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Dynamics of High-modulus Belts with Distributed Mass and Friction

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Abstract – *The power required to drive long conveyors is determined by calculating the rolling resistance and the gravity influences of the system masses which include the conveyed bulk material. Minimising rolling losses is a significant research activity worldwide by those involved in mechanical conveyor design. A belt's stationary tension distribution is usually significantly different to the running tension distribution as a result of rolling forces. The paper provides a practical method for applying rolling losses to predict steady-state belt forces, with application to dynamic tension simulation during starting and stopping.*

Keywords: Conveyors, friction, design, dynamics

1.0 INTRODUCTION

Rolling losses in long conveyor belts cause an accumulated strain increase in the belt reinforcement, leading to higher power consumption and belt tension. Complex interactions between visco-elastic indentations of the rubber belt on idler rollers, coupled with belt and bulk material flexing, generate drag forces that need to be calculated in order to design a long conveyor. As belt tension increases with length, long conveyors usually require high tension belts containing steel cable reinforcement [1]. Minimising the incremental addition of drag along a belt can have significant cost reduction in belt rating over long distances.

Accelerating a conveyor with distributed mass and elasticity to a steady-state velocity can result in large dynamic forces at drives, as a result of elastic wave propagation. Extended starting times can reduce dynamic starting forces. However, stopping or braking the conveyor in a required time (required by safety codes) can generate dangerous dynamic forces on drive and counterweight structures [2].

The larger the differential tension distribution on a drive drum, the larger are the potential dynamic forces. The problem is even more complex since design has to take into account a higher initial rolling drag until all the components wear in. A designer would not want to design to a friction factor that is low, resulting in insufficient torque to move the loaded conveyor when it was new. A practical solution is to derive and test rolling resistance predictions against real-world examples, as will be used in this paper.

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Various mechanical models of moving conveyors occur. For industry applications, CEMA [1] calculation methods are widely used but conservative, while the German Standard DIN 22101 and its various related standards are widely referenced by mechanical engineers.

2.0 ROLLING FORCES

For each element of belt X being part of a total conveyor length L, let $L = \Sigma X$, with distributed forces defined for each belt / idler interface element length X using the model shown in Figure 1. The rolling loss RC or RR (C=Carry, R = Return) at each interfaces between rubber belt and rolling idler increases the force in the belt FC or FR at element X, defined as follows :

Belt tension at carry run X1: $FCx1 = FCx + RCx$ (1a)

Belt tension at return run X1: $FRx1 = FRx + RRx$ (1b)

The definition of each FCx or FRx is compound and not linear [2]. For example, losses at the return run interface depend on rubber hardness, visco-elastic indentation and rebound time constants, belt flexural stiffness, belt sag inducing more contact wrap (which is belt tension dependent), rubber speed influences and sliding friction. In addition, if horizontal or vertical curves exist, then there is a general increase in frictional loss by a factor $K = FR(1 + A\theta)$ where A = arc radius of the curve and θ = the arc angle. The same formula is used to predict any sag / wrap effect on indentation at idler rolls when there is significant belt sag above 5%. For the carry run, all these factors also apply with the addition of material flex resistance.

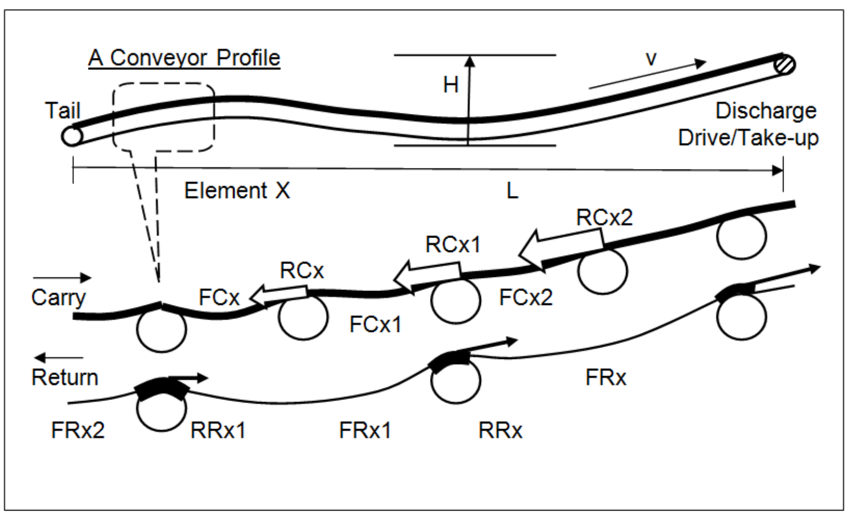

Figure 1: Contact rolling loss diagram

The computational problem can be further expanded to define parameters used in calculations and modelling of running resistances. After all summations are made along the entire belt carry and return run, a belt effective tension Fe has a form defined by Equation (2) :

$$Fe = L\, g\, [\, I + B + R + V + K\,] + Q\, v + P + O \qquad (2)$$

where

L = Length of conveyor (m),

g = gravitational acceleration = 9.81 m/s/s.

I = Rotating resistance of all idlers (kg/m), normalized for span length between idlers [1].

M = Mass (kg/m) to be lifted a height H ; (H can be negative or variable with load).

B = Belt mass carry and return strands (kg/m)

V = M*H/L

R = (BI + BF + MF + CF) (kg/m) = Belt indentation + belt flexure + material flexing + curve loss.

v = belt speed (m/s).

Q = Force to accelerate material (N), = M (v - loading speed)^2 .

P = Force to rotate all Pulleys (N).

O = Forces for all other accessories, scrapers and special losses (eg. turnovers) (N).

The power required to drive the conveyor is Pe = Fe . v. The maximum steady state running belt tension is T1 = (T2 + Fe) where T2 is the belt tension exiting the drive, set at a value to prevent drive slip or to minimize belt sag to at most 2.5 %.

Considering each element of the model in Figure 1, values for a practical design would use:

a) I : 0.3 < I < 1 (similar to CEMA Kx) – obtain bearing and idler manufacturer values.
b) Carry side drag force = (BI + BF + MF + CF) = 18 - 36 N/m (similar to CEMA Ky) [1] .
c) Return side drag force (MF = 0) ~ 1/3 of the carry run (with a triple span length).

Previous research shows that rolling loss is slightly dependent on belt speed, and almost linearly dependent on M (kg/m on the belt) [2]. Idler curvature K can be added with only a small effect for typical roller diameters between 127 mm and 152 mm. A good approximation between these interdependent parameters is:

$$RR\,(M, v) = 1.62\;\; M\;\; (5\,v/4 + 36) \qquad 10^{-3}\;\; (N/m) \tag{3}$$

3.0 DYNAMIC FORCES

Once the steady-state or static conditions are set to develop a set of design parameters, the dynamic model needs to be actioned in a way that retains static conditions during parked, starting, running and stopping. A discrete element model (DEM) may be applied, with each mass at a position x_j and displacements computed for the high-modulus springs between masses using a time step h. Elastic forces have to be defined for each mass M_j in the general elastic model of Figure 2 [3].

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Each mass element "M_j" has an associated acceleration "a_j", velocity "v_j" and displacement x_j". Externally applied forces include drive or booster drive force F_d and brake force F_b which can be activated at any element along the chain.

As discussed in the previous section, non-conservative forces include rolling friction, belt indentation, belt and material flexing, and velocity damping. Conservative forces acting on each element include spring forces "k" and gravity forces $M_j g \, Sin\Phi_j$ for each distributed mass at a particular slope Φ_j. Velocity and force histories at any element location along the discrete model are used to assess a conveyor design for low and high tensions that can affect mechanical equipment sizing. Preventing feed-forward errors from accumulating between time-step iterations is essential for model stability.

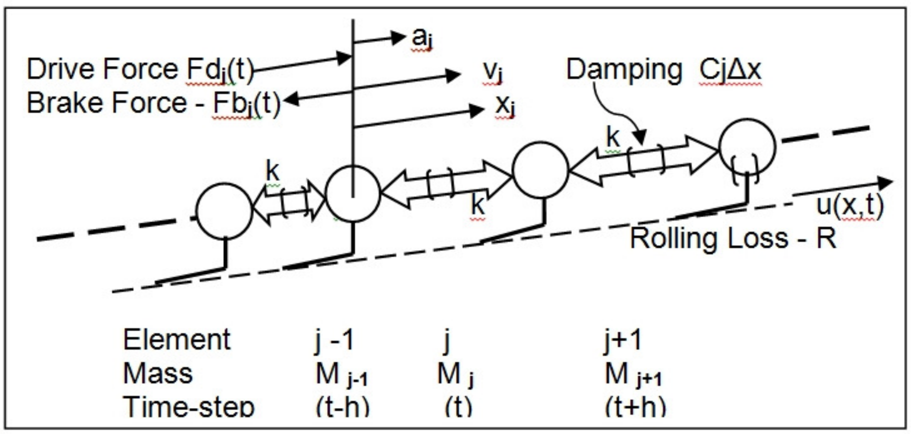

Figure 2: Elements of an elastic model with losses and externally applied forces.

In a simulation, the dynamic behavior of each element at time (t+h) is computed from the acceleration at the previous time t. In the case of spring forces on element j, the mass displacements either side of element j need to be considered. Forces are computed from spring strain and are processed in an iterative program loop for each mass, using the following:

a)	Inertia force	$M_j \ddot{x}_j$	Inertial force on a mass being accelerated
b)	Spring force	$k\,(2x_j - x_{j-1} - x_{j+1})$	as a conservative force, a belt dynamic tension
c)	Gravity force	$M_j\, g\, Sin\, \theta_j$ (lift)	as a conservative force on each mass at slope θ
d)	Take-up force	$M_{tu}\, g / 2$	as a offset force in the belt – single loop
e)	Damping force	$C_v\,(2v_j - v_{j-1} - v_{j+1})$	as a non-conservative force : Cv in N-s/m
f)	Rolling and Drag	$gL_j(A + R)$	as non-conservative, defined by Equation (3)
g)	Winch pull force	$Fw\,(x_j, t) / 2$	as an external force in a belt – single loop
h)	Drive /Brake forces	$Fb\,(x_j, t)$	as externally applied forces

Velocity and force histories at any element of a mass-spring model are used to determine if a conveyor design is dynamically suitable. Simulating the motion of any system with distributed mass and elasticity using a discrete-element model requires an exact execution of applied forces on a free-body, otherwise errors and instabilities perpetuate as the computation runs.

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A general force-balance method is used that externally applies forces to the model, including drive and braking functions. A force-balance method provides a tractable way of quickly establishing applied force values for both static and dynamic model stability. By this method, both static or running tensions of the belt are automatically produced by the dynamic analysis, without the need for a separate static analysis.

However, any errors produced when calculating the displacement at any element will accumulate and can cause instabilities and incorrect force histories. Eliminating error in computation is essential if steady state solutions are to mimic real conditions [4].

Using the fact that dynamic velocity is approximated in the limit by $v = v_o + a*h$ (h = time step), a simulation loop can process the evolving force on each mass in the following order:

1. Firstly, allow all body forces (conservative) to settle to steady-state conditions.
2. Calculate acceleration $a_j(0)$ using velocity $v_j(0)$ and $x_j(0)$.
3. Calculate $v_j(t + h)$, then $x_j(t + h)$
4. Calculate tension in each spring element $k * (x_{j+1} – x_j)$: damping is assumed active.

Figure 3 shows the timing regime for a belt start and stop simulation, in which the above calculations are looped progressively until the simulation has completed. A linear start ramp is shown for the impulse.

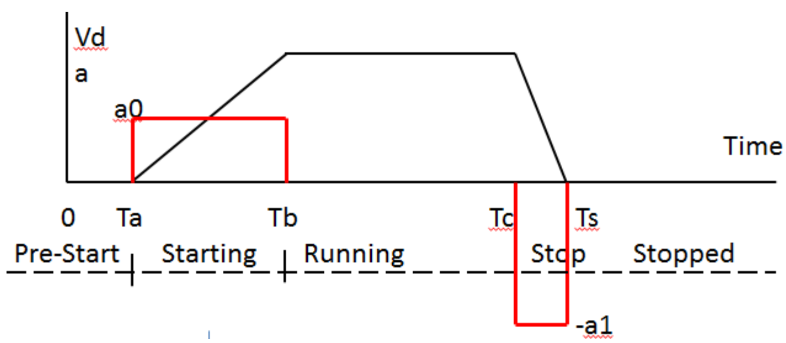

Figure 3: Accelerations for a belt start and stop

4.0 ADDING A DRIVE FORCE WITH IMPULSE BALANCE

Existence of non-conservative or dissipative forces that are non-repeating or have an externally applied component that does not repeat (like friction in a winch take-up), can cause feed-forward errors in a simulation if not properly accommodated. Wave motion itself can perturb a solution when dissipative forces act on large displacements. Initial conditions for each mass in the system of masses are set at time step t = 0 with $x_j = L_j$ (element length), $v_j = 0$ and a_j = gravity acting the mass at some slope angle.

If there are any small errors ε in the initial calculations of a(t) after the simulation starts at time t = Ta, acceleration can drift slowly to (a0 +/- ε) at each successive time step, whereas its value should be a0. Conservation of momentum must apply at all times, requiring a force balance condition throughout the entire simulation cycle. The best way to eliminate feed-forward errors

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is by ascribing the correct force to the model during transient simulation states, until steady-state conditions are reached (the static solution).

4.1 Example Model Set-up

The solution can best be described by way of an example. Figure 4 shows a typical declined conveyor profile with masses distributed as shown.

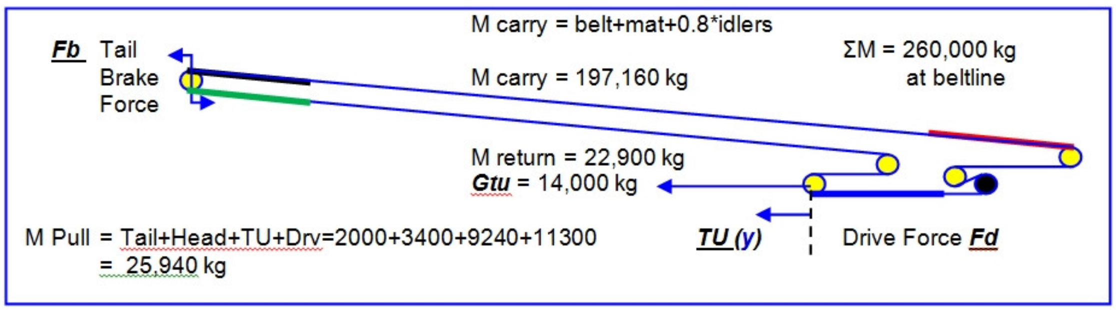

Figure 4: Diagrammatic representation of a conveyor profile (not to scale).

Parameters for the model are Lcc = 1,000 m, H = -33 m, with masses distributed as shown in the figure. The total system mass is ΣM = 260,000 kg. For the belt to accelerate to a speed of 4.2 m/s in 20 s, the effective force on the drive mass is Fd = 260,000 *0.21 = 54,600 N, without any allowance for dissipative forces. To stop the belt in 10 s, the effective braking force is Fb = 260,000*(-0.42) = -109,200 N.

To stop the system of moving masses, a brake force Fb is applied at one or more of the masses in the system. Irrespective, the system impulse for a complete simulation cycle ($Fd.\Delta t_1$ + $Fb.\Delta t_2$) = 0, i.e. momentum is conserved otherwise feed-forward errors in displacement and velocity will propagate during the simulation.

4.2 Force Balance Including Carry-side Rolling Loss

Rolling loss is a dissipative or non-conservative force that is never regained in the system. In order to ensure that the belt model runs at the required speed, externally applied forces will need to change in the presence of non-conservative forces including rolling losses. A force-balance method was adopted to minimize simulation errors.

Idler rolling force selected from CEMA [1] for a 1 m carry spacing is A = Ai/Si = 6.7 N/m (0.68 kg/m, not allowing for grease breakaway). Typical values used for BI, BF and MF are published elsewhere (see Ref. [3], Fig. 2). Adding idler and belt rolling loss from Equation (2) for the carry-side drag:

Ac = A (carry) = 0.68 kg/m (6670 N for the carry side)

Rc = (BI+BF+MF) = (0.93 + 0.127 + 1.42) kg/m = 2.48 kg/m [1]

Φ1 = (Rc + Ac) = 3.16 kg/m (31,000 N distributed over carry-side mass elements)

4.3 Adding Return-side Rolling Loss

For a return belt strand with a 3 m idler spacing:

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Ar = A (return) = 0.226 kg/m (2,223 N for the return run)

Rr = (BI+BF+MF) = (0.245 + 0.14 + 0) kg/m = 0.385 kg/m (3,777 N)

Φ2 = (Rr + Ar) = 0.611 kg/m (6,000 N distributed over return-side mass elements)

4.4 Force Balance Table

Total distributed rolling resistance R = (Φ1 + Φ2) = 37,000 N. The losses R are included in the force balance of Table 1 to generate an applied set of forces Fd and Fb to accommodate these losses.

Table 1: Adjusted force balance to conserve impulse due to non-conservative forces

Condition	0 < t < Ta settling	Ta < t < Tb starting	Tb < t < Tc At speed	Tc < t < Ts braking	T > Ts stopped	BALANCE Tests
SimTime (s)	t<100 s	100<t<120 s	120<t<190	190 < t < 200	t>200 s	
Impulse time (s)	0	$\Delta t_1 = 20$	0	$\Delta t_2 = 10$	0	
V (m/s)	0	0 to v = 4.2	v	4.2 to 0	0	
Rolling Force **R** (N)	0	-37,000	-37,000	-37,000	0	
New Start Force **Fd** (N)	109,200	91,600	37,000	0	109,200	
New Brake Force **Fb** (N)	-109,200	0	0	-72,200	-109,200	
Σ Forces	0	**54,600**	0	**-109,200**	0	**Checks**
Impulse F.Δt (kg. m/s)	0	1,092,000	0	-1,092,000	0	**Conserved**

4.5 Final Adjustment for a Brake or Holdback

From Table 1, the effective braking effort during the time interval Tc < t < Ts is 72,200 N, however at t > Ts the force jumps to 109,200 N as retained from Table 1. The difference in force will induce an artificial shock wave in the belt if not corrected. Table 2 shows the finally corrected model taking into account rolling and brake holdback levels, where Fb is held constant at 72,200 N when t > Tc.

For the data in Table 2, control equations used in the simulation to satisfy the force balance criteria are:

Rolling Loss : Carry side if(t<100,0,if(t<200,-31000/j,0))

Return side if(t<100,0,if(t<200,-6000/j,0))

Drives and Brakes : Fd if(t<100,72200,if(t<120,91600,if(t<190,37000,if(t<200,0,72200))))

Fb if(t<100,-72200,if(t<190,0,if(t<200,-72200,-72200)))

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Figure 5 shows belt element tensions at the drive and tail in the presence of rolling losses with a holdback brake at the tail. An important outcome of the dynamic analysis is that the steady-state tensions just before time t = Tc represent the static design tensions, obtained without conducting a separate belt static analysis.

Table 2: Force-balance to generate a design solution to accommodate rolling and brake holdback.

Condition	0 < t < Ta settling	Ta < t < Tb starting	Tb < t < Tc At speed	Tc < t < Ts braking	T > Ts stopped	BALANCE Tests
SimTime (s)	t<100 s	100<t<120 s	120<t<190	190 < t < 200	t>200 s	
Impulse time (s)	0	$\Delta t_1 = 20$	0	$\Delta t_2 = 10$	0	
V (m/s)	0	0 to v = 4.2	v	4.2 to 0	0	
Rolling Force **R** (N)	0	-37,000	-37,000	-37,000	0	
Start Force **Fd** (N)	72,200	91,600	37,000	0	72,200	
Brake Force **Fb** (N)	-72,200	0	0	- 72,200	- 72,200	
Σ Forces	0	**54,600**	0	**-109,200**	0	**Checks**
Impulse F.Δt (kg. m/s)	0	1,092,000	0	- 1,092,000	0	**Conserved**

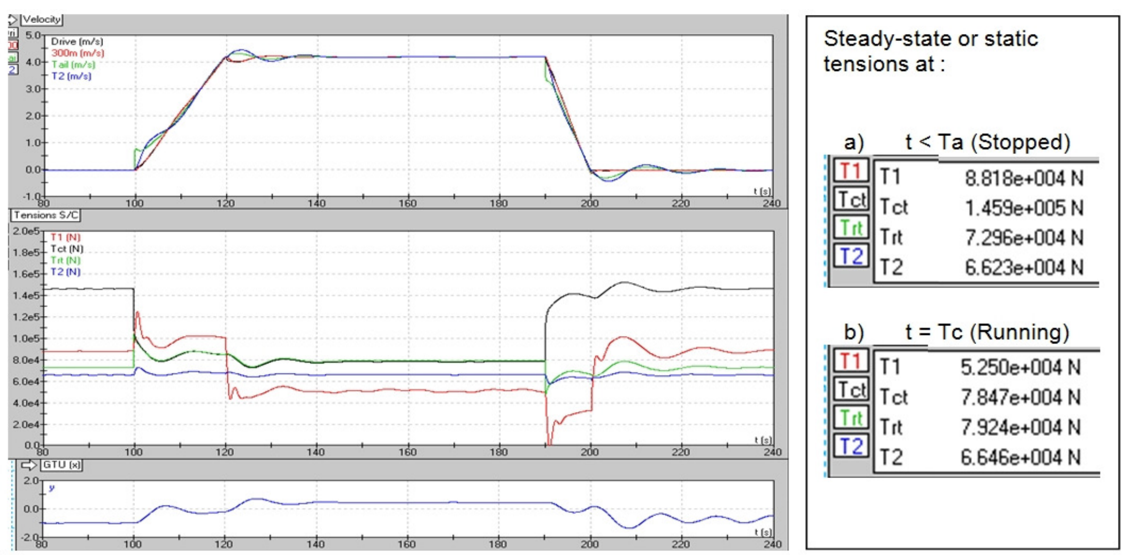

Figure 5: Rolling losses added to produce static and dynamic tensions for the model in Figure 2.

In the initial calculation of the force balance required to conserve the impulse, the holding force was equated to the maximum braking force applied at the tail, i.e. 109,200 N. The procedure

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elevates all element tensions. A better solution was given in Table 3 where the stopped holdback force was held at 72,200 N.

From Figure 5, a low tension is predicted on stopping just ahead of the head drive, and so the take-up mass should be increased to elevate all element tensions by about 10 kN to avoid sag problems. If this were the case, the tail pulley holdback force would increase, but the brake force would not change. The T2 tension at the take-up loop is then governed by the mass. In this scenario, any brake holding force reduction from 109.2 kN to 72.2 kN is mechanically beneficial to the design, i.e. reduced brake sizing. Adding counterweight will raise belt tensions overall but not affect the effective tensions around the tail pulley.

5.0 CONCLUSIONS

Ensuring impulse conservation during the starting and stopping simulation of a conveyor belt will remove computation feed-forward errors. The procedure shows that if all non-conservative forces are neglected in the initial calculation of the applied external forcing functions, accelerations and decelerations of the simulated belt will follow stable laws of mechanics. In other words, momentum conservation at each time step and the model will be stable.

Adding dissipative forces such as sliding friction in take-ups, rolling resistance and velocity-dependent damping requires adjustment of the model's forcing function as shown in Table 1. In all cases, the sum of forces needs to produce an impulse value identical to the original loss-free simulation. Application of these procedures is general and has been used to model stable belt conveyor dynamics. In addition, if the dynamic forces are not erroneous, there can be confidence that these curves may be used for design, brake and drive component sizing, belt ratings and pulley loads.

Static and dynamic tensions are both generated in a single simulation, eliminating the need for a separate static tension analysis. The paper has not addressed computation of stiffness elements, however belt moduli values can be used to determine the spring constants. Sag evolution in the presence of wave action in the belt affects the effective stiffness and damping, but has no influence on the impulse. Effects of non-linear breakaway and sag distribution can be included to ensure that momentum is conserved during starting and stopping.

A procedure for ensuring zero feed-forward errors during simulations greatly speeds up dynamic analysis. The advantage of the method is that brakes, holdbacks and take-up positions and magnitudes can be rapidly evaluated with the certainty that the force-balance method will not allow errors at each time-step of a simulation. Both static and dynamic tensions are produced in a single simulation by the above methods.

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