



## Distribution of Normal Stress under the Effect of Temperature for a Functionally Graduated Beam

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### ABSTRACT

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In this paper, the properties of the material are proposed according to the temperature and vary continuously in the direction of the thickness according to a law of power, the distribution of the stress along the thickness for functionally graduated beam was studied, the choice of materials is taken according to the demand of the industrial sector, the expression of the stresses has been studied analytically, the effects of the material distributions are presented, The results show that the previously mentioned effects play a very important role in the dynamic behavior of the FG beams.

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## 1. Introduction

A new class of composite materials, which is known as functionally graded material (FGM), has been drawn considerable attention. Functionally graded materials (FGMs) characterize a class of materials where the microstructures are spatially graded to achieve specific thermal and/or mechanical properties to suit the functionality of the structure [1]. Material properties are varied continuously in the thickness direction according to a simple power law distribution. A three-dimensional solid element is used for more accurate modeling of material properties and temperature field in the thickness direction. The Green–Lagrange nonlinear strain-displacement relation is used to account for large deflection due to uniform pressure and thermal loads and the incremental formulation is applied for nonlinear analysis [2].

Structural elements subjected to high temperature, severe temperature gradient, and uneven heating rates are unavoidable during the operation of gas turbines, nuclear reactors, castings, forgings, radiant burners, pipes in heat exchangers, artillery barrels, etc. Sharp temperature gradients in the structural elements arise due to sudden exposure to very large amount of heat

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which is observed during launching of rocket, space craft structural components subjected to radiant solar heat [3] and [4].

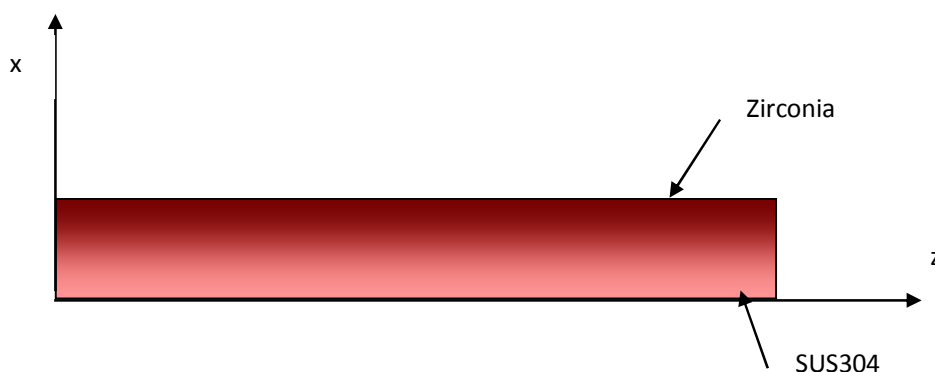
Functionally graded material (FGM) is a new kind of inhomogeneous composite. It possesses continuously varying microstructure and mechanical properties. The main advantage of FGM is that no internal boundaries exist and the interfacial stress concentrations can be avoided. Furthermore, functionally graded materials (FGMs) can be designed to achieve particular desired properties and the gradation in properties of the material can optimize stress distribution. Nowadays, there have been increasingly many modern engineering applications of FGMs, such as space shuttle, rocking-motor casings, and packaging materials in microelectronic industry [5].

Composite materials are those formed by combining two or more materials on a macroscopic scale such that they have better engineering properties than the conventional materials, for example, metals. Most man-made composite materials are made from two materials: a reinforcement material called fiber and a base material, called matrix material. The stiffness and strength of fibrous composites come from fibers which are stiffer and stronger than the same material in bulk form. The matrix material keeps the fibers together, acts as a load-transfer medium between fibers, and protects fibers from being exposed to the environment [6]. To overcome the limitations stated above, a lot of research has been conducted to increase the thermal carrying capacity of convectional heat transfer fluid and one of the ideal ways is to disperse nano-size particles in to the base fluid [7].

The natural convection heat transfer is one of the classical heat transfer problems that can narrate the development of modern understanding of heat transfer. Literature records include research on such problem since 1969 [8] until today [9-11].

Most of the recent studies showed that, the enhancement of heat transfer coefficient can be increased by adding solid metallic or nonmetallic nanoparticles with a high thermal conductivity of the base fluid [12-14]. Nanofluids effects have investigated by many researchers in the enhancement of heat transfer and the fluid flow [15-19].

The displacement field for this study, of which  $u$  and  $w$  are the two displacement components of a point located in the neutral axis,  $\phi$  is the rotation of the normal around the neutral axis, expressed as follows:



**Fig. 1.** FGM Beam

$$u_x(x, z, t) = u_0(x, t) + z\phi(x, t) - \alpha z^3(\phi(x, t) + w_{0,x}(x, t)) \quad (1)$$

Reproduction of this equation in matrix form

$$u_x = [1 \quad 0 \quad -\alpha z^3 \quad z - \alpha z^3] \begin{Bmatrix} u_0 \\ w_0 \\ w_{0,x} \\ \phi \end{Bmatrix} \quad (2)$$

The deformation-displacement relation is given by the following expression

$$\varepsilon_x = \partial u_x / \partial x \quad (3)$$

Substituting Equation (1) in (3) we obtain

$$\varepsilon_x = \partial u_x / \partial x = \partial u_0(x, t) / \partial x + z \partial \phi(x, t) / \partial x - \alpha z^3 (\partial \phi(x, t) / \partial x + \partial^2 w_0(x, t) / \partial x^2) \quad (4)$$

On the other hand the expression of the deformation in matrix form is given by

$$\varepsilon_x = [1 \quad z - \alpha z^3 \quad -\alpha z^3 \quad 0] \begin{Bmatrix} \partial u_0(x, t) / \partial x \\ \partial \phi(x, t) / \partial x \\ \partial^2 w_0(x, t) / \partial x^2 \\ \phi(x, t) + w_{0,x}(x, t) \end{Bmatrix} \quad (5)$$

The relation stress strain

$$\sigma_x = E \varepsilon_x - \alpha \Delta T \quad (6)$$

For graded materials evaluated deformations expressions and Young modules is follows a power law

$$E(z) = \begin{cases} E_1 & \text{pour } 0 < z < h_1 \\ E_1 \left( \frac{h_1 + h_2 - z}{h_2} \right) + E_2 \left( \frac{z - h_1}{h_2} \right) & \text{pour } h_1 < z < h_1 + h_2 \end{cases} \quad (7)$$

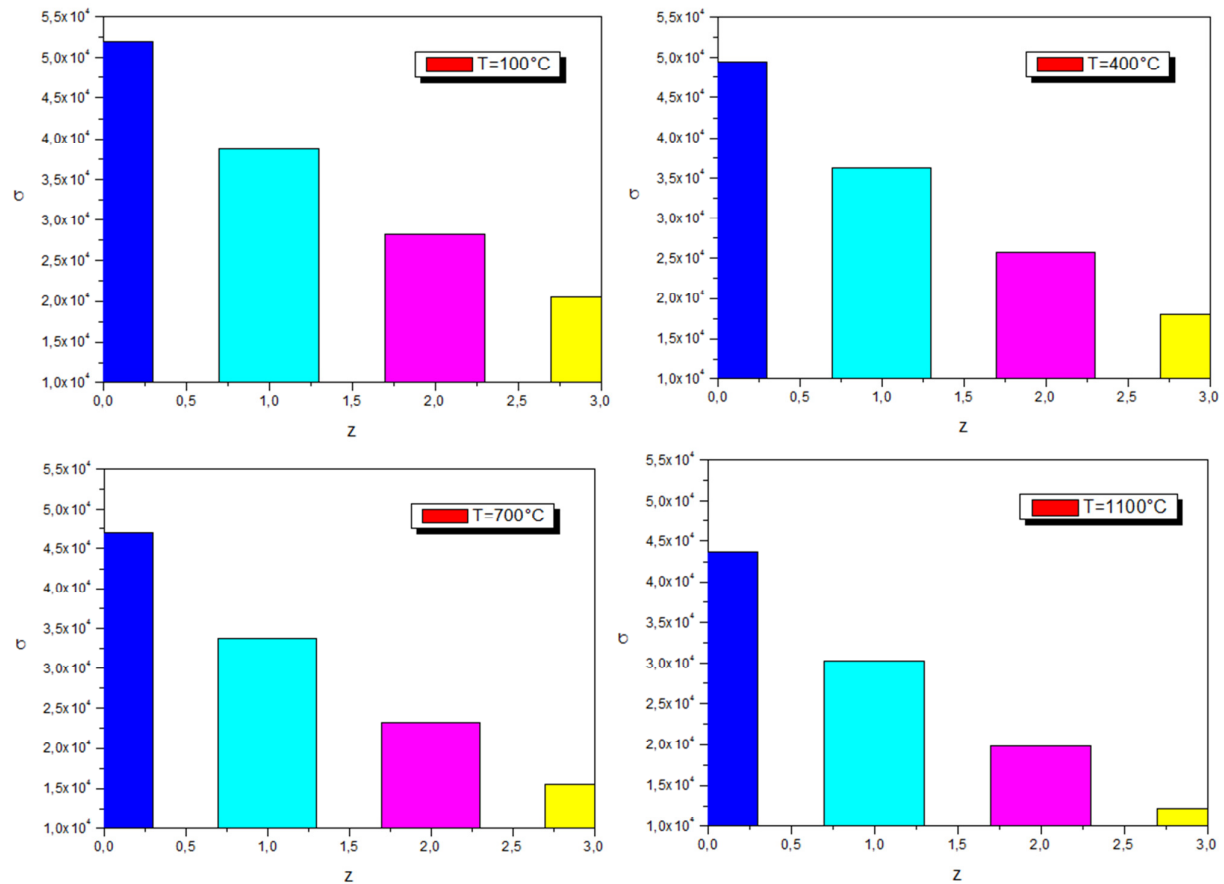
$$\varepsilon(z) = \begin{cases} \varepsilon_1 & \text{pour } 0 < z < h_1 \\ \varepsilon_1 \left( \frac{h_1 + h_2 - z}{h_2} \right) + \varepsilon_2 \left( \frac{z - h_1}{h_2} \right) & \text{pour } h_1 < z < h_1 + h_2 \end{cases} \quad (8)$$

$$\sigma_x = E_1 \varepsilon_1 + \left( E_1 \left( \frac{h_1 + h_2 - z}{h_2} \right) + E_2 \left( \frac{z - h_1}{h_2} \right) \right) \left( \varepsilon_1 \left( \frac{h_1 + h_2 - z}{h_2} \right) + \varepsilon_2 \left( \frac{z - h_1}{h_2} \right) \right) - \alpha \Delta T \quad (9)$$

## 2. Results and Discussion

Consider a FGM beam, the parameter values used are

$$\varepsilon_1=0.098799, \varepsilon_2=0.074799, h_1=1m, h_2=1,5m, E_1=204,04.10^9 Pa \quad E_2=224,26.10^9 Pa, \alpha = 4 / 3h^2$$



**Fig. 2.** Distribution of normal stress following z as a function of temperature

Figure 2 shows the influence of the temperature variation on the stress distribution for a gradually evaluated beam, the concentration is very clear in the lower part of the beam, the stress decreases along the thickness of the beam, the intensity of the stress decreases with the increase of the temperature.

## 3. Conclusion

In this study, the effect of the temperature increase, is shown in the normal stress distribution for a gradually graded beam, for this type of materials, the maximum value occurs only on the upper part of the beam, the properties of the materials change through the thickness of the beam according to the power law that gives a gap in the stress results, we find that the choice of material is necessary in the construction of structures, the change in temperature makes material to expand and if this expansion is restrained, stresses are induced which affect expected performance of structure.

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