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# Optimization of Two-dimensional Wedge Flow Field at Supersonic Mach Number



Sher Afghan Khan<sup>1,\*</sup>, Abdul Aabid<sup>1</sup>, Imran Mokashi<sup>1</sup>, Abdulrahman Abdullah Al-Robaian<sup>2</sup>, Ali Sulaiman Alsagri<sup>2</sup>

<sup>1</sup> Department of Mechanical Engineering, Faculty of Engineering, International Islamic University Malaysia, 53100, Kuala Lumpur, Malaysia

<sup>2</sup> Department of Mechanical Engineering, Qassim University, Saudi Arabia

ARTICLE INFO	ABSTRACT
Article history: Received 31 March 2019 Received in revised form 3 May 2019 Accepted 16 May 2019 Available online 20 May 2019	In the present condition, there is a high demand for exact solutions through analytical, computational and measurements in the field of aerospace engineering. For wind tunnel testing it is difficult to bear the cost because of highly expensive materials needed for wind tunnel testing and the time needed to accomplish the experiments. Therefore, in this paper, analytical and numerical methods are used to evaluate the flow field over a wedge at supersonic Mach numbers for attached as well as detached shock cases. The wedge with the various half-wedge angle at Mach number 2 has been considered for the simulation. Closed form solutions are obtained for the various semi-vertex angle of the wedge. Supersonic similarity parameter has been used to obtain the pressure distribution over wedge at a different angle of attack with attached and detached shock-expansion theory. The analytical results are compared with those obtained by simulation. The results are obtained for pressure, temperature, Mach number, and the density using ANSYS code are in good agreement with the results obtained analytically.
Keywords:	
CFD method, Supersonic flow, Wedge,	
FEM, and Mach number	Copyright © 2019 PENERBIT AKADEMIA BARU - All rights reserved

# 1. Introduction

Earlier attempts were made for theoretical analyses for an oscillating wedge by the researchers; for the unsteady supersonic/hypersonic flow. Tsien was the first to give the similarity law that applies to a wide range of Mach numbers [1]. Piston theory was developed by Lighthill for an oscillating airfoil in pitch at high Mach numbers [2]. Crasta *et al.*, [3] used unified hypersonic similitude to compute the stability derivatives in pitch with an attached shock at high incidence in hypersonic flow for a planar wedge. Renita *et al.*, [4] studied an effect of sweep angle on roll damping derivative for a delta wing with curved leading edges in unsteady flow.

A similitude was used to compute the stability derivatives for an oscillating two-dimensional (2-D) wedge with attached shock case in a pitch at a high angle of incidence at supersonic and hypersonic Mach number. For the 2-D flow, from the nose of the body, a normal shock wave is formed which

\* Corresponding author.

E-mail address: sakhan@iium.edu.my (Sher Afghan Khan)



extend around the body and form curved oblique shock depends on the flow deflection angle [5–13]. Singh *et al.*, [14] and Khan *et al.*, [15] optimized the flow parameters as pressure, temperature and density for a planar wedge using CFD method and validated their results with those obtained theoretically. The pressure and Mach number effects in a convergent-divergent nozzle to control the base pressure for suddenly expanded ducts using the FE method was identified, and results were compared with experimental results [16–24]. The CFD analysis over a CD nozzle with axisymmetric enlarged duct has been reported by Pathan *et al.*, [25-28] for different parametric studies through ANSYS simulation, and the primary consideration in all the studies is the effectiveness of micro-jets in order to control the base pressure in the suddenly expanded duct. The objective of this paper is to use the FE method to validate the theoretical results by computing the different flow parameters for the planar wedge. The parameters considered are pressure, temperature, Mach number, and density.

# 2. Methodology

The finite element method is used to simulate the flow field by the numerical method. In this method, all of the problems, such as structural, fluent, and thermal solutions can be obtained with a reasonable level of accuracy. The variables which are common in the engineering problems are displacements in solid mechanics, velocities in fluid mechanics, electric, and magnetic potentials in electrical engineering and temperatures in heat flow problems [29].

In this problem ANSYS was used to simplify the solution, ANSYS Fluent runs comprehensive modeling capabilities for a wide range of compressible and incompressible, laminar and turbulent fluid flow problems. Steady-state or transient analyses can be performed. Robust and accurate turbulence models are a vital component of the ANSYS Fluent suite of models. The turbulence models provided have a broad range of applicability, and they include the effects of other physical phenomena, such as buoyancy and compressibility. Particular care has been devoted to addressing issues of near-wall accuracy via the use of extended wall functions and zonal models [30].

Finite element methods involve a lot of numerical calculations. The solution runs in this method with the three necessary steps: Pre-Processing; in this case the inputs are: Solver: steady, absolute, 2D planar pressure-based; Model: Inviscid, Energy equation; Fluid: air, ideal gas; Boundary conditions: far field, pressure far field (Pa); symmetry, symmetry wall; wedge, wall; and the Solution method: Pressure (standard); density, momentum, turbulence kinetic energy, turbulence dissipation rate, energy (second-order upwind). Processing Solution, in this case, the solution steps are Solution initialization: standard, from pressure-far-field; Reference value: pressure-far-field (solid surface body); Solution run: Up to solution converged. Post-Processing/Results; many values calculated after the analysis, may be printed out. Suitable changes are to be made to incorporate various elements and for the calculations other than static linear analysis for this step to analyses the different results of a given problem. The results are shown in section 3.

#### 2.1 Geometry and Modelling of Wedge

The modeling of the wedge was done with the ANSYS geometry modeling software. With the standard procedure of the ANSYS workbench created the exact dimension of the given problem of the wedge. In the present solution, two-dimensional geometry modeling has been performed as shown in Figure 1.





Fig. 1. Two-dimensional shape of a wedge

The modeling of the wedge has been drawn in ANSYS workbench with the specified dimension, and there is an external surface has been created to model fluid area ahead of the wedge to show the fluid process from the wedge. The model is symmetric; therefore, only symmetry of the model was used as shown in Figure 2. The dimension; Horizontal Distance (H1) is 0.5 mm, Horizontal Distance (H2) is 1.5 mm, Vertical Distance (V3) is 1.259, and the Vertical Distance (V4) depends on Half-wedge angle (degree).



Fig. 2. Two-dimensional geometry of the wedge

# 2.2 Meshing and Boundary Conditions

The meshing has been performed for a given problem using ANSYS Meshing. Meshing is all about converting an infinite number of particles model of a finite number of particles. To obtain accurate solutions fine mesh has been created with a structured mesh grid. In order to obtain fine mesh, controlled the sizing by applying the curvature size with a coarse mesh and the element size with face meshing. Total 6761 binary nodes have been generated for all zone of the wedge. Figure 3 shows the type of mesh in a two-dimensional shape.

The 2D wedge is symmetry; therefore, only the symmetry of the model has been designed and modeled. The applied boundary condition in the present work is wedge, symmetry, and far field which is defined by selecting edges of the model.





Fig. 3. Finite element meshing

#### 2.3 Shockwaves

The shockwave created from the notch of the wedge when the fluid passes near to the wedge. It is an extremely thin region, across the wedge which the flow properties can change drastically.

#### 2.3.1 Oblique shockwaves

When a shockwave makes an oblique angle with the upstream flow, it is called oblique shockwave. Oblique shockwave occurs when a supersonic flow is encountered at the wedge that effectively turns the flow. Downstream of the oblique shock the properties change drastically. In Figure 4 shows the properties variation of oblique shock.



Fig. 4. Oblique Shockwave Properties Variation

For the oblique shocks, the different equation performance has been specified to optimize the fundamental study of fluid flows. They are:

**Continuity Equation** 

$$\rho_1 u_1 = \rho_2 u_2 \tag{1}$$

Momentum Equation  $p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$ 

Energy Equation  $h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}$  (2)

(3)

Oblique shock in terms of the normal component of the upstream Mach number Mn, 1

$$Mn, 1 = M_1 \sin\beta \tag{4}$$

Thermodynamic properties of density, pressure, and temperature across a normal shockwave

$$M_{n,2}^2 = \frac{1 + \left[\frac{\gamma - 1}{2}\right] M_{n,1}^2}{\gamma M_{n,1}^2 - \left(\frac{\gamma - 1}{2}\right)}$$
(5)

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)M_{n,1}^2}{2+(\gamma-1)M_{n,1}^2} \tag{6}$$

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1} \left( M_{n,1}^2 - 1 \right) \tag{7}$$

$$\frac{T_2}{T_1} = \frac{p_2 \rho_1}{p_1 \rho_2} = \left[1 + \frac{2\gamma}{\gamma+1} \left(M_{n,1}^2 - 1\right)\right] \frac{2 + (\gamma-1)M_{n,1}^2}{(\gamma+1)M_{n,1}^2} \tag{8}$$

Mn, 2 is the normal Mach number behind the shock wave. The downstream Mach number M2 can be found from Mn, 2 and the geometry of Figure 4 as

$$M_2 = \frac{M_{n,2}}{\sin(\beta - \theta)} \tag{9}$$

The changes across an oblique shock depend on two parameters, M1 and  $\beta$ .

$$\tan \theta = 2 \cot \beta \ \frac{M_1^2 \sin^2 \beta - 1}{M_1^2 (\gamma + \cos 2\beta) + 2}$$
(10)

This is called  $\theta - \beta - M$  relation and it specifies  $\vartheta$  as a unique function of M1 and  $\beta$ . This relation is vital to the analysis of oblique shock waves and results from it are plotted in Ref. [14].

#### 2.3.2 Validation

To validate the results obtained from theory and numerical are compared. A fundamental study has been considered as shown in Figure 1. The study proved that the fluid flows across the wedge for the different Mach number has a different flow regime. Figure 5 shows two different shocks for a given Mach number and angle.



Fig. 5. Attached and detached shock [14]



The finite element results are validated with the analytical results and obtained an excellent agreement for a different angle of the wedge. Table 1 shows the validation of the given results for attached shock and Table 2 for a detached shock.

Table 1								
Validation of results for attached shock								
Parameter	Mach Number M = 2							
	$\theta = 5 \ degree$		$\theta = 10 \ degree$		$\theta = 15 \ degree$		$\theta = 20 \ degree$	
	$\beta = 34 \ degree$		$\beta = 40 \ degree$		$\beta = 45 \ degree$		$\beta = 53 \ degree$	
	Analytical	Numerical	Analytical	Numerical	Analytical	Numerical	Analytical	Numerical
$M_2$	1.76	1.81	1.59	1.61	1.36	1.40	1.18	1.16
$\frac{\rho_2}{ ho_1}$	1.19	1.22	1.49	1.48	1.88	1.78	2.06	2.15
$rac{p_2}{p_1}$	1.58	1.66	1.76	2.53	2.69	3.61	2.88	5.27
$\frac{T_2}{T_1}$	1.10	1.08	1.18	1.18	1.31	1.29	1.38	1.42

Table 2						
Validation of results for detached shock						
Parameter	$\theta = 25 \ degree$		$\theta = 30 \ degree$			
	Analytical	Numerical	Analytical	Numerical		
$ \begin{array}{c} M_2 \\ \underline{\rho_2} \\ \overline{\rho_1} \\ \underline{p_2} \\ p_1 \\ \underline{T_2} \\ \underline{T_1} \end{array} $	Detached s	hock	Detached s	hock		

#### 3. Results and Discussion

In this section different results which are obtained from the ANSYS Fluent are presented in the section to follow.

# 3.1 Solution Convergence

The first step to solve the given problem the file should run until the solution is converged. Therefore, in this section, the solution convergence is shown for a different semi-vertex angle of the wedge are shown in Figure 6 to 11 for the angles in the range from 5 to 30 degrees and the solution is converged for the angle of thirty degrees. The convergence of solution also depends on the type of mesh, some elements applied and geometry of the model "maximum element maximum iteration." For the present case Table, 3 shows the iteration for each angle for conversation.

Table 3						
Number of iterations						
Angle of wedge	5	10	15	20	25	30
Number of Iterations	48	60	84	210	649	416





# 3.2 Effect of Mach Number

The input value of Mach number given in supersonic flow is M = 2. From the Figure 12 to 15, it is seen that in the downstream of the wedge there is sudden decrease in the Mach number, and the decrease in the Mach number is associated with the strength of the shock wave. With further increase in the flow deflection angle after certain limit the shock detaches with nose of the wedge as shown in Figure 16 and 17.













Fig. 14. Mach number variation for wedge angle 15-degree (a) Contour (b) Plot









# 3.3 Effect of Pressure

From the simulation results the static pressure was also extracted for better understanding of the flow variation across the wedge. From the results it has been observed that, the static pressure is very high near the nose of the wedge. The variation of the pressure with the angle of wedge are shown in Figure 18 to 21. When angle increases after a certain the limit the shock detached from the wedge as shown in Figure 22 and 23.



Fig. 18. Static pressure variation for wedge angle 5-degree (a) Contour (b) Plot



Fig. 19. Static pressure variation for wedge angle 10-degree (a) Contour (b) Plot











Fig. 22. Static pressure variation for wedge angle 25-degree (a) Contour (b) Plot





Fig. 23. Static pressure variation for wedge angle 30-degree (a) Contour (b) Plot

# 3.4 Effect of Temperature

The static temperature also was considered for the flow variation across the wedge length. From the results it is seen that the static temperature is high near the nose of the wedge up to certain angle of wedge as shown in from Figure 24 to 27. When the angle has reached a certain limit the shock wave detaches from the wedge as shown in Figure 28 and 29.



Fig. 24. Static temperature variation for wedge angle 5-degree (a) Contour (b) Plot





Fig. 26. Static temperature variation for wedge angle 15-degree (a) Contour (b) Plot



Fig. 27. Static temperature variation for wedge angle 20-degree (a) Contour (b) Plot



Fig. 29. Static temperature variation for wedge angle 30-degree (a) Contour (b) Plot

# 3.5 Effect of Density

Finally, the density plots of the fluid for all the angle of the wedge. From the results, it has been found that the density also increases up to a certain angle of wedge shows in from Figure 30 to 33. When angle increases to a limiting value the shock wave is detached from the wedge as shown in Figure 34 and 35.





(a) (b) Fig. 32. Density variation for wedge angle 15-degree (a) Contour (b) Plot

1 (m)





Fig. 35. Density variation for wedge angle 30-degree (a) Contour (b) Plot



# 4. Conclusions

From the CFD simulations, the solutions are optimized for the flow for different parameters. The present study considers both the cases of the attached and detached shock waves. The upstream Mach number considered is M = 2 for a fixed wedge angle and simulation was carried out. Further, the wedge angle was varied keeping the Mach number fixed, and the results were obtained. The simulation results were obtained using ANSYS code, for various flow and geometric parameters such as pressure, density, temperature, and Mach number are validated with the theoretical study of second-order shock-expansion theory. Hence, obtained results from CFD simulation are in good agreement with theoretical results. The discrepancy in the simulation results and the analytical results are within the acceptable limit.

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