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Numerical Study Model for Solidification and Melting of Nano-enhanced Phase Change Material by Enthalpy Porosity Method

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ABSTRACT

This paper presents numerical work on solidification and melting of nano-enhanced phase change material. In this work, we propose an improvised numerical solution for solidification and melting of nano-enhanced phase change material (NEPCM) by allowing different thermophysical properties in solid and liquid phase in enthalpy-porosity method. The solidification and melting of NEPCM will be conducted in a two-dimensional computational domain with adiabatic horizontal walls and isothermal vertical walls. In this approach, a single-domain formulation is coupled with porosity source term to the momentum equations to differentiate liquid and solid regions during the phase change process. A source term is also added to the energy equation via enthalpy terms to account for latent heat evolution during the same process. This research is expected to produce results closer to the experimental results relative to previous numerical works. This will indicate that the proposed model has contributed to the improvement on existing NEPCM solidification and melting numerical model.

Keywords:
Nano-enhanced phase change material; PCM; enthalpy-porosity method

1. Introduction

Over the recent years, researchers have dedicated their works to enhance the effectiveness of Phase Change Materials (PCM) as thermal energy storage system for various purposes. PCM relies on its ability to absorb and remove latent heat during its phase changing process in order to store and release heat energy as required. Such latent heat storage system is attractive as it offers high energy storage capacity during the process within acceptable range of temperature change [1, 2]. Although phase change process may occur in other different phase combinations, the most practical phase changes for PCM applications are liquid-solid (Solidification) and solid-liquid (Melting) [3]. In other words, PCM solidification allows for latent heat 'discharging' while PCM melting enables latent heat 'charging'. Figure 1 shows the principle of PCM application to achieve its function.

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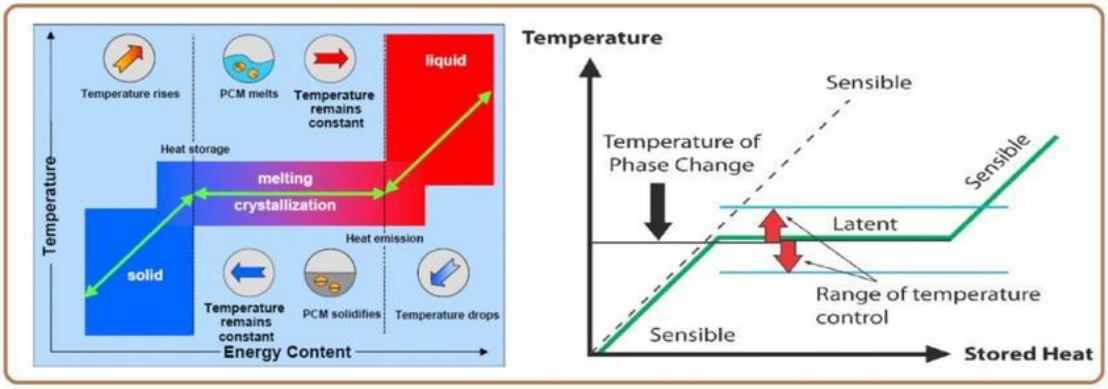

Fig. 1. Principle of Phase Change Materials (PCM) [4]

Nanotechnology is manipulation of nano-sized particles to achieve new technological innovations. Choi *et al*., [5] proved that thermal conductivity of fluids may be enhanced by dispersing nanosized particles. In the following years, many studies have been performed on nanofluids which confirms its ability to enhance liquid thermophysical properties [6, 7]. Parallel with this advancement, nanoparticles have also been applied to improve the performance of PCM thus the term Nano-Enhanced Phase Change Materials (NEPCM) is introduced. Figure 2 shows the typical composition of NEPCM which is a nanofluid substance which consist of base-fluid (PCM) and nanoparticles.

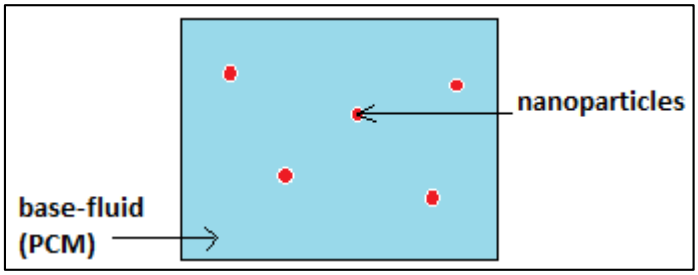

Fig. 2. NEPCM Composition

In 2007, Khodadadi and Hosseinizadeh [8] added nanoparticles in PCM to investigate these fine particles influence on its performance during phase change process and the result was encouraging. The introduction of NEPCM calls for the need to further analyse the balance in between the benefits offered and the cost of developing the material. For such detail analysis and study, numerical simulation is a convenient and economical approach that have been widely used over the years. It allows various different conditions to be simulated by substituting the variables in the models used.

Currently, NEPCM phase change numerical simulations can be performed by macroscale or mesoscale approach where the latter approach is relatively more complex [9]. In macroscale, NEPCM phase change can be modelled in one domain or two domains. Single-domain modelling is simpler compared to multi-domain modelling because only one domain needs to be solved at each iteration. Therefore, this approach is widely used and preferred by most researchers. The enthalpy-porosity method is commonly used for this approach. However, in most applications involving this method for phase change, the thermophysical properties for each solid and liquid phase is simplified to be similar or averaged to a single value [10].

In the 19th century, some studies on modelling solidification and melting have been recorded. The process was understood as a transient problem where a boundary in between solid and liquid develops and moves in the domain throughout the process. The position of this boundary is unknown in advance thus need to be determined as function of time and space as part of the solution. Although

Lame and Clapeyron [11] initiated this study, it was made well known by Stefan later through his works thus the name; Stefan Problem [12]. Many of the early works in this area solved pure heat conduction equation which neglects the existence of convective effect. In reality however, convective effects were proven to significantly influence the phenomena or in some cases dominate the overall heat transfer process [13, 14]. In addition to this fact, early models were also unable to solve multidimensional problems.

A more practical approach was developed later to avoid tracing the moving boundary explicitly. The moving boundary is indirectly embedded in a different form of equation. The methods under this category is known as methods of latent-heat evolution. They are widely known as weak solutions for the problem as the special attention required on the moving boundary is no longer required. One of the prominent methods under this category is enthalpy method [15]. Compared to the other methods, the main characteristic of enthalpy method is the evolution of latent heat is accounted by enthalpy as well as the correlation in between enthalpy and temperature; in terms of the latent heat characteristics of the material under phase change.

With the advancement of numerical methods and computational capability in the mid-20th century, solution to Navier-Stokes equations which were earlier limited and deemed to be tedious received more attention. Newtonian thermal-fluid flow problems were able to be predicted quantitatively by solving the set of mass, momentum and energy conservation equations. This scenario enabled for solidification / melting simulations to be conducted by considering both conductive and convective heat transfer mechanisms. As a result, enthalpy-porosity method (EPM) was developed [16, 17] . This approach is a single-domain formulation which adds porosity source term to the momentum equations to differentiate liquid and solid regions during the phase change process. A source term is also added to the energy equation via enthalpy terms to account for latent heat evolution during the same process.

In the 21st century, progress have been made to develop other approach which are able to solve solidification and melting phenomena. Mesoscale methods such as Lattice Boltzmann Method (LBM) have been proven to be able to yield acceptable results by modelling fluid flow at mesoscopic level in terms of local interactions between particles. LBM was first implemented for phase change material simulations in 2001 by Miller *et al*., [18].

Numerical simulations on NEPCM solidification & melting are mainly performed by researchers to study the parameters which influence the process such as nanoparticle fraction in the mixture, nanoparticle size, boundary conditions applied, and geometry. These factors have been investigated by many researchers by applying several different numerical models. Details on the numerical models used and findings of each research are summarized in Table 1 and Table 2. All works were validated by either previous experimental (E) or numerical (N) works and showed acceptable agreement.

Table 1 shows summary of numerical simulations for NEPCM solidification. The simulations were performed at various different nanoparticle characteristics (particle size and particle volume concentration) and different geometries. All of the works applied enthalpy-porosity method coupled with finite volume method (EPM-FVM) [8], [19-25] which is a single domain approach. All of the works also applied linear function mixture model [26] to obtain values for density, heat capacity and thermal expansion coefficient. Most of the works if not all, obtained values of dynamic viscosity and thermal conductivity using Brinkman [27] and Maxwell [28] correlation respectively.

Table 2 shows summary of numerical simulations for NEPCM melting. Similar to the previous phase change process, the simulations were also performed at various different nanoparticle characteristics and different geometries. Most of the works also applied enthalpy-porosity method coupled with finite volume method (EPM-FVM) [29-38] except for others which did not specify specifically the method applied [39] or applied mesoscale simulation by applying Lattice Boltzmann

Method (LBM) [40-43]. Other correlation [44-47] besides the mostly applied Brinkman and Maxwell were also used to represent the values of dynamic viscosity and thermal conductivity of the substance while all models still resorted to linear function mixture model to obtain values for density, heat capacity and thermal expansion coefficient.

Table 1
Summary of numerical simulations for NEPCM solidification

Ref	NEPCM	Particle size (nm)	Particle Volume (%)	Geometry	μ and k correlation	Properties value for solid / liquid	Findings
[8]	H_2O + Cu	10	0, 10, 20	Square	Brinkman[27] Maxwell [28]	Similar	**↑ Particle fraction results in:** ↑ heat release rate & k ↓ latent heat of fusion
[19]	H_2O + Cu	Not reported	0, 2.5, 5	Annulus	Brinkman[27] Maxwell [28]	Similar	**↑ Particle fraction results in:** ↑ heat transfer rate & k ↑ ΔT: ↓solidification time
[20]	n-Octadecane + Cu	45	0, 2, 8	Rectangle	Brinkman[27] Maxwell [28]	Similar	**↑ Particle fraction results in:** ↑ solidification & heat discharge rate
[21]	Paraffin + Al_2O_3	Not reported	0, 2, 5, 10	Annulus	Vajjha [44]	Different k f(T) = ρ & μ	**↑ Particle fraction results in:** ↑ charge-discharge rate ↑ heat transfer rate & k ↓ energy storage capacity
[22]	n-Hexadecane + Cu	10	0, 3, 8	Square	Corcione [45] Maxwell [28]	Similar except k	**↑ Particle fraction results in:** ↑ heat transfer rate & k + ↓wall T =↑solid fraction
[23]	H_2O + Cu	10	0, 10, 20	Trapezium	Brinkman[27] Maxwell [28]	Similar	**↑ Particle fraction &ΔT results in:** ↑ solidification & heat transfer rate **↑ Incline angle results in:** ↑ solidification rate
[24]	H_2O + Cu	2, 5	10 (wt%)	Square	Brinkman[27] Maxwell [28]	Similar	**↓ Size particles results in:** ↑ thermal conductivity ↓ frozen region thickness ≠ phase interface shape
[25]	H_2O + Cu	1	0, 5, 10	Wavy	Brinkman[27] Maxwell [28]	Similar except k	**↑ Particle fraction & waviness:** ↑ solidification rate

It should be noted that most models used similar thermophysical properties values for both in solid and liquid region during the phase change process. Others have different values, but they are dependent on temperature [21], [33-36] and not the phase condition. The models which have different value of property for both phases [22,25,37,40] are only for thermal conductivity which also includes the previously mentioned works.

Table 2
Summary of numerical simulations for NEPCM melting

Ref	NEPCM	Particle size (nm)	Particle Volume (%)	Geometry	Numerical Technique	μ and k correlation	Properties value for solid / liquid	Findings
[29]	Eicosane + CNT	Not reported	0, 2, 10	Rectangle	EPM -FVM	Brinkman[27] Gharagozloo [46]	Similar	↑ **Particle fraction results in:** ↑ melting rate & k ↑ viscosity ↓ latent heat of fusion
[30]	n-octadecane + CuO	Not reported	0, 2.5, 5	Square	EPM - FVM	Brinkman[27] Maxwell [28]	Similar	↑ **Particle fraction results in:** ↑ melting rate & k ↑ viscosity
[31]	RT 50 + Cu	1	0, 3, 5	Annulus	EPM - FVM	Brinkman[27] Maxwell [28]	Similar	↑ **Particle fraction results in:** ↑ melting rate ↑ melt front penetration ↑ liquid fraction
[32]	Paraffin + Cu	Not reported	0, 2.5, 5	Square	EPM - FVM	Brinkman[27] Maxwell [28]	Similar	↑ **Particle fraction results in:** ↑ melting rate & k ↑ **ΔT results in:** ↓melting time
[39]	NaNO3+ Al_2O_3	Not reported	0, 3, 6	Circle	FVM	Brinkman[27] Maxwell [28]	Different k f(T)= ρ, μ, Cp	↑ **Particle fraction results in:** ↑ melting rate & k
[33]	Paraffin + CuO	10	0 – 7	Shell-tube	EPM - FVM	Vajjha [44]	Different k f(T) = ρ & μ	↑ **Particle fraction results in:** ↑ melting rate ↑ viscosity
[40]	H_2O + Cu	1	0, 5, 10	Square	EB - LBM - DDF	Brinkman[27] Maxwell [28]	Similar except k	↑ **Particle fraction results in:** ↑ melting rate & k
[41]	H_2O + Cu	100	0, 1, 2, 3	Square	EB - LBM - DDF	Brinkman[27] Patel et.al[47]	similar	↑ **Particle fraction results in:** ↑ melting rate & k ↓ latent heat of fusion
[42]	H_2O + Cu	100	0, 2, 4	Cylinder	EB - LBM - DDF	Brinkman[27] Patel et.al [47]	Similar	↑ **Particle fraction results in:** ↑ melting rate & k ↓ latent heat of fusion
[43]	H_2O + Cu	100	0, 2, 4	Semi circle	EB - LBM - DDF	Brinkman[27] Patel et.al [47]	Similar	↑ **Particle fraction results in:** ↑ melting rate & k ↓ latent heat of fusion ↑ viscosity
[34]	Paraffin + Al_2O_3	Not reported	0, 2, 5	Square	EPM - FVM	Vajjha [44]	Different k f(T) = ρ & μ	**Source-sink at dedicated walls and hot wall on top results in:** = Highest melting rate
[35]	Paraffin + Al_2O_3	Not reported	0, 2, 5	Square	EPM - FVM	Vajjha [44]	Different k f(T) = ρ & μ	**Source-sink with alternate location & order on vertical wall results in:** = Highest melting rate
[36]	Paraffin + CuO	29	0, 1, 3	Square	EPM - FVM	Vajjha [44]	f(T) = ρ,k &μ	**Bottom heating results in:** ↑ melting rate ↑ stored energy
[37]	P116 + Al_2O_3	45	0, 2, 6, 8	Rectangle	EPM - FVM	Brinkman[27]		↑ **Aspect ratio results in:** ↑ area for heat transfer

						Maxwell [28]	Similar except k	↑ melting / charging rate
[38]	RT 50 + CuO	1	0, 2, 4 (wt)	Annulus	EPM - FVM	Not indicated	Similar	↑ **Particle fraction results in:** ↑ melting rate & k ↑ **Inclination angle:** ↑ melting rate

As can be seen in the literature, most numerical models for simulation of solidification and melting process of NEPCM have been established mainly by applying enthalpy-porosity formulation coupled with finite volume method. Most models solved a single macroscale domain at each iteration which resulted in no attempt to solve it in two domains which requires more complex approach. Such dilemma is avoided when mesoscale methods (Lattice Boltzmann Method-LBM) are applied but this method is less favourable as it is also relatively more complex. Regardless of the chosen scale, most models were simplified by assuming that the thermophysical properties values are similar throughout the phase change process regardless for liquid or solid. Such oversimplified model may result in critical deviation of the result relative to physical or experimental solution [10].

2. Methodology

In this research, the solidification and melting of NEPCM will be conducted in a two-dimensional computational domain with adiabatic horizontal walls and isothermal vertical walls. Figure 3 shows an example of the numerical domain for NEPCM melting process.

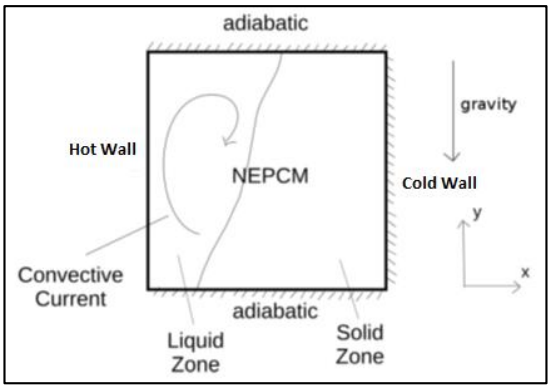

Fig. 3. NEPCM Melting Process Numerical Domain [65]

The system of equations for natural convection coupled with phase change will be applied to simulate NEPCM solidification and melting. These equations will be correlated with the phenomena by using FORTRAN code. The program will use the enthalpy-porosity method [16, 17] coupled with finite volume method (EPM-FVM) as this approach has been widely used by other researchers to represent the phenomena. This approach is a single-domain formulation which adds porosity source term to the momentum equations to differentiate liquid and solid regions during the phase change process. A source term is also added to the energy equation via enthalpy terms to account for latent heat evolution during the same process.

Assuming the mixture is incompressible, behaves as a Newtonian fluid; the continuity, momentum and energy conservation equations for 2-dimension are written as follows:

Continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

x-momentum equation:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho_{nf}}\left[-\frac{\partial P}{\partial x}+\mu_{nf}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-C\frac{(1-\varepsilon)^2}{\varepsilon^3+b}u\right] \quad (2)$$

y-momentum equation:

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=\frac{1}{\rho_{nf}}\left[-\frac{\partial P}{\partial y}+\mu_{nf}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-C\frac{(1-\varepsilon)^2}{\varepsilon^3+b}v+(\rho\beta)_{nf}g\left(h-h_{ref}\right)\right] \quad (3)$$

Energy equation:

$$\frac{\partial h}{\partial t}+u\frac{\partial h}{\partial x}+v\frac{\partial h}{\partial y}=\frac{k_{nf}}{(\rho C_p)_{nf}}\left(\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}\right)\pm\left[\frac{\partial[\rho\Delta H]}{\partial t}+\frac{\partial[\rho\Delta H]}{\partial x}+\frac{\partial[\rho\Delta H]}{\partial y}\right] \quad (4)$$

Subscript *nf* denotes nanofluid (NEPCM). Three distinct regions will be present during the phase change process: a solid region, a liquid region, and a mushy region consisting of liquid and solid.

In a system going through a phase change via heat transfer, the total enthalpy is indicated as:

$$H=h+\Delta H \quad (5)$$

where it is the sum of sensible enthalpy, h and latent heat, ΔH. The latent heat component is temperature dependent and the phase change process should occur within a temperature range since NEPCM is a non-pure substance:

$$\Delta H=\varepsilon(T)=\begin{Bmatrix} L\ for\ T>T_L \\ L(1-\varepsilon)\ for\ T_L>T>T_S \\ 0\ for\ T<T_S \end{Bmatrix} \quad (6)$$

where a parameter known as liquid fraction, ε is introduced to indicate the fraction of liquid in all cells and T_L and T_S is the melting and freezing temperature respectively while L is the latent heat of fusion. The liquid fraction is defined as follows:

$$\varepsilon=\frac{\Delta H}{L} \quad (7)$$

For linear phase change which should occur for NEPCM (non-pure substance):

$\varepsilon = 0$ if $T < T_S$ and $\varepsilon = 1$ if $T > T_L$ (8)

For $T_S < T < T_L$, liquid fraction, ε is

$$\varepsilon=\frac{T-T_S}{T_L-T_S} \quad (9)$$

This parameter is calculated for each cell in the domain at each iteration, based on an enthalpy balance. The mushy zone is a region in which the liquid fraction lies between 0 and 1. The mushy zone is modelled as a pseudo porous medium in which the porosity increases from 0 to 1 or decreases from 1 to 0 as the NEPCM liquefies or solidifies respectively. Porosity decreases to zero in the solid region and therefore the velocity in the region will also drop to zero as indicated in the source term.

The single value thermophysical properties will be estimated by linear function mixture model [26] for density, heat capacity and thermal expansion coefficient. Dynamic viscosity and thermal conductivity will be estimated using Brinkman [27] and Maxwell [28] correlations respectively.

Density of NEPCM will be defined by

$$\rho_{nf} = (1 - \emptyset)\rho_f + \emptyset\rho_s \tag{10}$$

Heat capacity of NEPCM will be defined by:

$$\left(\rho C_p\right)_{nf} = (1 - \emptyset)\left(\rho C_p\right)_f + \emptyset\left(\rho C_p\right)_s \tag{11}$$

Thermal expansion coefficient of NEPCM will be defined by:

$$(\rho\beta)_{nf} = (1 - \emptyset)(\rho\beta)_f + \emptyset(\rho\beta)_s \tag{12}$$

Dynamic viscosity of NEPCM will be defined by:

$$\mu_{nf} = \frac{1}{(1-\emptyset)^{2.5}}\mu_f \tag{13}$$

Thermal conductivity of NEPCM will be defined by:

$$k_{nf} = \frac{k_s+2k_f+2(k_s-k_f)\emptyset}{k_s+2k_f-2(k_s-k_f)\emptyset}k_f \tag{14}$$

Subscript *nf* denotes nanofluid (NEPCM), while *f* and *s* are the values for the base-fluid (PCM) and nanoparticle respectively. $\emptyset$ is the volume fraction of nanoparticle dispersed in the PCM.

For single domain solution which will allow two values of thermophysical properties for solid and liquid region to be considered, the continuity, momentum and energy conservation will be amended and improvised as one of the objectives in this research.

The widely known algorithm Semi-Implicit Method for Pressure Linked Equations (SIMPLE) [66] will be used to solve the equations that govern the fluid flow and heat transfer phenomena. Figure 4 shows the overall flow chart for the methodology approach to be applied in this research. Finally, in order to visualize all the numerical values obtained from the simulation, Tecplot 360 EX shall be used.

2.1 Model Validation

Several diagrams will be constructed from the calculated domain for validation purposes. Melting or solidification front progress with time will be compared with predictions from experimental work of Gau and Viskanta [49] and numerical work of Brent *et al*.,[17]. The shape of the phase change front must be in acceptable agreement with these previous works.

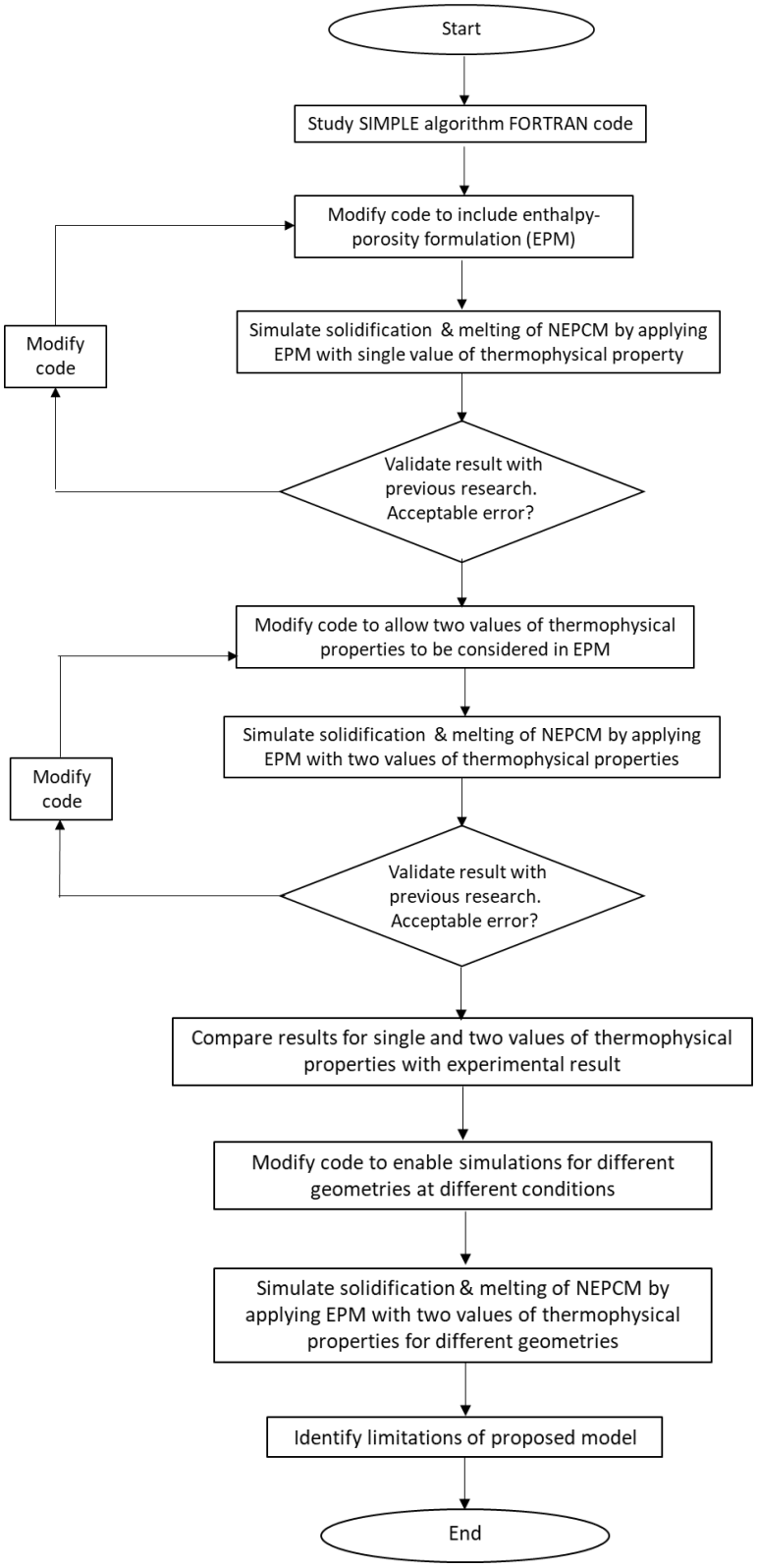

Fig. 4. Research Methodology Flow Chart

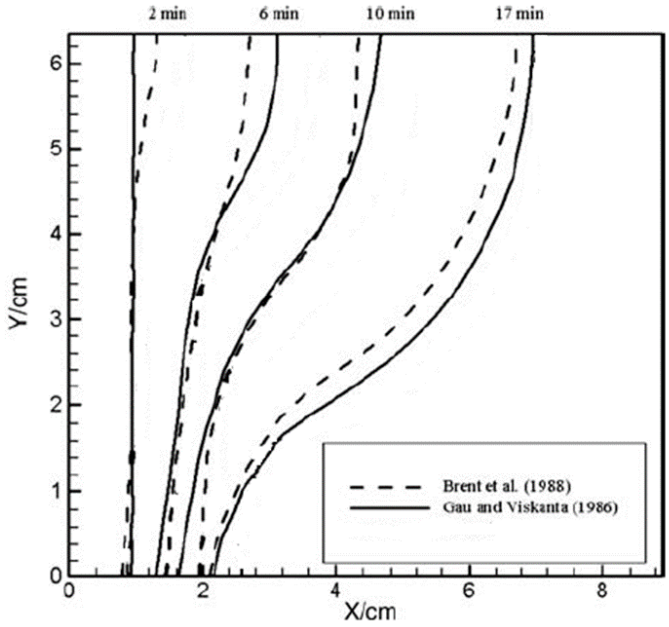

Fig. 5. Prediction of melting front with time [17], [49]

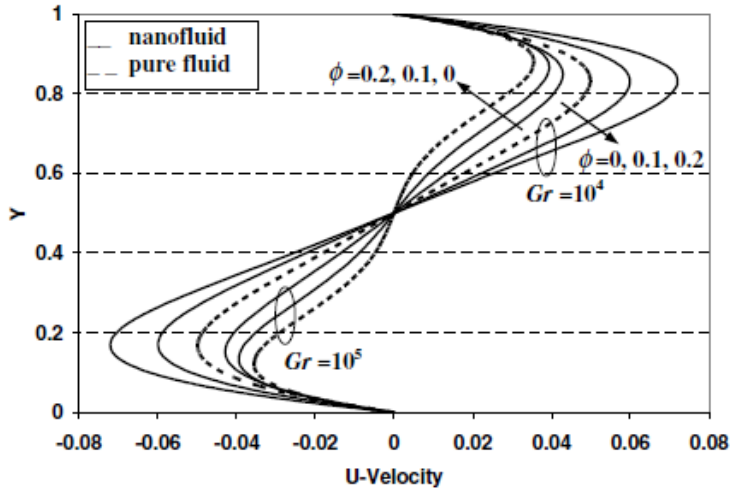

Fig. 6. Predicted horizontal velocity component on the vertical mid-plane of the square cavity [48]

Next, the horizontal velocity component on the vertical mid-plane of the square cavity will be compared with the works of Khanafer *et al*., [48]. This parameter must also be in acceptable agreement. The similarities mentioned above must be present to ensure the model satisfies the physics of the problem.

3. Conclusion

This research is expected to produce results which are even closer to the experimental results relative to previous numerical works. This will indicate that the proposed model has contributed to improvement on existing NEPCM solidification and melting numerical model. The limitations and

applicability of this proposed model will also be identified to avoid production of unacceptable or less accurate result during future model application. It is expected by completing this research, critical deviation of NEPCM phase change numerical simulation result relative to physical or experimental solution can be further reduced.

References

[1] Trelles, Juan P., and John J. Dufly. "Numerical simulation of porous latent heat thermal energy storage for thermoelectric cooling." *Applied Thermal Engineering* 23, no. 13 (2003): 1647-1664.

[2] Farid, Mohammed M., Amar M. Khudhair, Siddique Ali K. Razack, and Said Al-Hallaj. "A review on phase change energy storage: materials and applications." *Energy conversion and management* 45, no. 9-10 (2004): 1597-1615.

[3] Bruno, F., M. Belusko, M. Liu, and N. H. S. Tay. "Using solid-liquid phase change materials (PCMs) in thermal energy storage systems." In *Advances in Thermal Energy Storage Systems*, pp. 201-246. Woodhead Publishing, 2015.

[4] V. Majidzade, D. Babanly, S. V. Askerova, and D. Tagiyev, "Phase Relations and Some Properties of the System TI1 - T12Se TI2Te," *3rd International Turkic World Conference on Chemical Sciences and Technologies,* 2017.

[5] Choi, Stephen US, and Jeffrey A. Eastman. *Enhancing thermal conductivity of fluids with nanoparticles*. No. ANL/MSD/CP-84938; CONF-951135-29. Argonne National Lab., IL (United States), 1995.

[6] Yu, Wenhua, David M. France, Jules L. Routbort, and Stephen US Choi. "Review and comparison of nanofluid thermal conductivity and heat transfer enhancements." *Heat transfer engineering* 29, no. 5 (2008): 432-460.

[7] Özerinç, Sezer, Sadık Kakaç, and Almıla Güvenç Yazıcıoğlu. "Enhanced thermal conductivity of nanofluids: a state-of-the-art review." *Microfluidics and Nanofluidics* 8, no. 2 (2010): 145-170.

[8] Khodadadi, J. M., and S. F. Hosseinizadeh. "Nanoparticle-enhanced phase change materials (NEPCM) with great potential for improved thermal energy storage." *International communications in heat and mass transfer* 34, no. 5 (2007): 534-543.

[9] Tessarotto, Marco, Enrico Fonda, and Massimo Tessarotto. "The Computational Complexity of Traditional Lattice-Boltzmann Methods for Incompressible Fluids." In *AIP Conference Proceedings*, vol. 1084, no. 1, pp. 470-475. AIP, 2008.

[10] Belhamadia, Y. O. U. S. S. E. F., ABDOULAYE S. Kane, and A. N. D. R. É. Fortin. "An enhanced mathematical model for phase change problems with natural convection." *Int. J. Numer. Anal. Model* 3, no. 2 (2012): 192-206.

[11] Lamé, G., and B. P. Clapeyron. "Mémoire sur la solidification par refroidissement d'un globe liquide." In *Annales Chimie Physique*, vol. 47, pp. 250-256. 1831.

[12] Hu, Henry, and Stavros A. Argyropoulos. "Mathematical modelling of solidification and melting: a review." *Modelling and Simulation in Materials Science and Engineering* 4, no. 4 (1996): 371..

[13] Sparrow, Ephraim M., S. V. Patankar, and S. Ramadhyani. "Analysis of melting in the presence of natural convection in the melt region." (1977): 520-526.

[14] Lamberg, Piia, Reijo Lehtiniemi, and Anna-Maria Henell. "Numerical and experimental investigation of melting and freezing processes in phase change material storage." *International Journal of Thermal Sciences* 43, no. 3 (2004): 277-287.

[15] Voller, V., and M. Cross. "Accurate solutions of moving boundary problems using the enthalpy method." *International journal of heat and mass transfer* 24, no. 3 (1981): 545-556.

[16] Voller, Vaughan R., M. Cross, and N. C. Markatos. "An enthalpy method for convection/diffusion phase change." *International journal for numerical methods in engineering* 24, no. 1 (1987): 271-284.

[17] Brent, A. D., Vaughan R. Voller, and K. T. J. Reid. "Enthalpy-porosity technique for modeling convection-diffusion phase change: application to the melting of a pure metal." *Numerical Heat Transfer, Part A Applications* 13, no. 3 (1988): 297-318.

[18] Miller, W. "The lattice Boltzmann method: a new tool for numerical simulation of the interaction of growth kinetics and melt flow." *Journal of crystal growth* 230, no. 1-2 (2001): 263-269.

[19] Sebti, S. S., S. H. Khalilarya, I. Mirzaee, S. F. Hosseinizadeh, S. Kashani, and M. Abdollahzadeh. "A numerical investigation of solidification in horizontal concentric annuli filled with nano-enhanced phase change material (NEPCM)." *World Applied Sciences Journal* 13, no. 1 (2011): 09-15.

[20] Elbahjaoui, Radouane, and Hamid El Qarnia. "Thermal analysis of nanoparticle-enhanced phase change material solidification in a rectangular latent heat storage unit including natural convection." *Energy and Buildings* 153 (2017): 1-17.

[21] Arasu, Amirtham VALAN, Agus P. Sasmito, and Arun S. Mujumdar. "Numerical performance study of paraffin wax dispersed with alumina in a concentric pipe latent heat storage system." *Thermal science* 17 (2013): 419-430.

[22] Kashani, S., A. A. Ranjbar, M. M. Madani, M. Mastiani, and H. Jalaly. "Numerical study of solidification of a nano-enhanced phase change material (NEPCM) in a thermal storage system." *Journal of Applied Mechanics and Technical Physics* 54, no. 5 (2013): 702-712.
[23] Sharma, R. K., P. Ganesan, J. N. Sahu, H. S. C. Metselaar, and T. M. I. Mahlia. "Numerical study for enhancement of solidification of phase change materials using trapezoidal cavity." *Powder technology* 268 (2014): 38-47.
[24] El Hasadi, Yousef MF, and J. M. Khodadadi. "Numerical simulation of the effect of the size of suspensions on the solidification process of nanoparticle-enhanced phase change materials." *Journal of Heat Transfer* 135, no. 5 (2013): 052901.
[25] Kashani, S., A. A. Ranjbar, M. Abdollahzadeh, and S. Sebti. "Solidification of nano-enhanced phase change material (NEPCM) in a wavy cavity." *Heat and Mass Transfer* 48, no. 7 (2012): 1155-1166.
[26] Vanaki, Sh M., P. Ganesan, and H. A. Mohammed. "Numerical study of convective heat transfer of nanofluids: a review." *Renewable and Sustainable Energy Reviews* 54 (2016): 1212-1239.
[27] Brinkman, H. C. "The viscosity of concentrated suspensions and solutions." *The Journal of Chemical Physics* 20, no. 4 (1952): 571-571.
[28] Maxwell, James Clerk. *A treatise on electricity and magnetism*. Vol. 1. Clarendon press, 1881.
[29] Fan, Li-Wu, Liang Zhang, Zi-Tao Yu, Xu Xu, Ya-Cai Hu, and Ke-Fa Cen. "A numerical investigation of constrained melting of nanostructure-enhanced phase change materials in a rectangular cavity heated from below." In *ASME 2012 Heat Transfer Summer Conference collocated with the ASME 2012 Fluids Engineering Division Summer Meeting and the ASME 2012 10th International Conference on Nanochannels, Microchannels, and Minichannels*, pp. 283-290. American Society of Mechanical Engineers Digital Collection, 2012.
[30] Sushobhan, B. R., and S. P. Kar. "Thermal modeling of melting of nano based phase change material for improvement of thermal energy storage." *Energy Procedia* 109 (2017): 385-392.
[31] Hosseini, Seiyed Mohammad Javad, Ali Akbar Ranjbar, Kourosh Sedighi, and Masoumeh Rahimi. "Melting of nanoprticle-enhanced phase change material inside shell and tube heat exchanger." *Journal of Engineering* 2013 (2013).
[32] Sebti, Seyed Sahand, Mohammad Mastiani, Hooshyar Mirzaei, Abdolrahman Dadvand, Sina Kashani, and Seyed Amir Hosseini. "Numerical study of the melting of nano-enhanced phase change material in a square cavity." *Journal of Zhejiang University SCIENCE A* 14, no. 5 (2013): 307-316.
[33] Parsazadeh, Mohammad, and Xili Duan. "Numerical and statistical study on melting of nanoparticle enhanced phase change material in a shell-and-tube thermal energy storage system." *Applied Thermal Engineering* 111 (2017): 950-960.
[34] Mostafavinia, Nasser, Saman Eghvay, and Amir Hassanzadeh. "Numerical analysis of melting of nano-enhanced phase change material (NePCM) in a cavity with different positions of two heat source-sink pairs." *Indian Journal of Science and Technology* 8, no. S9 (2015): 49-61.
[35] Ebrahimi, Aziz, and Abdolrahman Dadvand. "Simulation of melting of a nano-enhanced phase change material (NePCM) in a square cavity with two heat source–sink pairs." *Alexandria engineering journal* 54, no. 4 (2015): 1003-1017.
[36] Arıcı, Müslüm, Ensar Tütüncü, and Antonio Campo. "Numerical investigation of melting of paraffin wax dispersed with CuO nanoparticles inside a square enclosure." *Heat Transfer Research* 49, no. 9 (2018): 847-863.
[37] Elbahjaoui, Radouane, and Hamid El Qarnia. "Transient behavior analysis of the melting of nanoparticle-enhanced phase change material inside a rectangular latent heat storage unit." *Applied Thermal Engineering* 112 (2017): 720-738.
[38] Pahamli, Y., M. J. Hosseini, A. A. Ranjbar, and R. Bahrampoury. "Effect of nanoparticle dispersion and inclination angle on melting of PCM in a shell and tube heat exchanger." *Journal of the Taiwan Institute of Chemical Engineers* 81 (2017): 316-334.
[39] Bechiri, M., K. Mansouri, N. Hamlet, and S. Amirat. "Numerical solution of NEPCM melting inside spherical enclosure." In *2015 3rd International Conference on Control, Engineering & Information Technology (CEIT)*, pp. 1-4. IEEE, 2015.
[40] Feng, Yongchang, Huixiong Li, Liangxing Li, Lin Bu, and Tai Wang. "Numerical investigation on the melting of nanoparticle-enhanced phase change materials (NEPCM) in a bottom-heated rectangular cavity using lattice Boltzmann method." *International Journal of Heat and Mass Transfer* 81 (2015): 415-425.
[41] Darzi, AA Rabienataj, M. Farhadi, and M. Jourabian. "Lattice Boltzmann simulation of heat transfer enhancement during melting by using nanoparticles." *Iranian Journal of Science and Technology. Transactions of Mechanical Engineering* 37, no. M1 (2013): 23.
[42] Jourabian, Mahmoud, Mousa Farhadi, Korush Sedighi, Ahmad Ali Rabienataj Darzi, and Yousef Vazifeshenas. "Melting of NEPCM within a cylindrical tube: numerical study using the lattice Boltzmann method." *Numerical Heat Transfer, Part A: Applications* 61, no. 12 (2012): 929-948.

[43] Jourabian, Mahmoud, and Mousa Farhadi. "Melting of nanoparticles-enhanced phase change material (NEPCM) in vertical semicircle enclosure: numerical study." *Journal of Mechanical Science and Technology* 29, no. 9 (2015): 3819-3830.
[44] Vajjha, Ravikanth S., Debendra K. Das, and Praveen K. Namburu. "Numerical study of fluid dynamic and heat transfer performance of Al2O3 and CuO nanofluids in the flat tubes of a radiator." *International Journal of Heat and fluid flow* 31, no. 4 (2010): 613-621.
[45] Corcione, Massimo. "Heat transfer features of buoyancy-driven nanofluids inside rectangular enclosures differentially heated at the sidewalls." *International Journal of Thermal Sciences* 49, no. 9 (2010): 1536-1546.
[46] Gharagozloo, Patricia E., John K. Eaton, and Kenneth E. Goodson. "Diffusion, aggregation, and the thermal conductivity of nanofluids." *Applied Physics Letters* 93, no. 10 (2008): 103110.
[47] Patel, Hrishikesh E., K. B. Anoop, Thirumalachari Sundararajan, and Sarit Kumar Das. "A micro-convection model for thermal conductivity of nanofluids." In *International Heat Transfer Conference 13*. Begel House Inc., 2006.
[48] Khanafer, Khalil, Kambiz Vafai, and Marilyn Lightstone. "Buoyancy-driven heat transfer enhancement in a two-dimensional enclosure utilizing nanofluids." *International journal of heat and mass transfer* 46, no. 19 (2003): 3639-3653.
[49] Gau, Chie, and R. Viskanta. "Melting and solidification of a pure metal on a vertical wall." (1986): 174-181.
[50] Kuehn, T. H., and R. J. Goldstein. "An experimental and theoretical study of natural convection in the annulus between horizontal concentric cylinders." *Journal of Fluid mechanics* 74, no. 4 (1976): 695-719.
[51] Duan, Q., F. L. Tan, and K. C. Leong. "A numerical study of solidification of n-hexadecane based on the enthalpy formulation." *Journal of materials processing technology* 120, no. 1-3 (2002): 249-258.
[52] Duggirala, R. K., C. X. Lin, and C. Ghenai. "Investigation of double-diffusive convection during the solidification of a binary mixture (NH 4 Cl–H 2 O) in a trapezoidal cavity." *Experiments in fluids* 40, no. 6 (2006): 918.
[53] Hannoun, N., V. Alexiades, and T. Z. Mai. "A reference solution for phase change with convection." *International Journal for Numerical Methods in Fluids* 48, no. 11 (2005): 1283-1308.
[54] Hosseinizadeh, S. F., AA Rabienataj Darzi, and F. L. Tan. "Numerical investigations of unconstrained melting of nano-enhanced phase change material (NEPCM) inside a spherical container." *International Journal of Thermal Sciences* 51 (2012): 77-83.
[55] Agyenim, Francis, Philip Eames, and Mervyn Smyth. "A comparison of heat transfer enhancement in a medium temperature thermal energy storage heat exchanger using fins." *Solar Energy* 83, no. 9 (2009): 1509-1520.
[56] Tan, F. L. "Constrained and unconstrained melting inside a sphere." *International Communications in Heat and Mass Transfer* 35, no. 4 (2008): 466-475.
[57] Sciacovelli, Adriano, Francesco Colella, and Vittorio Verda. "Melting of PCM in a thermal energy storage unit: Numerical investigation and effect of nanoparticle enhancement." *International Journal of Energy Research* 37, no. 13 (2013): 1610-1623.
[58] Aydın, Orhan, Mithat Akgün, and Kamil Kaygusuz. "An experimental optimization study on a tube-in-shell latent heat storage." *International journal of energy research* 31, no. 3 (2007): 274-287.
[59] Lacroix, M. "Predictions of natural-convection-dominated phase-change problems by the vorticity-velocity formulation of the Navier-Stokes equations." *Numerical Heat Transfer, Part B Fundamentals* 22, no. 1 (1992): 79-93.
[60] Gong, Zhen-Xiang, and Arun S. Mujumdar. "Flow and heat transfer in convection-dominated melting in a rectangular cavity heated from below." *International Journal of Heat and Mass Transfer* 41, no. 17 (1998): 2573-2580.
[61] de Vahl Davis, G. "Natural convection of air in a square cavity: a bench mark numerical solution." *International Journal for numerical methods in fluids* 3, no. 3 (1983): 249-264.
[62] Huber, Christian, Andrea Parmigiani, Bastien Chopard, Michael Manga, and Olivier Bachmann. "Lattice Boltzmann model for melting with natural convection." *International Journal of Heat and Fluid Flow* 29, no. 5 (2008): 1469-1480.
[63] Arasu, A. Valan, and Arun S. Mujumdar. "Numerical study on melting of paraffin wax with Al2O3 in a square enclosure." *International Communications in Heat and Mass Transfer* 39, no. 1 (2012): 8-16.
[64] Hosseini, M. J., A. A. Ranjbar, K. Sedighi, and M. Rahimi. "A combined experimental and computational study on the melting behavior of a medium temperature phase change storage material inside shell and tube heat exchanger." *International Communications in Heat and Mass Transfer* 39, no. 9 (2012): 1416-1424.
[65] Mahato, Ankit, Devendra Kumar, and Arvind Kumar. "Modelling of melting/solidification behaviour of nanoparticle-enhanced phase change materials." In *Proceedings of the 22nd National and 11th International ISHMT-ASME Heat and Mass Transfer Conference. Kharagpur: ASME*. 2013.
[66] Patankar, Suhas V., W. J. Minkowycz, and E. M. Sparrow. "Series in computational methods in mechanics and thermal sciences." *McGraw-Hill Book Company, New York* (1980): 1-197.