Numerical Simulation of the Dynamics of a Droplet in a Low-gravitational Field

S. Sharafatmandjoor[*,1], *C. S. Nor Azwadi*[2]

[1]Department of Mechanical and Aerospace Engineering, Science and Research Branch, Islamic Azad University, Tehran, Iran.
[2]Faculty of Mechanical Engineering, Universiti Teknologi Malaysia, 81310 Johor, Malaysia.
[*]*sharafatmand@srbiau.ac.ir*

Abstract – *The dynamics of a liquid droplet in a low gravitational field is examined using a finite difference/front tracking projection-based numerical technique. The unsteady, incompressible, viscous, immiscible multi-fluid, two-dimensional Navier-Stokes equations for both the liquid and the gas are solved using the single-fluid model. The droplet deformation and the gas wake generation and oscillations phenomena is studied.* **Copyright © 2016 Penerbit Akademia Baru - All rights reserved.**

Keywords: Droplet Dynamics, Two-phase Flows, Direct Numerical Simulation (DNS), Single-fluid Model, Front Tracking Scheme, Projection Method

1.0 INTRODUCTION

In this study, we revisit the nearly zero-gravity fields which are decisive since they deal with problems that are of very significant concern in many modern technological fields [1]. However in small volumes of flow gravitational effects play negligible roles in dynamics of the system [2-4] in the absence of gravity or its equivalent acceleration, in a two-phase system, often a small wall-bound volume of one phase impedes the other one. This phenomenon is usually a source of obstruction of the flow [5-8].

Many investigators have been trying to develop numerical methods to simulate moving of one fluid in another in free fall. Most of them consider the 2D axisymmetric model for the droplet, due to plausible impulsive accelerations. For example, we can refer to the work of [9] who investigated droplets at several Reynolds and Weber numbers using a finite difference method. Also, [10-12] studied the deformation and breakup of two-dimensional droplets. Only few works *eg.* [1,13,14] deal with very small values of gravity as the exclusive governor of the droplet.

In this work, five two-dimensional test cases using modest variations of the parameters and effects presumed to be important for the flow modulations and droplet dispersion and dynamics are studied. A one-fluid model has been employed in the current study to approach the physics of the problem.

2.0 FORMULATION AND NUMERICAL METHODS

The unsteady, incompressible, viscous, immiscible multi-fluid, two-dimensional Navier-Stokes equations in conservative form are written as:

$$\frac{D(\rho \vec{V})}{Dt} = -\tilde{\nabla}P + \rho g + \tilde{\nabla} \cdot (2\mu T) + s\,k\,n\,\delta(X - X^f) \tag{1}$$

where T is the rate of transformation tensor, with components $T_{ij} = (v_{i,j} + v_{j,i})/2$, V is the velocity vector, s is surface tension coefficient, k is curvature, g is gravity acceleration, and n is a normal to the droplet surface. Here, surface tension forces have been added as a delta function to provide the proper interface boundary conditions. These equations are supplemented by the incompressibility condition:

$$(\tilde{\nabla} \cdot V) = 0. \tag{2}$$

We also have equations of state for the discontinuous density and viscosity fields:

$$\frac{\partial(\rho)}{\partial t} + V \cdot \tilde{\nabla}\rho = 0$$
$$\frac{\partial(\mu)}{\partial t} + V \cdot \tilde{\nabla}\mu = 0. \tag{3}$$

The governing equations are solved by a relatively standard finite difference projection method (Peyret and Taylor 1986) on a staggered Cartesian grid. The pressure equation, which is non-separable due to the variable density, is solved by a Black and Red SOR iterative technique. In this method, rewriting equation (1) reads:

$$\frac{\partial(\rho \vec{V})}{\partial t} = \tilde{\nabla}P + E(\vec{V}) \tag{4}$$

$$E(V) = -(\tilde{\nabla} \cdot \rho V)V + \rho g + \tilde{\nabla} \cdot (2\mu T) + s\,k\,n\,\delta(X - X^f) \tag{5}$$

Now, if we discretize equation (4) with respect to time, the following equation is obtained:

$$\frac{\rho^{n+1}V^{n+1} - \rho^n V^n)}{Dt} = -\tilde{\nabla}P + E(V) \tag{6}$$

In the adopted projection method, equation (6) is split into the following two equations:

$$\frac{\rho^{n+1}V^* - \rho^n V^n)}{Dt} = E(V) \tag{7}$$

$$\frac{\rho^{n+1}V^{n+1} - \rho^* V^*)}{Dt} = -\tilde{\nabla}P \tag{8}$$

Where V^* is called provisional velocity. The incompressibility condition, equation (2) can also be written as:

$$(\tilde{N}.\overset{\text{\tiny ur}\,n+1}{V}) = 0 \tag{9}$$

Here, one can get the Poisson equation for pressure from equations (6) and (9), which leads to:

$$\tilde{N}.\frac{1}{r^{n+1}}\tilde{N}P^{n+1} = \frac{1}{Dt}\tilde{N}.\overset{\text{\tiny ur}\,*}{V}. \tag{10}$$

We first find $\overset{\text{\tiny ur}\,*}{V}$ from equation (7) (r^{n+1} is known from front tracking approach), then equation (10) is solved for pressure and finally $\overset{\text{\tiny ur}\,n+1}{v}$ is found from equation (8).

To advect the discontinuous density and viscosity field and to compute surface quantities, we use a technique introduced by [15], which is called Immersed Boundary Method. The surface tension is represented by separate computational elements referred to as front. The front grid is one-dimensional and is advected by the fluid velocity, which is interpolated from the fluid grid. There is no numerical diffusion of this front since this thickness remains constant for all times. The technique, which is presented in more detail by [16] uses an indicator function I(x), which is zero for the ambient fluid phase and one for the drop phase. I(x) is constructed using the known position of the front points. Then, for density and viscosity fields we have:

$$r(x) = r_D + (r_D - r_o)I(x) \tag{11}$$

$$m(x) = m_D + (m_D - m_o)I(x) \tag{12}$$

3.0 TEST CASES

The resulting droplets dynamics was investigated by varying parameters for the five cases shown in Table 1.

Table 1: Test Cases.

Cases	gx(m/s2)	gy(m/s2)	σ(N/m)
A1	0.03	0	0.5
A2	0.015	0	0.05
A3	0.03	0	0.05
A4	0.06	0	0.05
A5	0.06	0.03	0.05

In all cases, downward gravity is selected to be very small in order to allow drag force to be significant. Density ratios and viscosity ratios are fixed to be 40 and 3.22 respectively according to computational experiments performed in [17]. To ensure the grid resolution, the location of droplets is predicted by three computational mesh sizes in three different times for case A4 (Fig.1). As the time increases, the results obtained by the 90×150 grid differ from the other two mesh sizes, which are very close to each other. Therefore a 120×200 grid is selected.

A droplet with $D = 0.2$ is initially placed in $X = 0.5$ and $Y = 0.8$ in a 1×2 domain of quiescent gas (Fig. 2a). All the boundary conditions are periodic to avoid wall effects and to enable the code to simulate a set of equal distance droplets as well.

Case A1 is set to represent moderate gravity and low surface tension (most likely a solid cylinder in a cross flow). Less gravity and more surface tension are studied in case A2. Cases A3 and A4 reveal the effects of increasing gravity and case A5 represents the influence of adding gravity in the vertical direction.

4.0 RESULTS

To start dealing with the kinematics simulations, deformation maps are presented in Figs.2. In the maps, we mark the interface location of droplets in each case using different symbols. In Fig. 2a, the droplets are overlapped initially in their cylindrical shape. Fig.2b shows the droplets at $t = 1$. The droplet of case A4 is placed under the other ones and experiences the most deformation, due to its higher gravity force. This phenomenon is qualitatively the same as results of [16]. The droplet in case A2 lies behind and despite of differences in surface tension, the ones of cases A1 and A3 match each other, due to their relatively equal gravity forces. Fig.2c illustrates the droplet at $t = 1.5$. All the effects described in Fig. 2b are more amplified within the increased time. Here the droplet in case A3 gets more deformed than the one in Case A1, because it has less surface tension.

Figures 3-6 show the relative pressure shadowgraphs of inside and outside of the droplets for different times and cases. Because of the front tracking method used, these figures demonstrate modest variations of parameters (e.g. pressure) from inside to outside of the droplets.

The droplet in case A1 is shown in Figs. 3a and 3b. One can observe the beginning of wake generation in the rear side of the droplet in Fig.3a at $t = 1.8$. The spreading of the wake is shown in Fig. 3b for $t = 3.2$.

Fig. 4 represents relative pressure field of the droplet in case A2. Decreasing the surface tension forces the droplet to move slowly and the wake generation is not observed even after a long time ($t = 3$). Internal pressure difference and circulation is also illustrated in this figure. Obviously, the pressure of the button of the droplet is more than the other parts of the droplet.

Figs.5a to 5d show the different sequences of wake generation and shedding for case A3. In Fig.5a, at $t = 1.4$, the wake is starting to be generated more strongly than the previous cases, due to its larger gravity acceleration. The internal pressure difference is also more evident. Fig.5b shows the path in which the wakes are about to shed symmetrically behind the droplet at $t = 2.8$. In Fig.5c, at $t = 3.2$, wakes are shed and the separating of two vertical regions is almost performed. Fig.5d shows the weakening of two wakes behind the droplet at $t = 4.3$. There is no significant observation of regenerated wakes, due to interacting of neighboring droplet's pressure fields.

Figs.6a to 6e, represent the pressure field for the droplet in case A4. In Fig. 6a at $t = 1.4$, a strong desire to wake generation is evident rear the droplet with a flatter shedding angle relative to those of case A3 (Fig. 5a). The screw wake shedding pattern is illustrated in Fig.6b at $t = 2$ and their weakening and spreading process is shown in Figs.6c and 6d at 3s. And $t = 3.7$ respectively. In Fig.6e, regenerating of wakes is shown at $t = 4.1$. The strength of these wakes is not the same as the first generated ones because of the complexity of the whole domain caused by periodic boundary conditions.

The effect of adding a gravity acceleration (case A5) in a vertical direction is shown in Figs 7a and 7b at $t = 1$ and $t = 2$ respectively. The wake generation process is almost identical to previous cases except the asymmetry between the two wakes and the asymmetric pressure field in the domain. Also, Fig 7a shows the complex flow field, effect of surrounding droplet and asymmetry of the pressure field.

Two important characteristics of a droplet are its normalized acceleration and vorticity. The evolution of such quantities (and those of the liquid rear the droplet) are plotted in figures 8 and 9 respectively. As one can observe in Fig 8, the acceleration oscillation of case A4 is less than for case A5, due to symmetry of the flow pattern. Note from Fig8 that the oscillation frequencies correspond closely to the drops response time in the early times. The zigzag pattern in acceleration showed in this figure is in agreement with [5]. Fig. 9 shows the evolution of the average vorticity (which influences lift) around the droplets for cases A2, A4 and A5. As expected, the vorticity level of case A2 is much less than those of the other cases. At later times the drop of cases A4 and A5 disperse differently and over time are subject to different vorticity fields due primarily to the relative position to their corresponding wakes.

5.0 CONCLUSIONS

The motion of a droplet in a low gravitational field was investigated within five test cases. The effect of surface tension and gravity on the kinematics of a droplet was also studied. One can observe that the droplet's deformation phenomenon has a definite effect on the wake dynamics and the surrounding gas. On the other hand, the co-effect of different droplets strongly affects their acceleration, which is observable in the long time behavior of the system.

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