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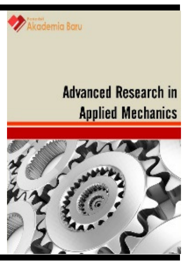

An RSM approach to maximize fundamental frequency of e-glass epoxy composite panels

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ABSTRACT

In the current paper, the response surface methodology has been applied to maximize the fundamental frequency of composite panels. In total 4 different input parameters namely aspect ratio (b/a), thickness ratio (h/a), number of plies (n) and ply angle (θ) are considered. The effect of these parameters on natural frequency is studied in detail. The dataset used for building the RSM model is obtained from FEA simulations. The FEA, in turn, are first compared with experimental data to validate the accuracy of the numerical schemes. It is found that for maximizing the fundamental frequency, thickness ratio should be high. Low aspect ratio and fewer number of plies are desirable.

1. Introduction

The vibration plates is a special case of the problem of more general mechanical vibrations. The equations that govern the motion of the layers are simpler than for general three-dimensional objects, for one of the dimensions of the plate is smaller than the other two. This suggests that a two-dimensional plate theory will give an approximation of the actual exercises in three plate-like objects, and indeed that is found to be true. Several theories have been developed to describe the motion of plates. The most commonly used is the Kirchhoff-Love theory and the theory of Mindlin-Reissner.

Due to their immense practical applications, a significant number of researchers have focused on understanding the dynamic behaviour of plates. Haldar and Sheikh [1] utilized a high precision composite plate bending element was applied by to the free vibration analysis of isotropic and fiber-reinforced laminated composite folded plates. Kalita and Haldar [2] used a nine-node isoparametric plate element in conjunction with first-order shear deformation theory is used for free vibration analysis of rectangular plates with central cutouts. Kalita *et al*. [3-6] also used the nine-node isoparametric plate element, in conjunction with first-order shear deformation theory, to free vibration analysis of rectangular plates. They found that rotary inertia significantly affects thick plates, while it can be ignored for thin plates. The numerical convergence was shown to be rapid and

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based on a comparison with data from the literature; it was proposed that their formulation can yield highly accurate results. Thai and Kim [7] used two variable refined plate theory to calculate the natural frequencies of laminated composite plates. Thai and Choi [8] conducted vibration analyses of thick rectangular plates with various boundary conditions using two variable refined plate theory. Xiang *et al*. [9] presented a meshless method based on thin plate spline radial basis functions and higher-order shear deformation theory (HSDT) to analyze the free vibration of clamped laminated composite plates. Adim et al. [10] recently demonstrated the utility of a simplified HSDT. Asadi et al. [11] demonstrated the use of FSDT for composite shells. In their work, first order polynomials for in-plane displacements in the z-direction were utilized allowing for the inclusion of shear deformation and rotary inertia effects. In a similar work, they [12] used generalized differential quadrature to solve free vibration problems of isotropic, cross-ply, angle-ply and general lay-up cylindrical shells. Higher-order shear deformation theory has been employed by Asadi and co-workers to calculate natural frequencies of composite plates [13] and shells [14]. Shi *et al*. [15] used the Galerkin method (taking the transverse shear effects) to perform free vibration analysis of an arbitrarily laminated plate with all four edges clamped.

In certain cases, it is desired to have composite laminates with maximum fundamental frequency. Some optimization methods like Genetic Algorithm (GA), Particle Swarm Optimization (PSO), Ant Colony Optimization (ACO), etc. have been used so far to maximize fundamental frequency. Composite plates are in high demand due to their high specific stiffness/strength especially for structural and aerospace-based applications wherein weight reduction is critical. Further reduction in weight of these composite plates is possible by optimizing the fiber orientations; ply thickness, stacking sequence, etc. A stochastic search algorithm called ‘‘direct search simulated annealing” (DSA), proposed by Ali *et al*. [16] was successfully used by Akbulut and Sonmez [17] to minimize the weight of a composite laminate by imposing a strength constraint in order to ensure that the resulting optimally designed laminate will carry the applied loads without failure. Later Akbulut and Sonmez [18] conducted a similar analysis for out-of-plane loading. Rettenwander *et al*. [19] presented their work on fiber-orientation optimization based on principal stress directions.

2. Methodology

In the present work, a finite element formulation has been developed using eight node shell element to study the free vibration characteristics of rectangular E-Glass Epoxy composite plates. The finite element program is developed in ANSYS APDL environment. An experimental study is carried out to validate the program. After this, a parametric study is designed to explore the effect of various geometric and material properties on the fundamental frequency. A response surface method is used for the design of experiments. A flowchart describing the basic workflow in presented in Fig. 1. The material properties considered for the composite laminate is shown in Table 1. There are different testing points and distance between each point. The transducer (accelerometer) is placed at one point, and the impulse force (hammer/force transducer) is applied on some points. The transducers (accelerometer and force transducer) are connected to the module as input signals. The power supply will do some signal processing on the input and then send the output to the input channel of Data Acquisition Hardware. The Data Acquisition Hardware is connected to the computer with USB cable and does modal analysis with PULSE Reflex Modal Analysis Pack software. A composite laminate of required aspect ratio is prepared. The length of the fixture that holds the steel bar is measured, and a margin is left of that length on the laminate. The accelerometer is then fixed to the laminate at one node but on the face of the laminate opposite to the markings. The necessary connection between the wires and cables are made. The data acquisition hardware, PC, accelerometer and the impact hammer connections are made. Necessary inputs are provided in the PULSE Reflex Modal Analysis Pack software. On providing impact by the impact hammer on the nodes marked on the cantilever one by one, the accelerometer records the signals. Signals from the impact hammer and the accelerometer are received by the data acquisition hardware for each impact provided one by one and are analyzed by the software. Curve known as Frequency Response Function (FRF) is generated by the software that is used to find the natural frequencies of the laminate.

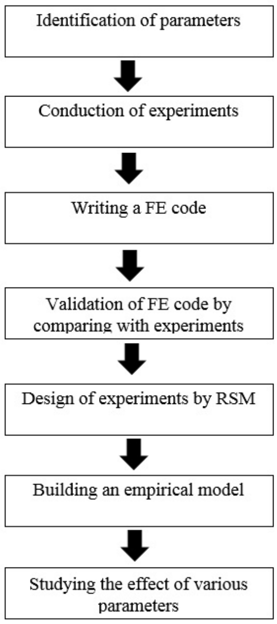

Fig. 1. Basic workflow of the study

Table 1
Material Properties of laminate considered for the study

E_x	E_y	E_z	G_{xy}	G_{xz}	G_{yz}	μ_{xy}	μ_{xz}	μ_{yz}
(in GPa)								
34	6.53	6.53	2.433	2.433	1.698	0.217	0.217	0.366

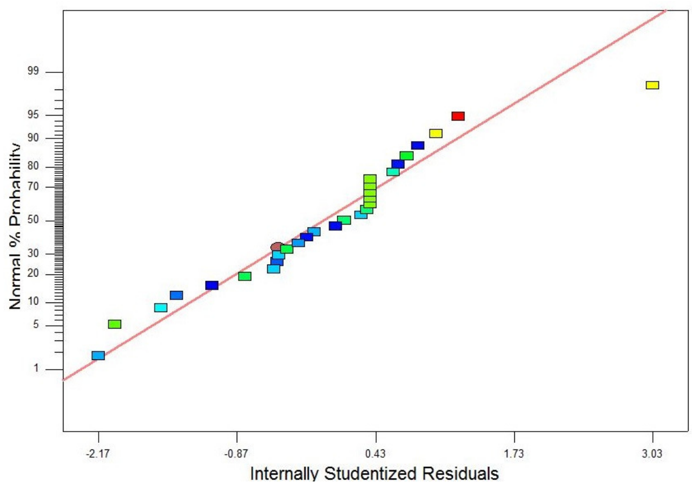

Fig. 2. Normal probability plot of residuals

Table 2
Parameters considered and recorded response

Run	Factor 1	Factor 2	Factor 3	Factor 4	Response 1
	b/a	h/a	No of plies	Ply angle	Nat. Frq. (1) Hz
1	1.5	0.055	4	90	66.528
2	1	0.055	4	45	300.57
3	1	0.055	4	45	300.57
4	0.5	0.1	4	45	465.2
5	1.5	0.055	6	45	191.19
6	0.5	0.055	6	45	127.94
7	1.5	0.01	4	45	16.999
8	1	0.055	4	45	300.57
9	1	0.1	6	45	226.52
10	1	0.055	6	0	110.49
11	1	0.055	2	0	110.5
12	1	0.055	4	45	300.57
13	1	0.01	6	45	24.92
14	1	0.01	4	0	20.551
15	0.5	0.01	4	45	56.718
16	0.5	0.055	4	0	205.78
17	1.5	0.055	4	0	97.076
18	1	0.055	2	90	350.41
19	1	0.1	2	45	209.66
20	1	0.1	4	90	94.311
21	1	0.055	4	45	300.57
22	0.5	0.055	4	90	350.41
23	1	0.01	2	45	22.756
24	1	0.1	4	0	191.19
25	1.5	0.055	2	45	87.122
26	0.5	0.055	2	45	283.69
27	1	0.055	6	90	110.49
28	1	0.01	4	90	20.551
29	1.5	0.1	4	45	158.12

Results from the ANSYS simulation are compared with the experiments to assure the validity of the finite element simulation. Based on literature survey and associated studies it is identified that aspect ratio (b/a), thickness ratio (h/a), the number of plies (n) and ply angle (θ) are the main parameters that affect the natural frequency. Response surface methodology is used to develop a design of experiments.

Table 3
Validation of FEA results

h/a	Source	1	2	3	Source	1	2	3
	b/a = 1				**b/a = 2**			
0.001	FEA	3.73	7.87	7.87	FEA	2.49	3.22	4.54
	Experimental	3.87	8.20	8.20	Experimental	2.52	3.26	4.72
	% variation	3.79	4.23	4.23	*% variation*	1.14	1.2	4.1
	Liew [20]	3.64	7.43	7.43	Liew [20]	2.49	3.22	4.53
0.1	FEA	3.29	6.29	6.29	FEA	2.31	2.95	4.07
	Experimental	3.45	6.41	6.41	Experimental	2.37	3.05	4.11
	% variation	*4.65*	*1.99*	*1.99*	*% variation*	*2.84*	*3.33*	*1.08*
	Liew [20]	3.29	7.43	7.43	Liew [20]	2.30	2.95	4.07
0.2	FEA	2.69	4.69	4.69	FEA	1.95	2.45	3.29
	Experimental	2.84	4.80	4.80	Experimental	1.97	2.49	3.36
	% variation	*5.81*	*2.39*	*2.39*	*% variation*	*1.07*	*1.64*	*2.23*
	Liew [20]]	2.68	4.96	4.96	Liew [20]	1.94	2.45	3.29

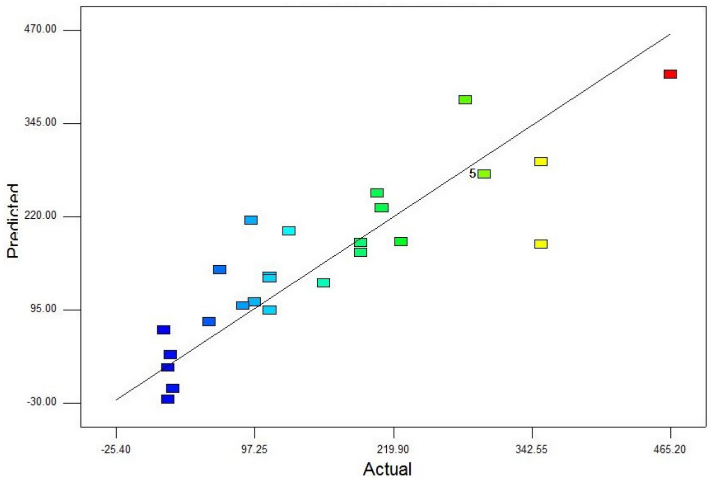

Fig.3. Predicted vs. actual responses

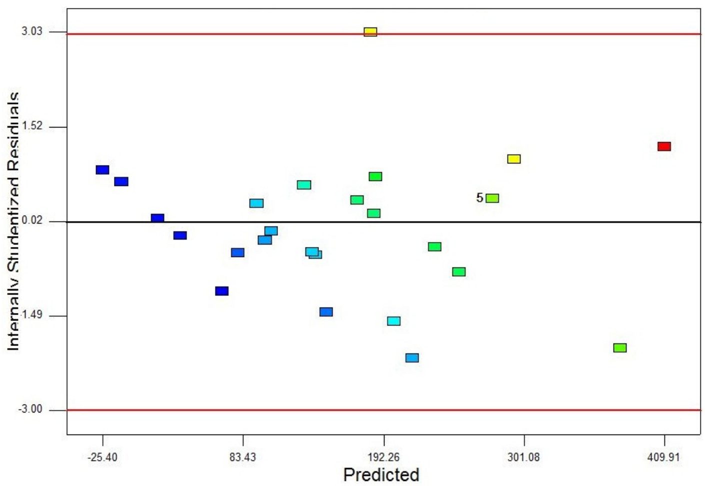

Fig.4. Residuals vs. Predicted responses

Table 4
Analysis of variance

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F
Model	334955.98	9	37217.33	8.21	< 0.0001
A-b/a	63467.54	1	63467.54	13.99	0.0014
B-h/a	116526.70	1	116526.70	25.69	< 0.0001
C-No of plies	6192.01	1	6192.01	1.365	0.2571
D-Ply angle	5508.92	1	5508.92	1.22	0.2842
AB	17870.47	1	17870.47	3.94	0.0618
AC	16876.34	1	16876.34	3.72	0.0688
B^2	73928.20	1	73928.20	16.30	0.0007
C^2	25283.95	1	25283.95	5.57	0.0291
D^2	40125.80	1	40125.80	8.85	0.0078
Residual	86172.88	19	4535.414		
Lack of Fit	86172.88	15	5744.858		
Pure Error	0	4	0		
Cor Total	421128.87	28		**R^2**	**0.8954**

Response Surface Methodology (RSM) is a statistical technique that explores the relationship between cause and effect parameters [21]. The RSM helps in design of experimentation and develop a quadratic empirical equation. For an experimentation design involving 4 parameters and 3 setting levels each, the total number of possible combinations would be $3^4 = 81$ trials. However, by using the RSM design this can be reduced to 29 trials. After determining the working range of these parameters, the upper limit of parameter was coded as 1; lower limit, as −1 and the intermediate parameter levels were taken as 0. This is detailed out in Table 3. Based on the RSM approach, a quadratic equation of the following nature is derived,

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \beta_{14} x_1 x_4 + \beta_{23} x_2 x_3 + \beta_{24} x_2 x_4 + \beta_{34} x_3 x_4 + \beta_{11} {x_1}^2 + \beta_{22} {x_2}^2 + \beta_{33} {x_3}^2 + \beta_{44} {x_4}^2 \quad (1)$$

The design of experiments is reported in Table 2. Analysis of variance or ANNOVA is then performed to understand the effect of this parameters better. Analysis of variance (ANOVA) is a collection of statistical models used to analyze the differences among group means and their associated procedures [22-24].

Table 5
Constraints and goals of optimization

Constraints	Goal	Lower limit	Upper limit
b/a	is in range	0.5	1.5
h/a	is in range	0.01	0.1
No of plies	is in range	2	6
Ply angle	is in range	0	90
Nat. Frq. (1)	maximize	15	500

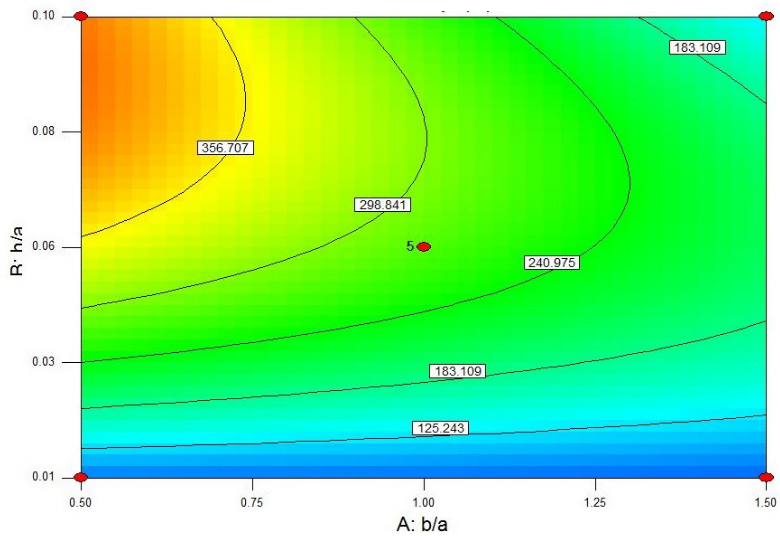

Fig. 5. Contour plot of fundamental frequency (thickness ratio vs. aspect ratio)

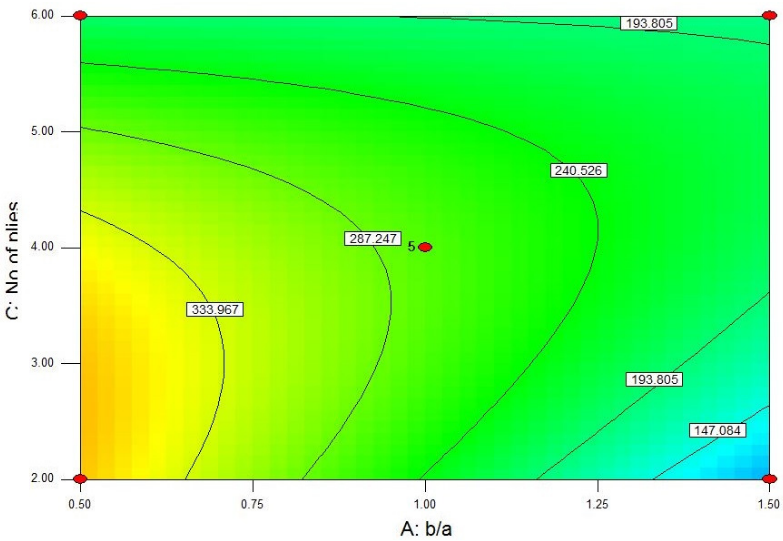

Fig. 6. Contour plot of fundamental frequency No. of plies vs. aspect ratio)

3. Results and Discussion

Since the present study is based on finite element simulation, the validation of the numerical schemes is extremely necessary. To do so, some numerical examples are compared with the experimental results. However, due to the lack of availability of composite laminates, isotropic plates are considered in the validation study. Thus, Table 3 reports several cases of numerical data comparison with the experimental data. The Poisson's ratio is taken as 0.3, thickness ratio (h/a) is taken as 0.001, 0.1 and 0.2, aspect ratio (a/b) is varied as 1 and 2. The first three non-dimensional parameters $\lambda = \left(\frac{\omega a^2}{\Pi^2}\right)\sqrt{\rho h/D}$ obtained by the present FEA simulation and are compared with the experimental results and available literature. All sides clamped boundary condition is considered. Liew [20] has used mathematically complete two-dimensional polynomials in the displacements and rotation functions to approximate the appropriate mode shapes with the Rayleigh-Ritz method. It is evident that the percentage error between the numerical and the experimental results is very less and hence the current FEA codes can safely used to predict the dynamic behavior of the composite plates.

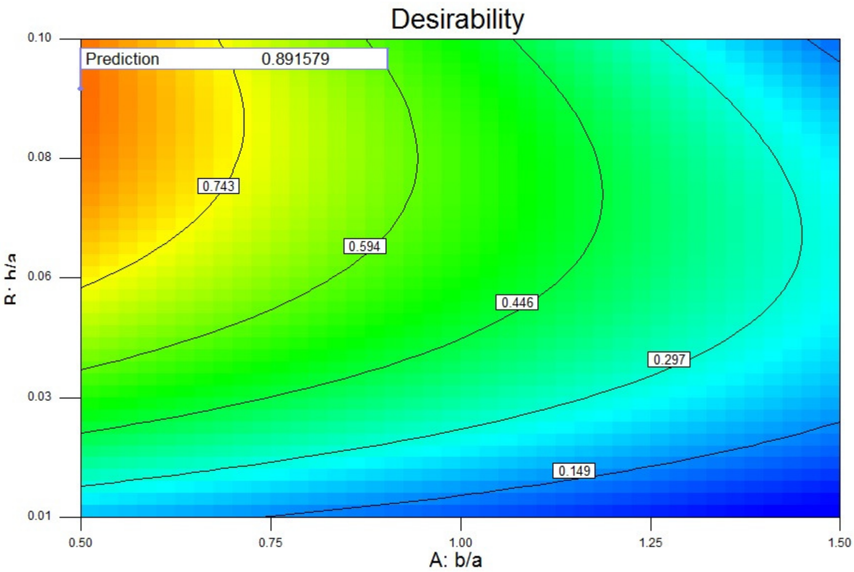

Fig. 7. Desirability contour plot for fundamental frequency maximization

Table 2 contains the design of experimentation. 4 parameters are used the input response, and the 1st natural frequency is used as the output response. In total 29 sets of numerical experiments are performed.

Analysis of variance (ANNOVA) is carried out, and the results are reported in Table 4. The Model F-value of 8.21 implies the model is significant. There is only a 0.01% chance that a "Model F-Value" this large could occur due to noise. Values of "Prob > F" less than 0.0500 indicate model terms are significant. In this case, A, B, B2, C2, D2 are most relevant model terms. Values greater than 0.1000 indicate the model terms are not so significant. If there are many insignificant model terms (not counting those required to support hierarchy), the model reduction may improve the model. This has been done in the current case as some terms like AD, BC, BD, CD and A2 are not significant and hence are removed to improve the model. For the sake of brevity, the ANNOVA test on the full quadratic model is not reported, and only the reduced quadratic model is reported herein. Lack of fit is the portion of the residual SS that is due to the model not fitting the data. It is the weighted sum of squared deviations between the mean response at each factor level and the corresponding fitted value. The R2 value of the current model is 89.54% indicating high predictability of the response variable by the current model.

Based on the regression analysis the following equation is formed,

Nat. Frq. (1) = -56.97 – 241.88*b/a + 10854.97* h/a +4 6.30* No of plies + 3.908629479* Ply angle – 2970.68*b/a*h/a + 64.95*b/a*No of plies – 51767.89*h/a^2 – 15.33 * No of plies^2 – 0.04*Ply angle^2 (2)

The normal probability of the studentized residuals is shown in Fig. 2. The studentized residuals are obtained by dividing the residual using an approximation of its standard deviation. It is observed that most of the residual data points lie on the line or very close to it which shows normal distribution of errors. Further, there is no cluster of residuals at one place which means that there are no ties in the data, thus implying that the measuring resolution is adequate. Also, no significant outliers are seen in the plot. Fig. 3 show the comparison of predicted and the actual responses. Fig. 4 demonstrates the variation of predicted response versus the externally studentized residuals for mark intensity. In Fig. 4, the data points are randomly scattered, and it advocates that in the present mode, the hypothesis of constant variance is not violated. Thus this further confirms that the proposed model is adequate. Fig. 5 and 6 show the variation of fundamental frequency with change in thickness ratio vs. aspect ratio and No. of plies vs. aspect ratio respectively.

Table 6
Predicted parameter settings for fundamental frequency maximization

b/a	h/a	No. of plies	Ply angle	Nat. Frq. (1)	Desirability
0.5	0.09	2.57	51.22	447.42	0.89

Table 7
Confirmation trials

b/a	h/a	No. of plies	Ply angle	Nat. Frq. (1)
0.5	0.09	3	50	444.51
0.5	0.09	2	50	442.36

The fundamental frequency is then maximized using desirability function. The constraints and the goals are listed in Table 5. The optimization is a constrained optimization problem. Based on the desirability criteria the fundamental frequency is maximized which is reported in Table 6. The contour plot for desirability is indicated in Fig. 7. The results of the confirmation trial are given in Table 7.

4. Conclusion

In the present work, the effect of various parameters (aspect ratio (b/a), thickness ratio (h/a), number of plies (n) and ply angle (θ)) on the fundamental frequency is investigated using FEA analysis and the data is analyzed statistically. Based on the study the following conclusions can be drawn-

- The FEA simulation is efficient and accurate. The comparison with experimental data showed that the deviation of FEA results is less than 5%.
- The RSM method is simple and can be used to form a 2nd order equation to describe the desired response.
- The natural frequency decreases with increase in thickness ratio.
- Frequency is found to be decreasing with increase aspect ratio.
- In general, growing number of plies would increase the natural frequency.
- For maximizing the fundamental frequency thickness ratio should be high. Low aspect ratio and fewer number of plies are desirable.

Thus the current methodology can be adopted for optimizing frequency responses. Future work will consider a larger number of input parameters and optimization problems like frequency separation maximization will be considered.

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