coefficients which are the skin friction, $f''(0)$, Nusselt number, $-\theta'(0)$ and Sherwood number, $-\phi'(0)$ which all the values accelerate when the buoyancy ratio parameter $\varepsilon$ is increased for both cases $\lambda = 0.5$ and $\lambda = 1.0$.

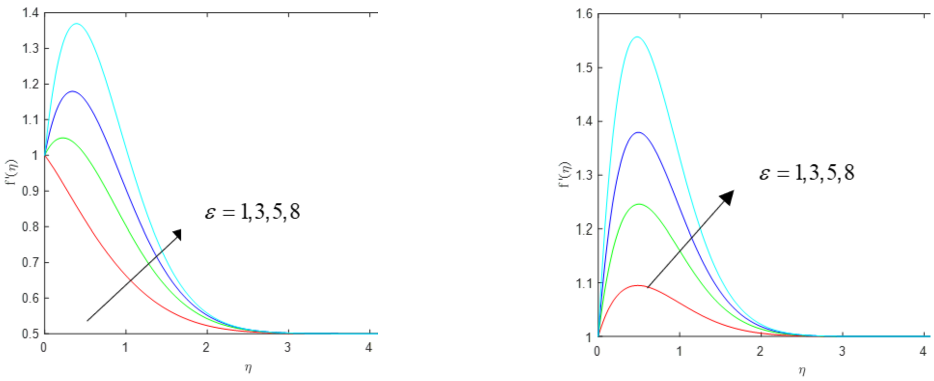
**Table 5**
Result of the Skin Friction, $f''(0)$, Nusselt Number, $-\theta'(0)$ and Sherwood Number, $\phi(\eta)$ at the various values of the Buoyancy Ratio parameter, $\varepsilon$

| $\varepsilon$ | $f''(0)$ | $-\theta'(0)$ | $-\phi'(0)$ |
|---|---|---|---|
| 1 | -0.29379586 | 3.93825272 | 0.27881377 |
| 3 | 0.48053829 | 4.00285751 | 0.37760719 |
| 5 | 1.18669109 | 4.05685063 | 0.44828706 |
| 8 | 2.16731012 | 4.12595081 | 0.52829616 |

**Table 6**
Result of the Skin Friction, $f''(0)$, Nusselt Number, $-\theta'(0)$ and Sherwood Number, $-\phi'(0)$ at the various values of the Buoyancy parameter, $\varepsilon$

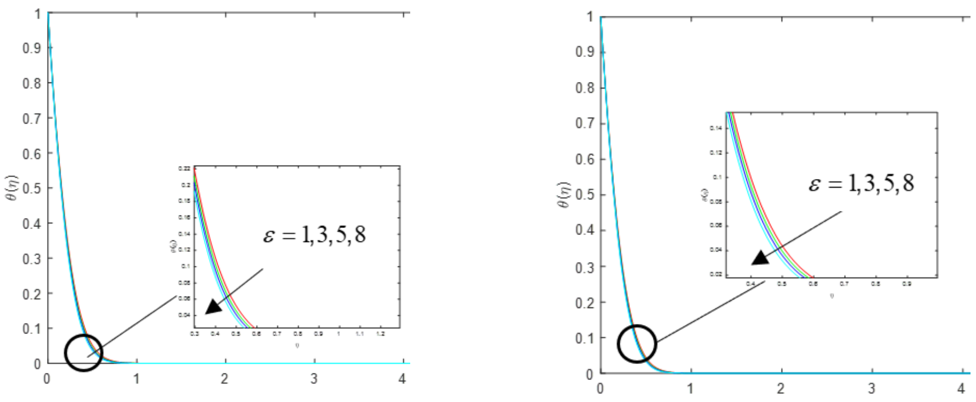
| $\varepsilon$ | $f''(0)$ | $-\theta'(0)$ | $-\phi'(0)$ |
|---|---|---|---|
| 1 | 0.51572068 | 4.01582577 | 0.46079816 |
| 3 | 1.21679096 | 4.06834421 | 0.51812792 |
| 5 | 1.87634963 | 4.11468694 | 0.56522954 |
| 8 | 2.80984137 | 4.17613017 | 0.62369005 |

Figure 6 shows a steep rise in all velocity profiles for the increasing values of $\varepsilon$ with $\lambda$=0.5 and 1 but the temperature and concentration profiles depicted in Figures 7 and 8 are decreasing gradually which are consistent with Figures 3 - 5. Evidently, at a low buoyancy ratio parameter for $\lambda$<1, species buoyancy force has a lesser influence on the flow characteristics and skin friction value is negative (flow reversal). The increment of buoyancy ratio parameter will increase the sheer stress on the wall which implies the hydrodynamic boundary layer thickness increases. The higher the buoyancy ratio parameter, the more the natural convection effect occurs, which follows the increment in the velocity gradient. Both free and forced convection mode contribute to the range of 0 < $\varepsilon$ < 10 which makes the flow strongly accelerated compared to [9], where the flow declines without the presence of buoyancy. This simultaneously encourages more efficient diffusion of heat and species resulting in a marked increase in heat and mass transfer rates at the wall.



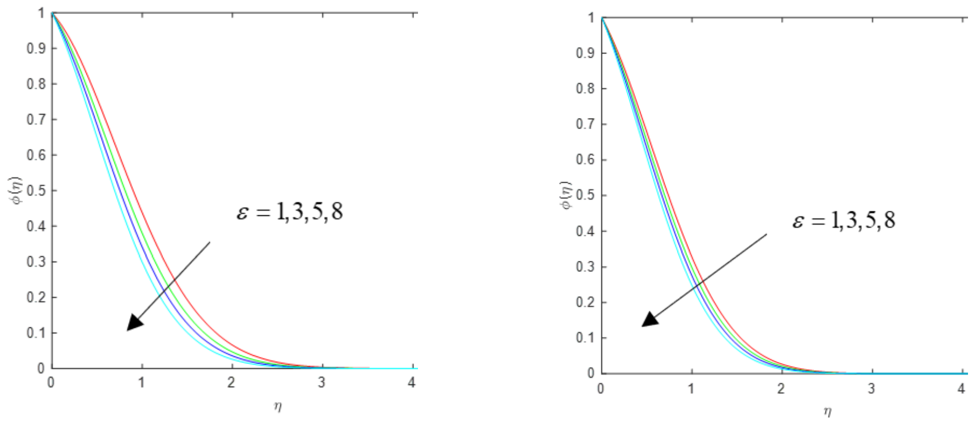

(a) (b)

**Fig. 6.** Effect of the buoyancy ratio parameter, $\varepsilon$ for values of (a) $\lambda$=0.5 and (b) $\lambda$=1.0 with $\Pr = 7$, $M = 1$, $S = 1$, $Sc = 1$, $1/D = 1$, $\gamma = 1$ and $\lambda_t = 1$ on Velocity profile, $f'(\eta)$




(a) (b)

**Fig. 7.** Effect of the buoyancy ratio parameter, $\varepsilon$ for values of (a) $\lambda$ =0.5 and (b) $\lambda$ =1.0 with $\Pr=7,\ M=1,\ S=1,\ Sc=1,\ 1/D=1,\ \gamma=1$ and $\lambda_t=1$ on temperature profile $\theta(\eta)$




(a) (b)

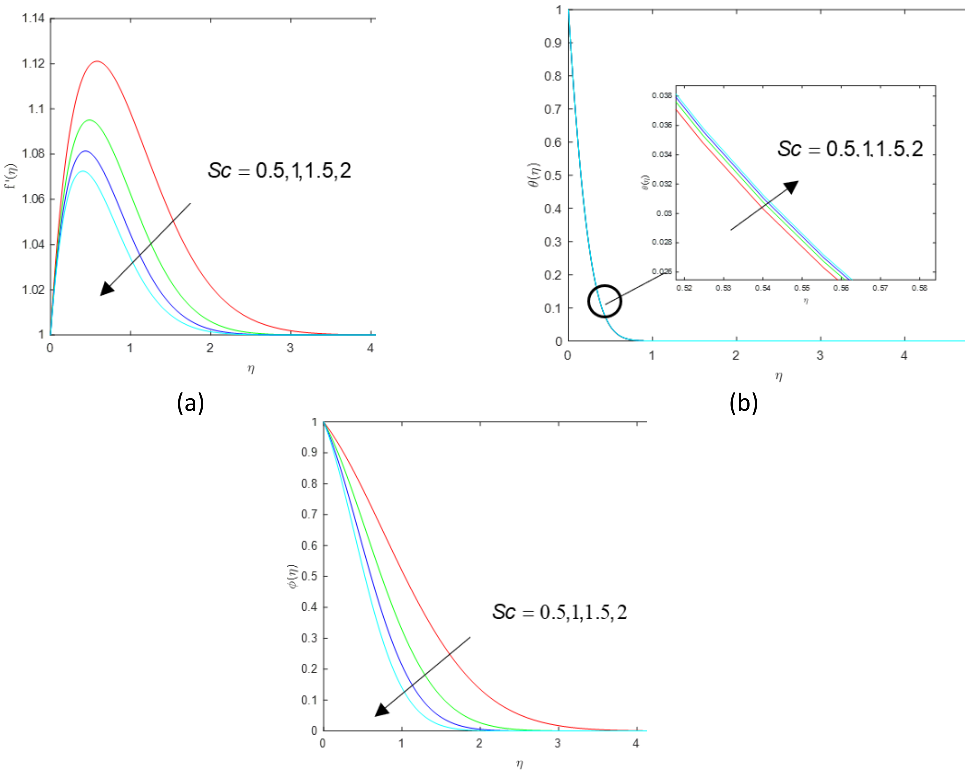
**Fig. 8.** Effect of the buoyancy ratio parameter, $\varepsilon$ for values of (a) $\lambda=0.5$ and (b) $\lambda=1.0$ with $\Pr=7,\ M=1,\ S=1,\ Sc=1,\ 1/D=1,\ \gamma=1$ and $\lambda_t=1$ on Concentration profile, $\phi(\eta)$

Table 7 displays the increase of Schmidt number *Sc* results in a slight decrease of the local skin friction and Nusselt number but a drastic increase is marked in local Sherwood number. Schmidt number refers to the ratio of viscous diffusion and mass diffusion rates. The higher Schmidt number represents the higher viscosity which ultimately increase the convective mass transfer on the surface and simultaneously escalates the Sherwood number.

**Table 7**
Result of the skin friction, $f''(0)$, Nusselt Number, $-\theta'(0)$ and Sherwood Number $-\phi'(0)$ at the various values of the Schmidt Number, *Sc*

| *Sc* | $f''(0)$ | $-\theta'(0)$ | $-\phi'(0)$ |
|---|---|---|---|
| 0.5 | 0.56884478 | 4.02151731 | 0.33225421 |
| 1.0 | 0.51572068 | 4.01582577 | 0.46079816 |
| 1.5 | 0.48290339 | 4.01233579 | 0.55801458 |
| 2.0 | 0.45932707 | 4.00984938 | 0.63935828 |

Figures 9(a) and 9(c) portray the velocity and concentration profiles decelerate strongly with an increase of *Sc* while temperature profile in Figure 9(b) increasing with *Sc*. From Figure 9(c), the concentration in boundary layer thickness retards due to the decreasing of molecular diffusivity since *Sc* is inversely varied to the diffusion coefficient. It can be concluded that the species concentration is low for superior values of *Sc* and high for smaller values of *Sc* throughout the boundary layer regime transverse to the wall.




(c)

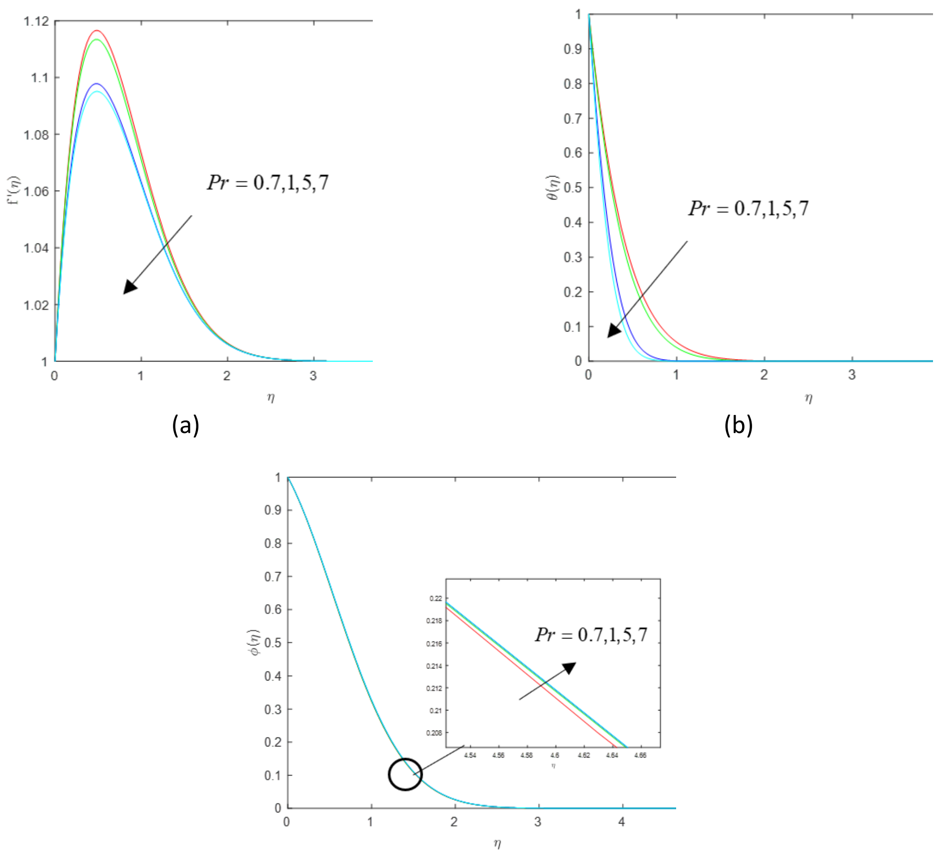
**Fig. 9.** Effect of the various values of the Schmidt Number, *Sc* with $\mathrm{Pr}=7,\ \lambda=1,\ M=1,\ S=1,\ \gamma=1,\ 1/D=1,\ \lambda_t=1$ and $\varepsilon=1$ on (a) Velocity profile $f'(\eta)$, (b) Temperature profile, $\theta(\eta)$ and (c) Concentration profile, $\phi(\eta)$

Table 8 presents the effect of Prandtl number towards the skin friction, $f''(0)$, Nusselt number, $-\theta'(0)$ and Sherwood number $-\phi'(0)$. It is observed $f''(0)$ and $-\phi'(0)$ decrease steadily while $-\theta'(0)$ increases remarkably with the increment of *Pr*.

**Table 8**
Result of the Skin Friction, $f''(0)$, Nusselt Number, $-\theta'(0)$ and Sherwood Number, $-\phi'(0)$ at the various values of the Prandtl Number, *Pr*

| *Pr* | $f''(0)$ | $-\theta'(0)$ | $-\phi'(0)$ |
|---|---|---|---|
| 0.7 | 0.59494906 | 2.40352644 | 0.46868937 |
| 1.0 | 0.58544147 | 2.53246980 | 0.46747596 |
| 5.0 | 0.52892221 | 3.63175866 | 0.46178696 |
| 7.0 | 0.51572068 | 4.01582577 | 0.46079816 |

Figures 10(a) - 10(b) perceived that the velocity and temperature profiles declined with an increase in Prandtl number. However, there is a minimal increment in the concentration profile as shown in Figure 10(c) as *Pr* values increasing. This is because the isotherms are almost linear due to the combined effect of conduction and forced convection thus makes the concentration profile increase gradually with the increment of *Pr*. The thickness of the thermal boundary layer will diminish since higher Prandtl number has lower thermal conductivity, reducing conduction and results in the increasing of heat transfer rate on the plate as displayed in Figure 10(b) and proven by Table 8.




(c)

**Fig. 10.** The effect of Prandtl Number, *Pr* with with $\lambda = 1$, $M = 1, S = 1, Sc = 1$, $\gamma = 1$, $1/D = 1$, $\lambda_t = 1$ and $\varepsilon = 1$ on (a) Velocity profile $f'(\eta)$, (b) Temperature profile $\theta(\eta)$ and (c) Concentration profile, $\phi(\eta)$

## 4. Conclusions

The present paper analysed the problem of the chemical reaction on unsteady MHD flow over a vertical stretching plate embedded in porous medium with Richardson number. Several parameters were considered which were chemical reaction rate $\gamma$, Schmidt number *Sc,* Prandtl number *Pr,* thermal Richardson number $\lambda_t$ and buoyancy ratio $\varepsilon$. The analyses focused on the skin friction coefficient, $f''(0)$, Nusselt number, $-\theta'(0)$ and Sherwood number $-\phi'(0)$ along with the velocity, temperature and concentration behaviours.

i. Temperature profile and Sherwood number increase as the chemical reaction parameter increase while skin friction, Nusselt number, velocity and concentration profile decrease.

ii. There is a decrease in the skin friction with the velocity, Nusselt number and concentration profile with the increment of Schmidt number, *Sc* while Sherwood number and temperature profile increase.
iii. Due to increasing Prandtl number *Pr*, the skin friction and the velocity behaviour, Sherwood number and temperature profile decrease. Meanwhile, the Nusselt number, as well as concentration profile increase.
iv. The skin friction coefficient, Nusselt number, Sherwood number and velocity profile increase with rising values of thermal Richardson number while temperature and concentration profiles decrease.
v. There is an increment in the skin friction coefficient, Nusselt number, Sherwood number and velocity profile as the buoyancy ratio parameter increase whereas temperature and concentration profiles decrease which were consistent with effect of thermal Richardson number.

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