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LATTICE BOLTZMANN METHOD: AN ALTERNATIVE CFD TECHNIQUE

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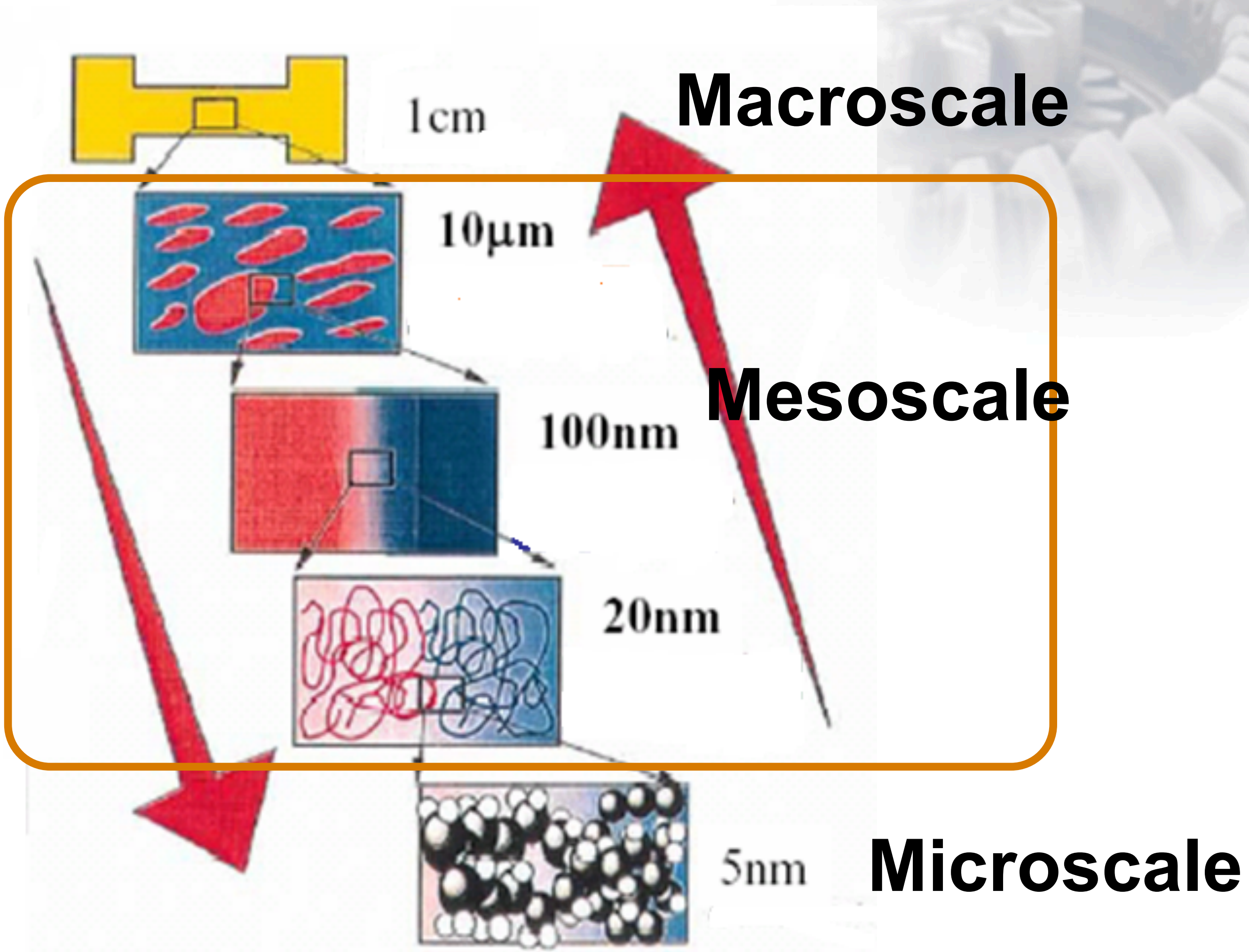
Have you ever experienced these problems in CFD

- Complicated Navier-Stokes equation
- Different governing equation for different flow phenomena.
- Lengthy program source code.
- Complicated boundary condition treatment.
- Very long simulation time.
- 100% computer software dependence, etc

Lattice Boltzmann method will solve your problem

Contents:

- What is lattice Boltzmann method (LBM)
- How LBM works
- Why we need LBM
- Simulation results
- Q & A



1cm
10μm
100nm
20nm
5nm
Macroscale
Mesoscale
Microscale

What is LBM

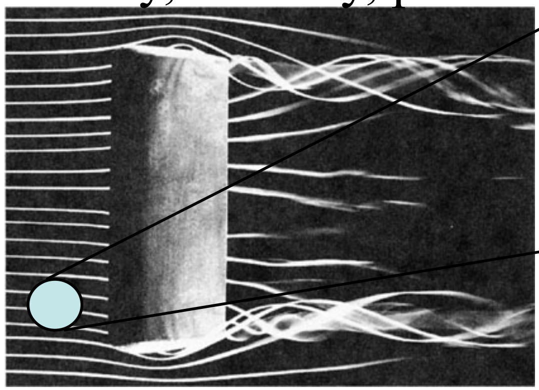
Statistical Mechanics

Density distribution function $f(\mathbf{x},t)$

Moment of distribution function

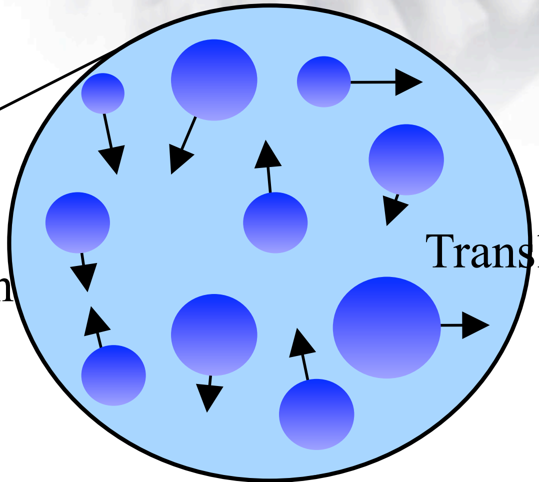
Macroscopic variables

Density, velocity, pressure, etc.



Homogeneous fluid

Collision



Translation

Top-down versus Bottom-up

Partial Differential Equations
(Navier-Stokes)

Discretization
Finite differences, finite element, finite volume or spectral methods

Difference Equations

Standard Numerical Methods

Simulate Fluid Flow

Simulate Fluid Flow

Partial differential Equations
(Navier-Stokes)

Multi-scale Analysis

Discrete Model
Lattice Boltzmann Equation

Lattice Boltzmann Approach

The primary goal of LB approach is to build a bridge between the microscopic and macroscopic dynamics rather than to dealt with macroscopic dynamics directly.

In other words, the goal is to derive macroscopic equations from microscopic dynamics by means of statistics rather than to solve macroscopic equations.

The Boltzmann Equation*

$$\underbrace{\frac{\partial f}{\partial t} + \mathbf{c}\frac{\partial f}{\partial \mathbf{x}}}_{\text{Advection term}} = \underbrace{\Omega(f)}_{\text{Collision term}}$$

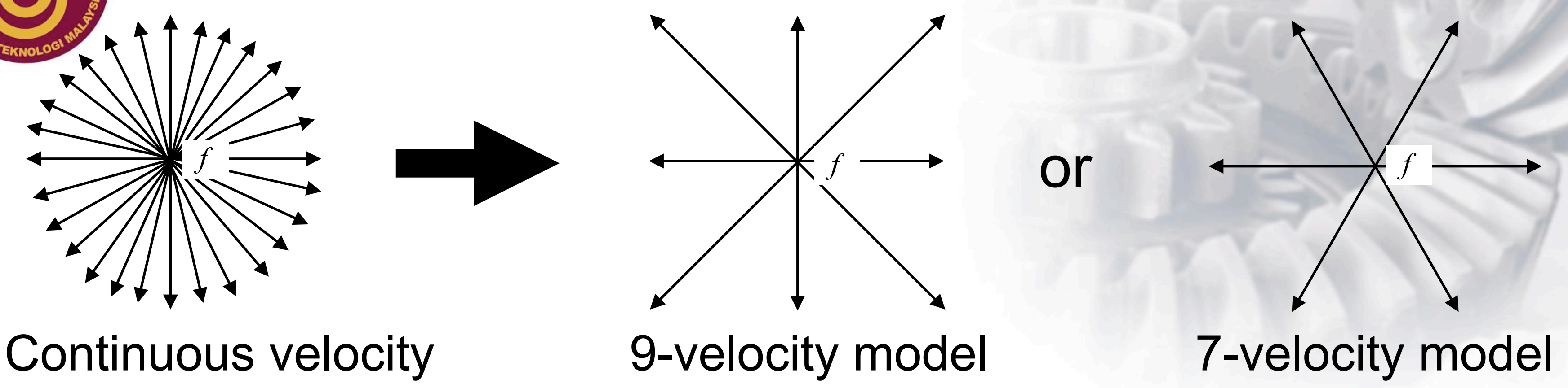
f : particle distribution function

$\mathbf{c}$: velocity of distribution function

Equation describes the evolution of groups of molecules

*H. Harris, *An introduction to the theory of the Boltzmann equation* (holt, Rinehart and Winston, New york, 1971)

Discrete velocity model



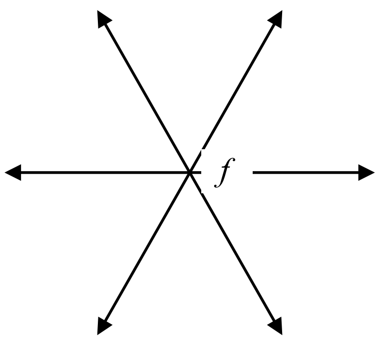
The direction of distribution function is limited to seven or nine directions

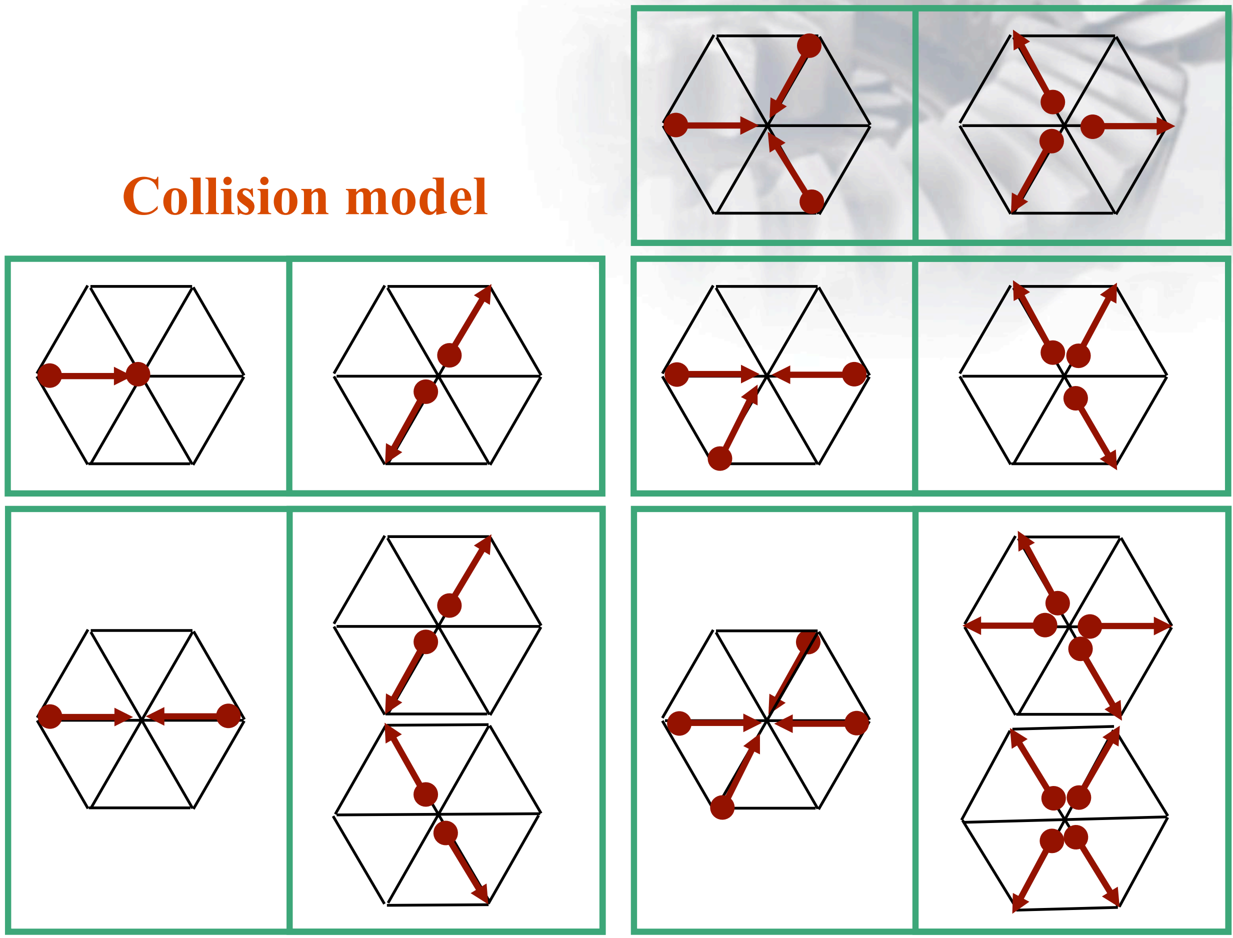
$$\frac{\partial f_i}{\partial t} + \mathbf{c}_i \frac{\partial f_i}{\partial \mathbf{x}} = \Omega(f_i)$$

$$i = 1,2,3,\ldots,9 \text{ or } 1,2,3,\ldots,7$$

Collision model

7-discrete velocity model





Bhatnagar-Gross-Krook(BGK) Collision model*

$$\Omega(f_i) = -\frac{f_i - f_i^{eq}}{\tau}$$

f_i^{eq} : equilibrium distribution function

τ : time relaxation

$\frac{1}{\tau} f_i^{neq}$ relax to equilibrium state during collision process

f_i^{neq} : nonequilibrium distribution function

*P. L. Bhatnagar, E. P. Gross and M. Krook, *Phys. Rev* **94** 511 (1954)

BGK Boltzmann equation

$$\frac{\partial f_i}{\partial t} + \mathbf{c}_i \frac{\partial f_i}{\partial \mathbf{x}} = -\frac{f_i - f_i^{eq}}{\tau}$$

Equilibrium distribution function (*Maxwell-Boltzmann distribution function**)

$$f_i^{eq} = \rho\omega_i\left[1 + 3(\mathbf{c}_i \cdot \mathbf{u}) + \frac{9}{2}(\mathbf{c}_i \cdot \mathbf{u})^2 - \frac{3}{2}\mathbf{u}^2\right]$$

Macroscopic parameters

Density

$$\rho = \sum_i f_i d\mathbf{c} = \sum_i f_i^{eq} d\mathbf{c}$$

Velocity

$$\mathbf{u} = \sum_i \mathbf{c} f_i d\mathbf{c} = \sum_i \mathbf{c} f_i^{eq} d\mathbf{c}$$

Macroscopic Equations

Macroscopic equations (continuity and N-S equation) are obtained using Chapmann-Enskog expansion procedure

$\nabla \cdot \mathbf{u} = 0$ continuity equation derived from BGK equation

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{\nabla P}{\rho} + \frac{2\tau - 1}{6} \nabla^2 \mathbf{u}$$

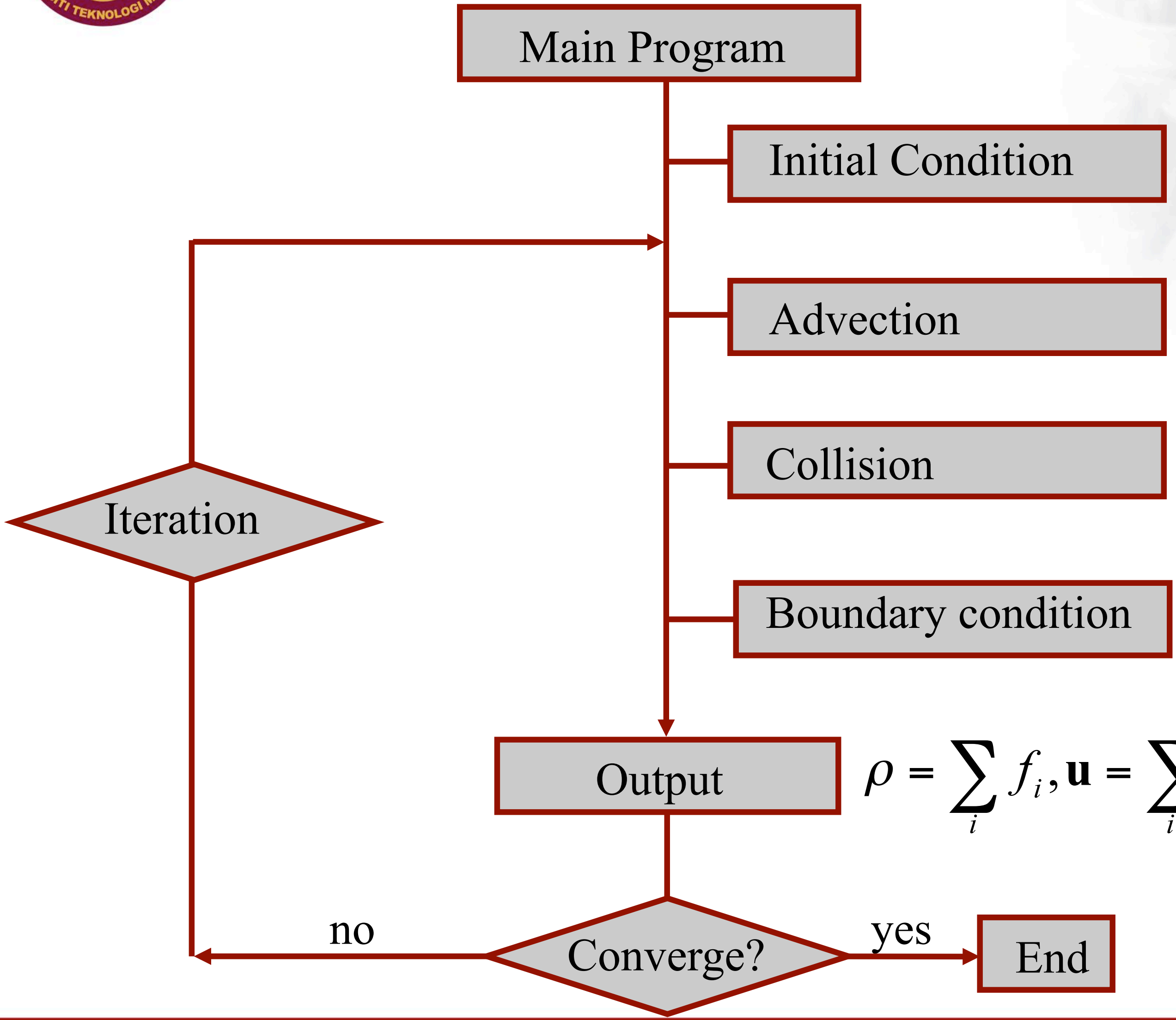
Momentum equation derived from BGK equation

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{\nabla P}{\rho} + \upsilon \nabla^2 \mathbf{u}$$

Momentum equation derived Newton's second law

Relation between time relaxation and fluid viscosity $\upsilon = \frac{2\tau - 1}{6}$

Simulation algorithm



from ρ, u, v
calculate f_i^{eq}
$f_i = f_i^{eq}$

$$f_i(\mathbf{x}, t) = f_i(\mathbf{x} + \Delta\mathbf{x}, t + \Delta t)$$

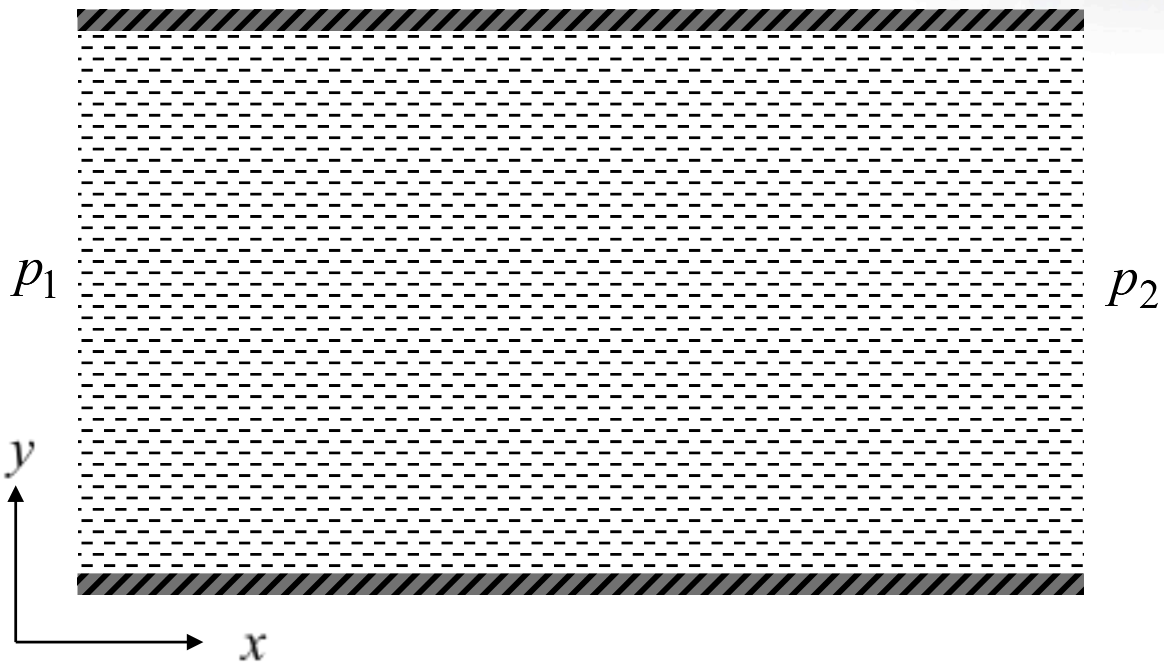
$$f_i = -\frac{1}{\tau}\left(f_i - f_i^{eq}\right)$$

$$\rho = \sum_i f_i, \mathbf{u} = \sum_i \mathbf{c} f_i \Big/ \sum_i f_i$$

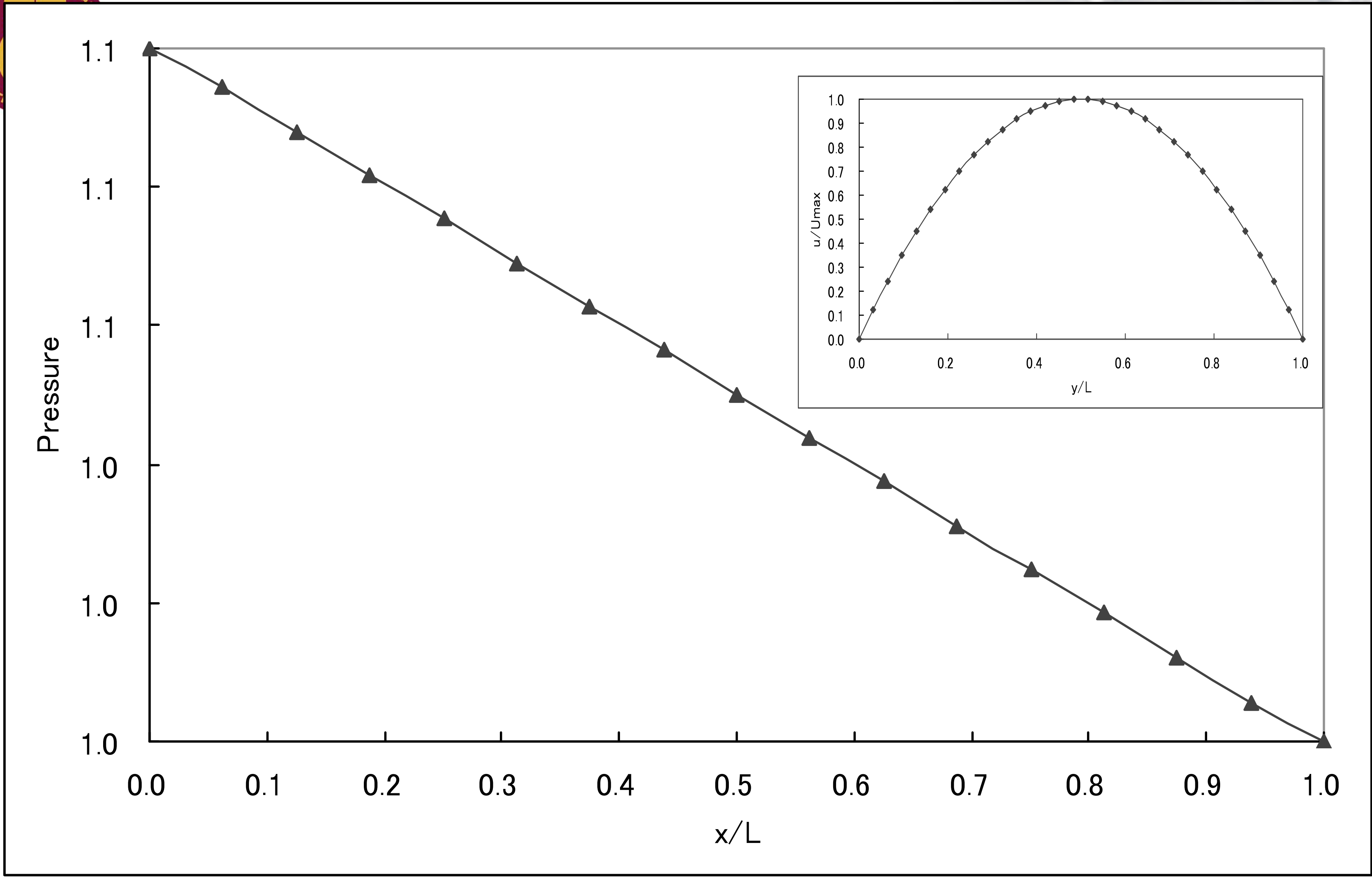
Numerical Test: Poiseulli Flow

Initial conditions

p_1	p_2	ρ	u	v	τ	**mesh**
1.1	1.0	1.0	0.0	0.0	1.0	180 x 30



Flow is driven by pressure gradient at inlet and exit channel $p_1 > p_2$

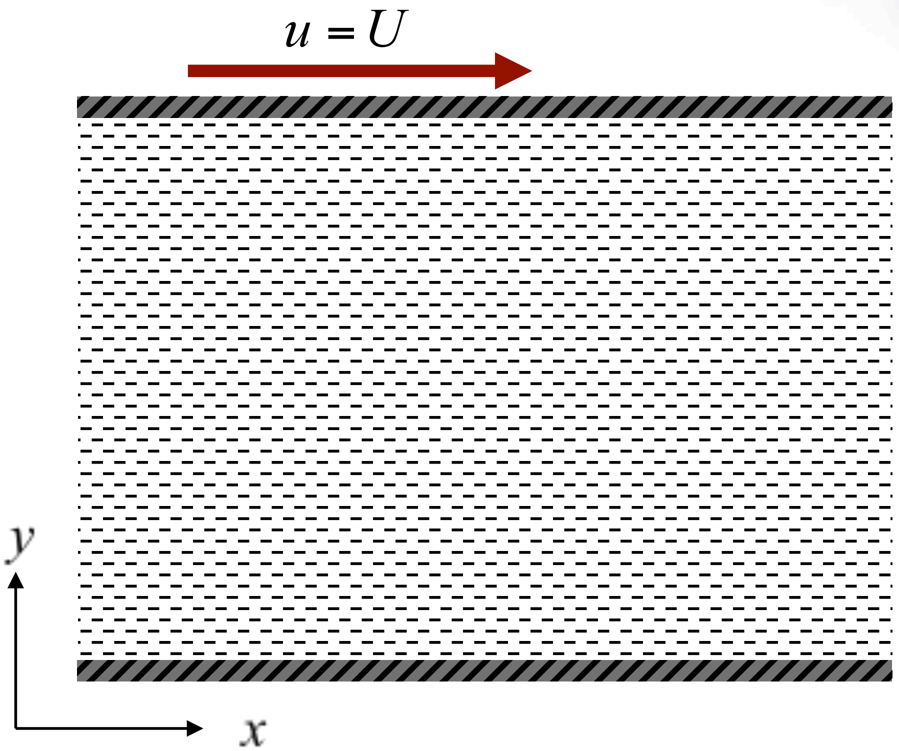


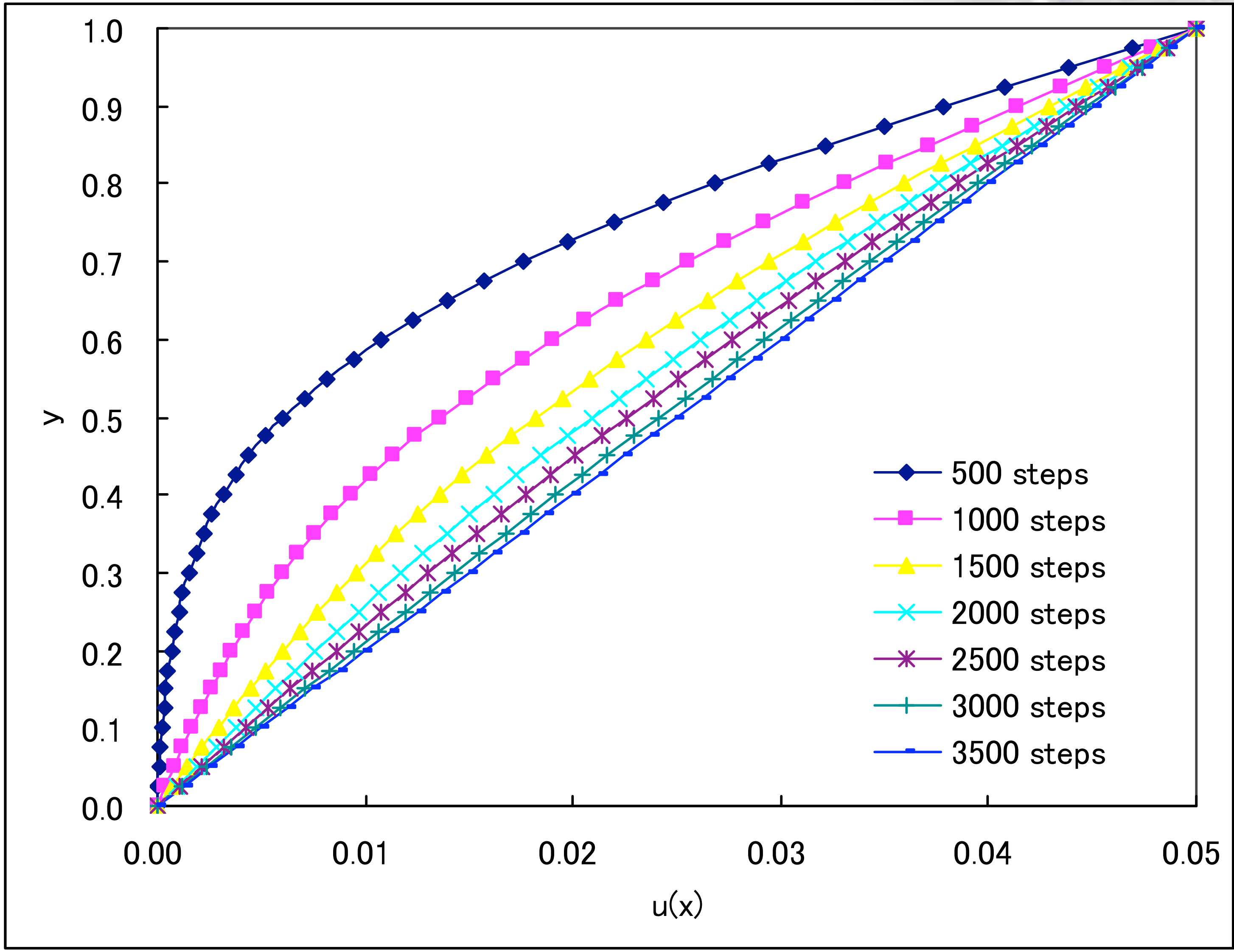
Graph: Centerline density profile along the channel. The inset displays the velocity profile across the channel at the steady state.

Numerical Test: Couette Flow

Initial conditions

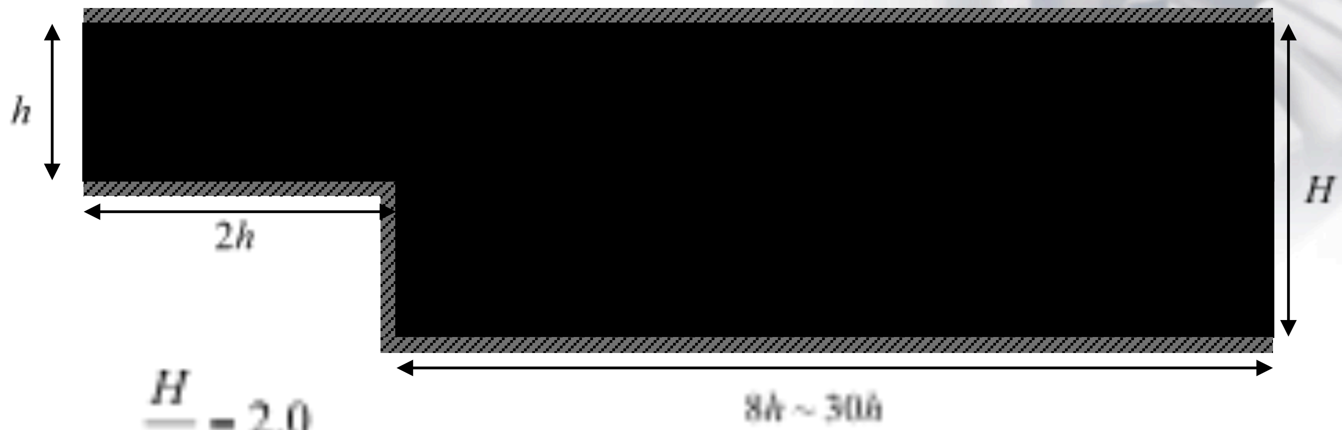
ρ	τ	U	u	v	**mesh**
1.0	1.0	0.05	0.0	0.0	80 x 40





Graph: Velocity profiles across the normalized channel width at different times.

Numerical Test: Back-Step flow



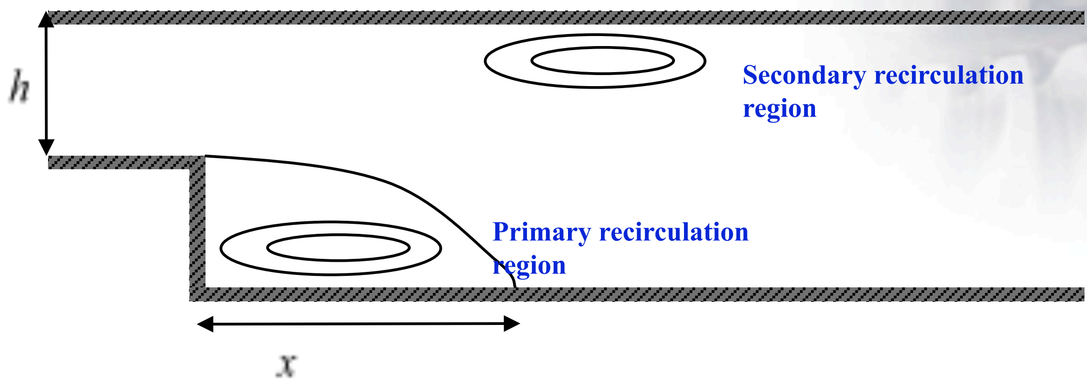
Case 1: $\frac{H}{h} = 2.0$

Case 2: $\frac{H}{h} = 2.5$

$h = 20$ lattice mesh

Case 3: $\frac{H}{h} = 3.0$

Predicted result



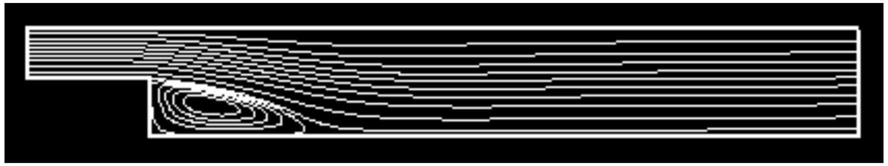
Reference

１．Armaly, Durst, Peireira, Schonung, 1983, *Experimental and theoretical investigation of backward-facing step flow*, J. Fluid Mechanics, 127, pp 473.

２．Biswas, Breuer, Durst, 2004, *Backward-facing step flows for various expansion ratios at low and moderate Reynolds numbers*, J. Fluids Engineering, 126, pp 362.

Streamline plot for expansion ratio 2.0

Re=50



Re=100

Re=300

Re=600

Time development of streamline plot for expansion ratio 2.0 at Reynolds number 300

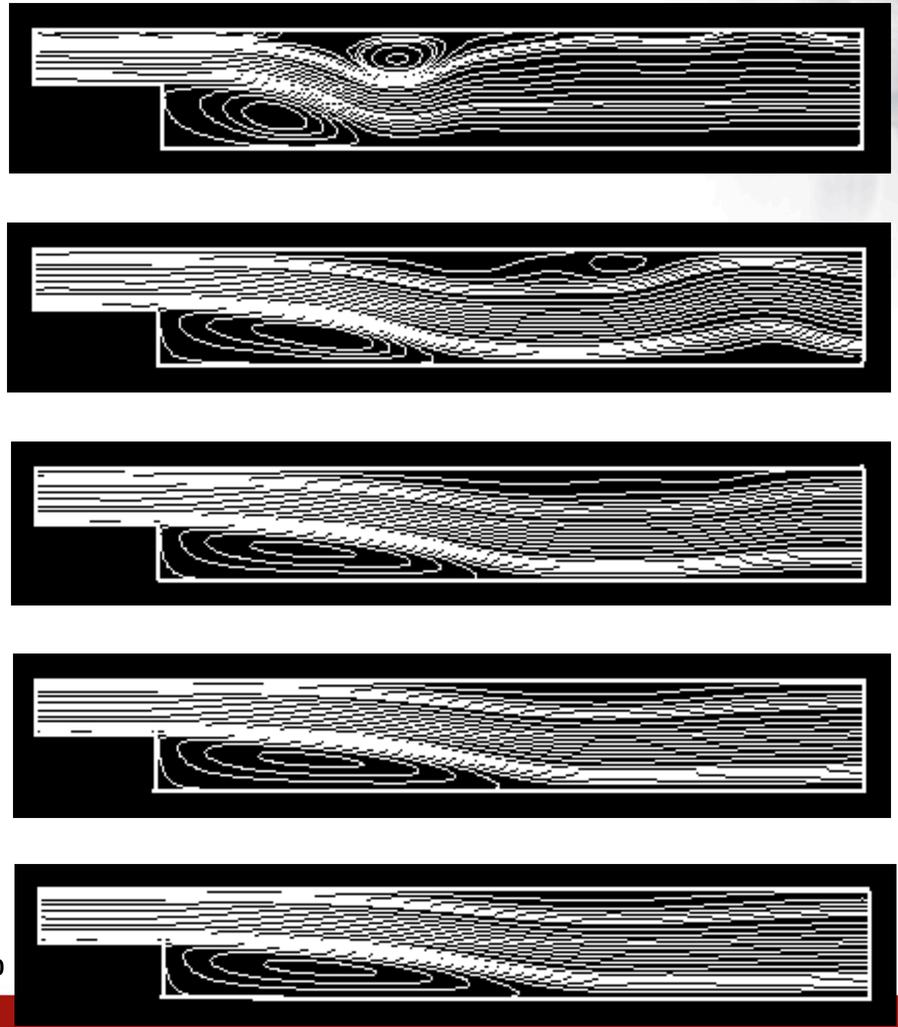
Iteration = 2000

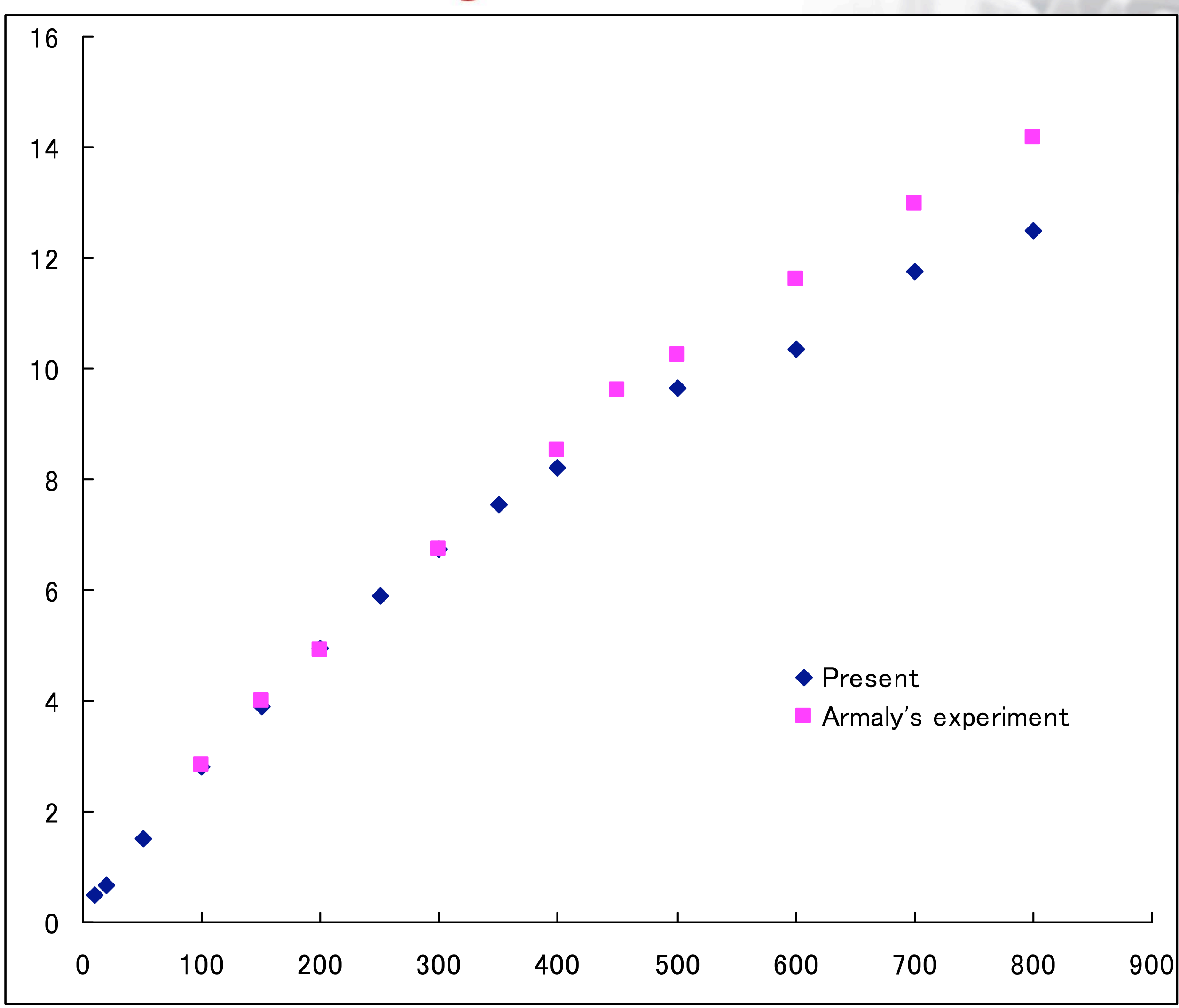
Iteration = 4000

Iteration = 6000

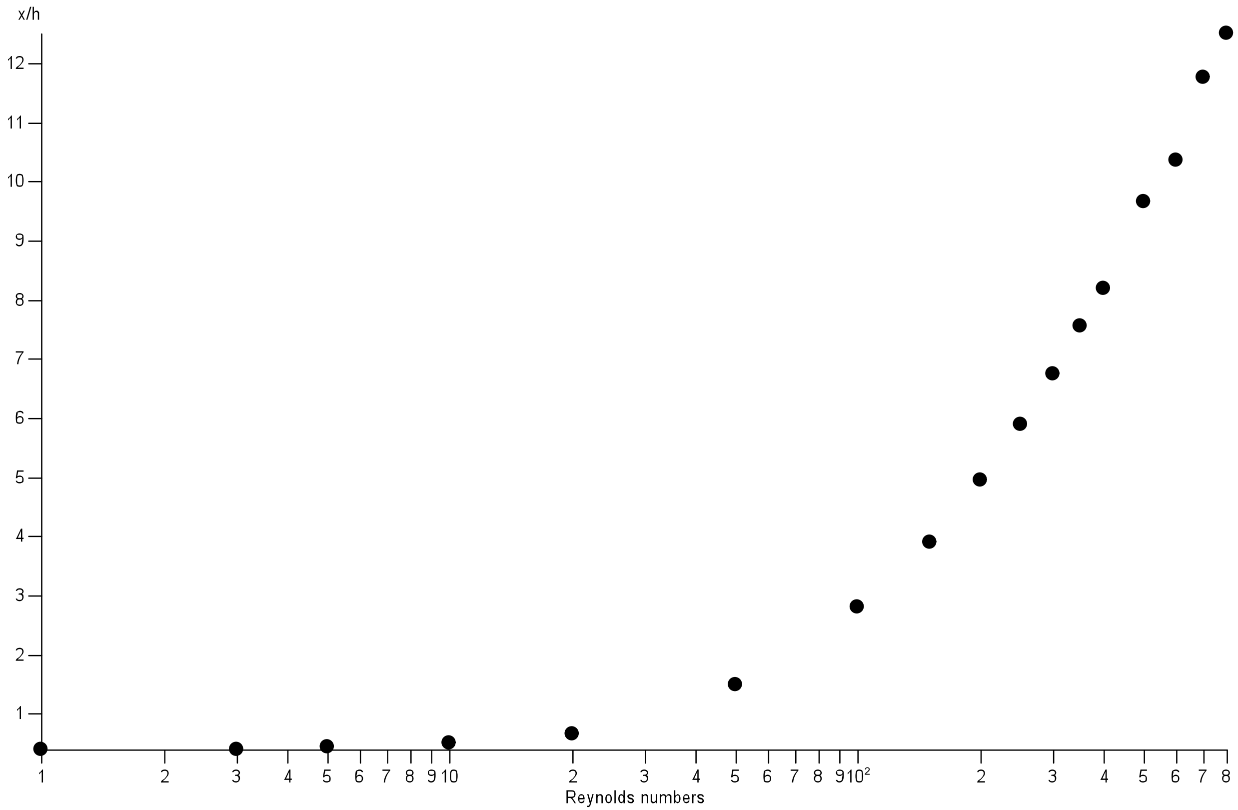
Iteration = 8000

Iteration = 10000





Graph shows the data obtained by Armaly and Lattice Boltzmann simulation for expansion ratio H/h = 2.0

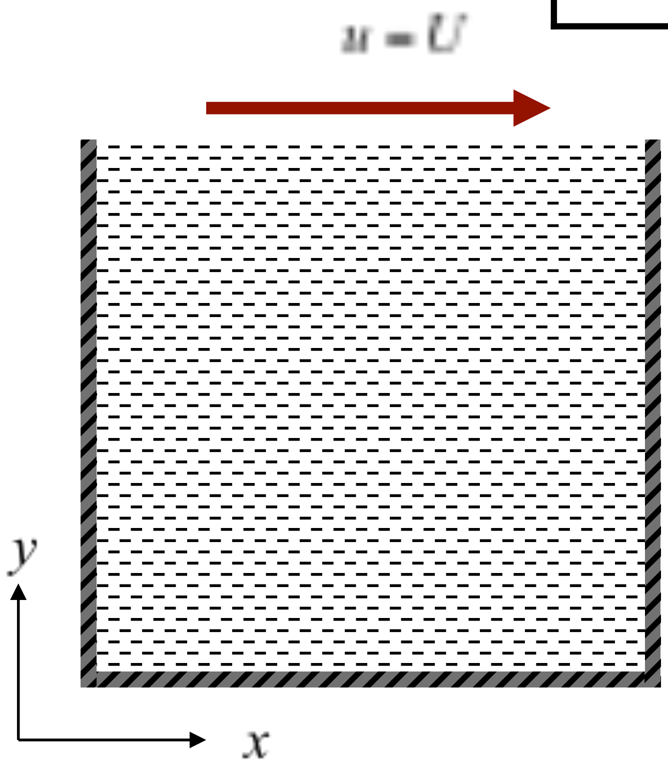


Graph shows the relationship between reattachment and Reynolds numbers for expansion ratio 2.0

Numerical Test: Lid-driven Cavity Flow

Initial conditions

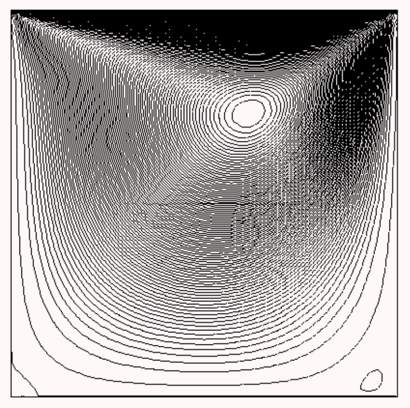
ρ	U	u	v
1.0	0.1	0.0	0.0



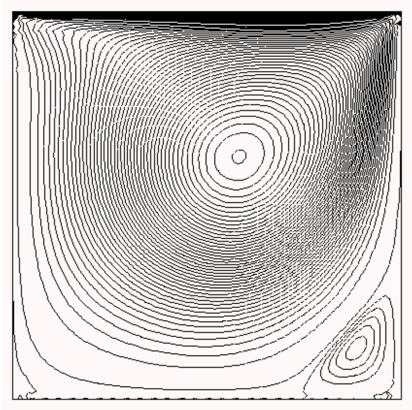
$$Re = \frac{UL}{\upsilon}$$

$$\tau = 3\upsilon + 0.5$$

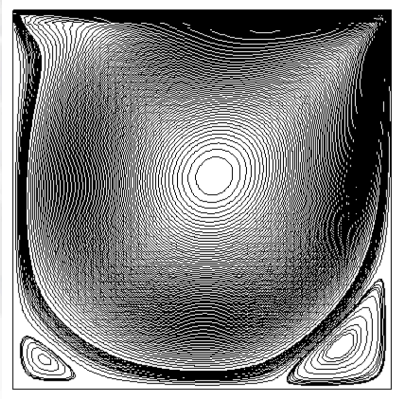
Re	Mesh	τ
100	45 x 45	0.635
400	45 x 45	0.511
1000	45 x 45	0.505
~~3200~~	~~45 x 45~~	~~0.504~~
3200	101 x 101	0.509



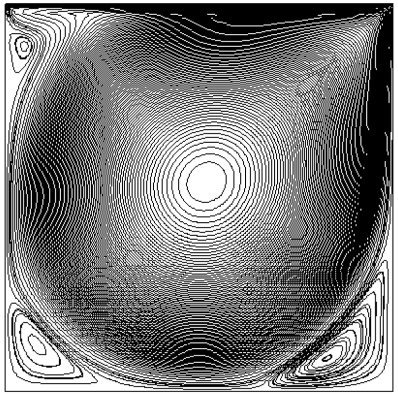
Re = 100



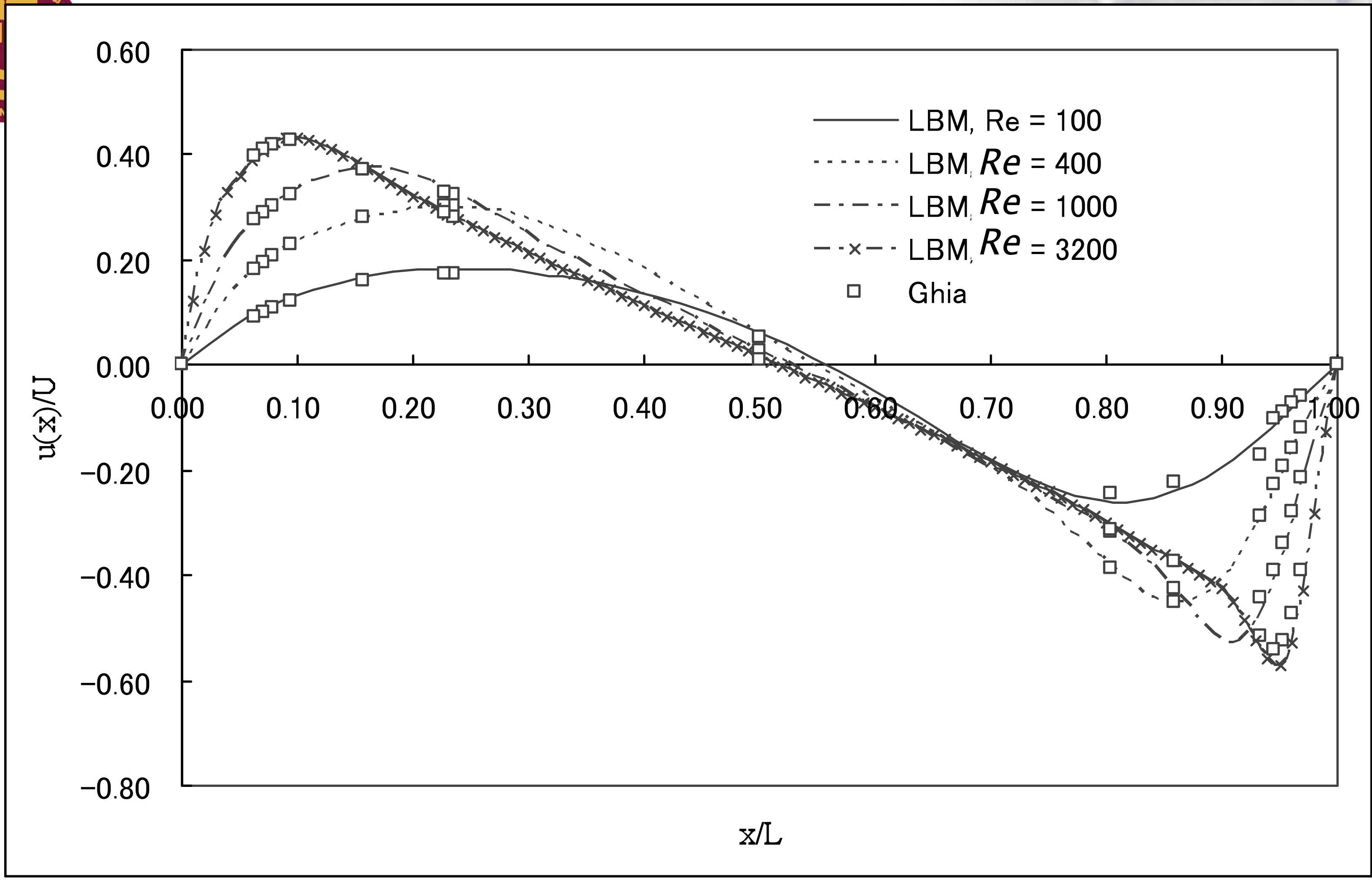
Re = 400



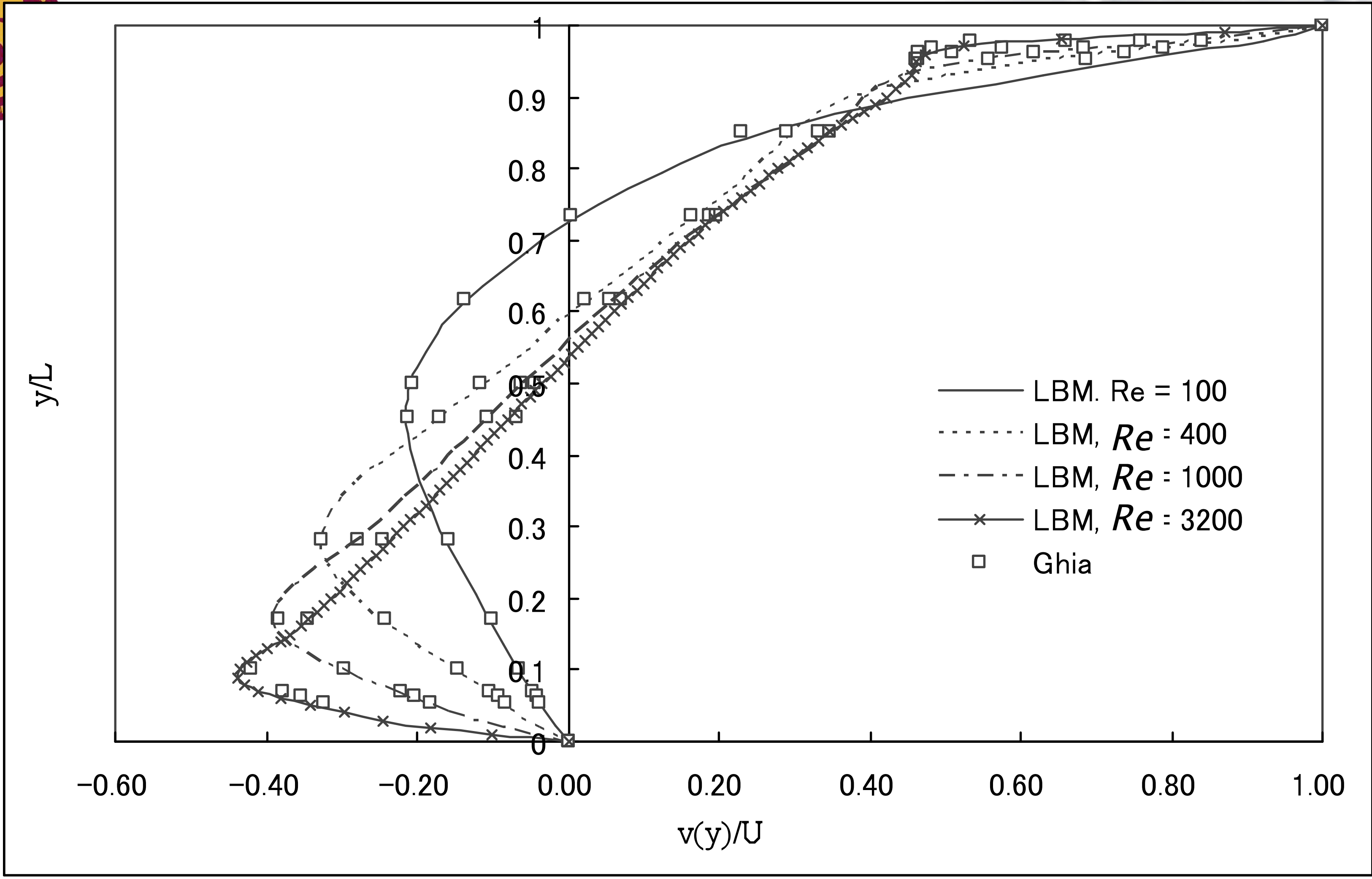
Re = 1000



Re = 3200

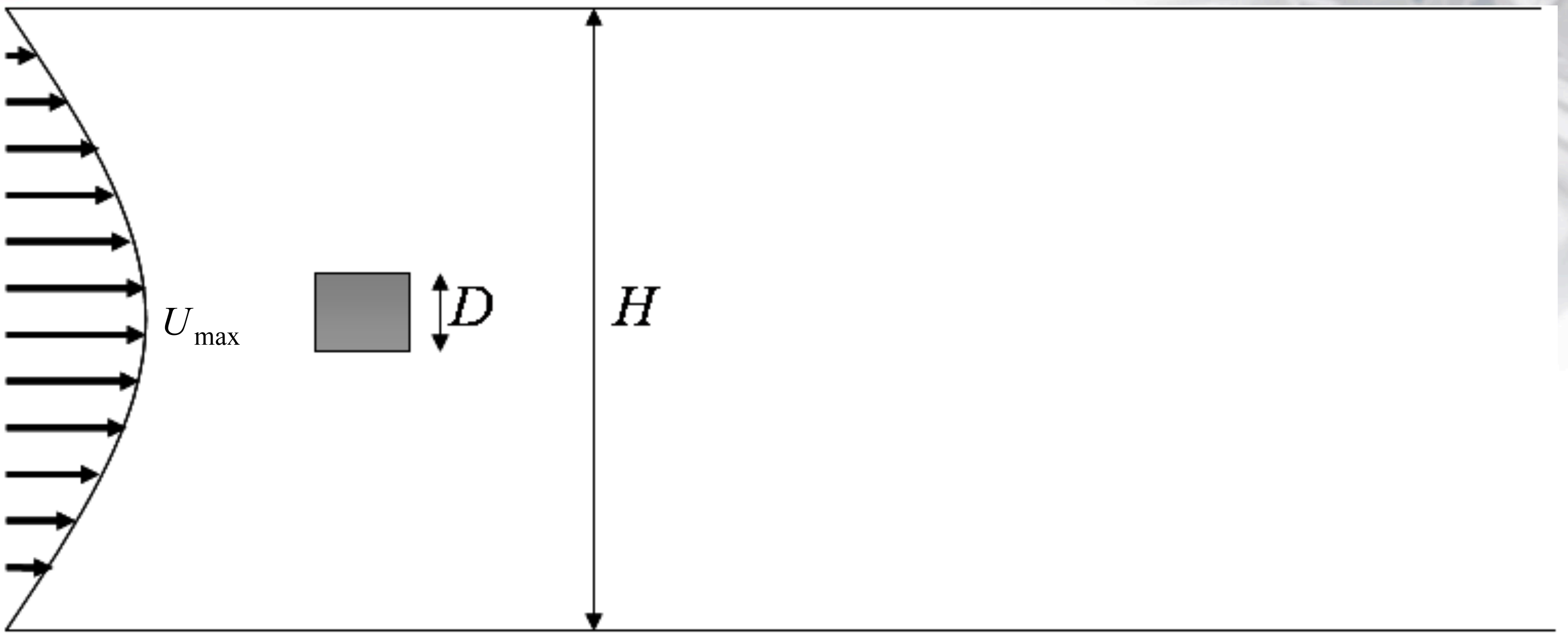


Graph: Plots of normalized vertical velocity.



Graph: Plots of normalized horizontal velocity.

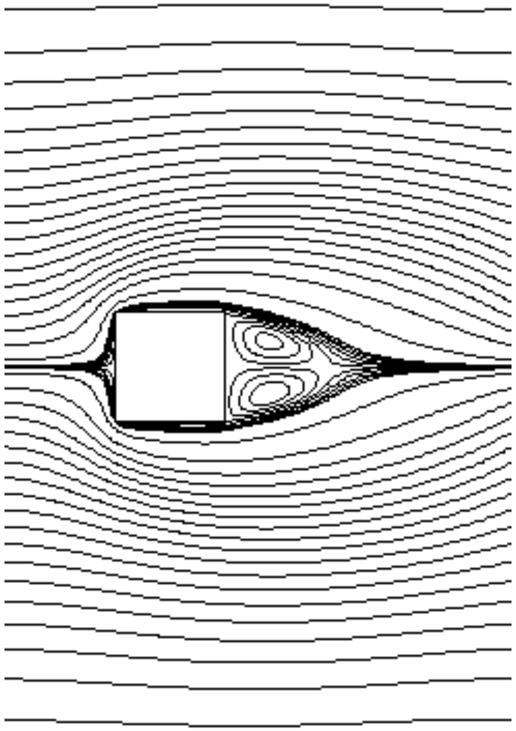
Numerical Test: Flow around rectangular cylinder



Blockage ratio: $\dfrac{D}{H} = 0.125$

Reynolds number: $\mathrm{Re} = \dfrac{U_{max} D}{\upsilon}$

i) Length of trailing recirculation



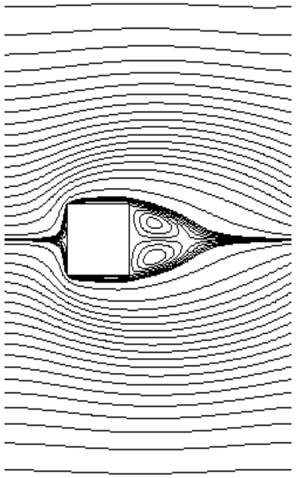
M. Breuer, Intl J. Heat and Fluid Flow, 21 (2000) pp. 186

$$\frac{L_r}{D} = -0.065 + 0.0554\,\mathrm{Re}$$

Present simulation

$$\frac{l_r}{D} = -0.2668 + 0.0692\,\mathrm{Re}$$

ii) Critical Reynolds number



M. Breuer, Intl J. Heat and Fluid Flow, 21 (2000) pp. 186

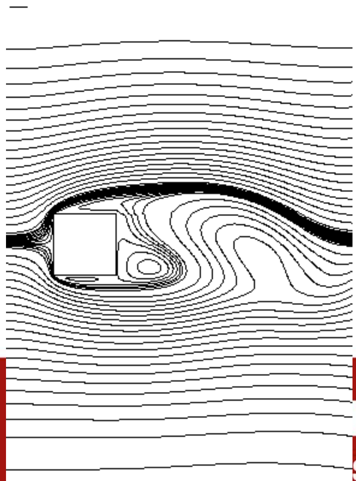
$$\mathrm{Re}_c = 60$$

A. Okajima, J. Fluid Mechanics, 33 (1990) pp. 171

$$\mathrm{Re}_c = 70$$

K. M. Klekar and S. V. Patankar, Intl. J. Numer. Method Fluid, 14, (1992) pp. 327

$$\mathrm{Re}_c = 54$$



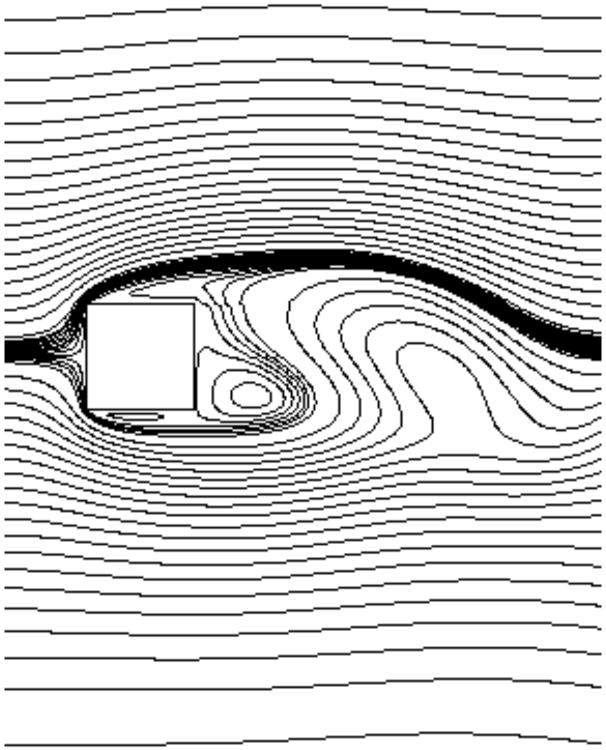
W. B. Guo et al, Chinese Physics ,12 (2003) pp.67

$$\mathrm{Re}_c = 60 - 62$$

Present simulation

$$\mathrm{Re}_c = 53$$

iii) Leading edge separation



M. Breuer, Intl J. Heat and Fluid Flow, 21 (2000) pp. 186 $\mathrm{Re} = 100 - 150$

W. B. Guo et al, Chinese Physics ,12 (2003) pp.67 $\mathrm{Re} = 133$

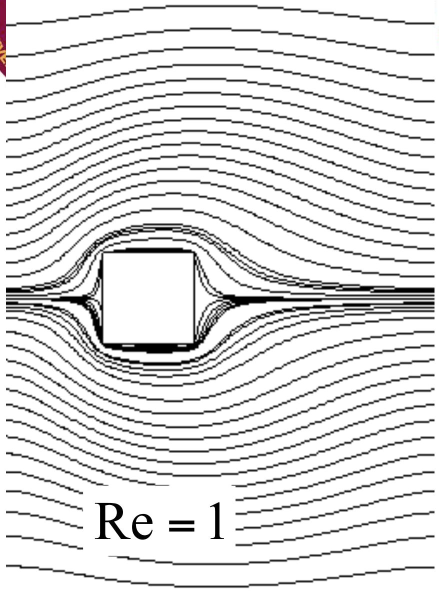
K. M. Klekar and S. V. Patankar, Intl. J. Numer. Method Fluid, 14, (1992) pp. 327

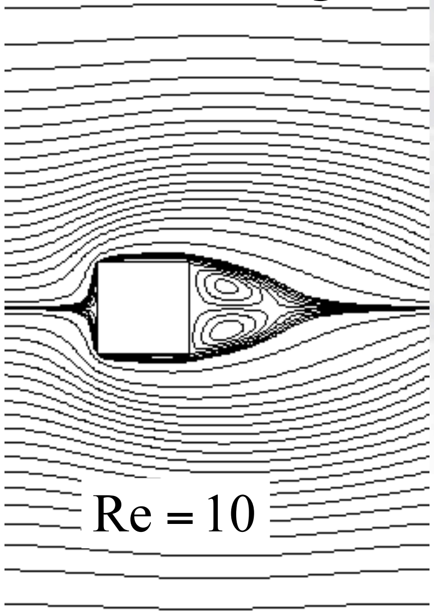
$\mathrm{Re} = 100 - 150$

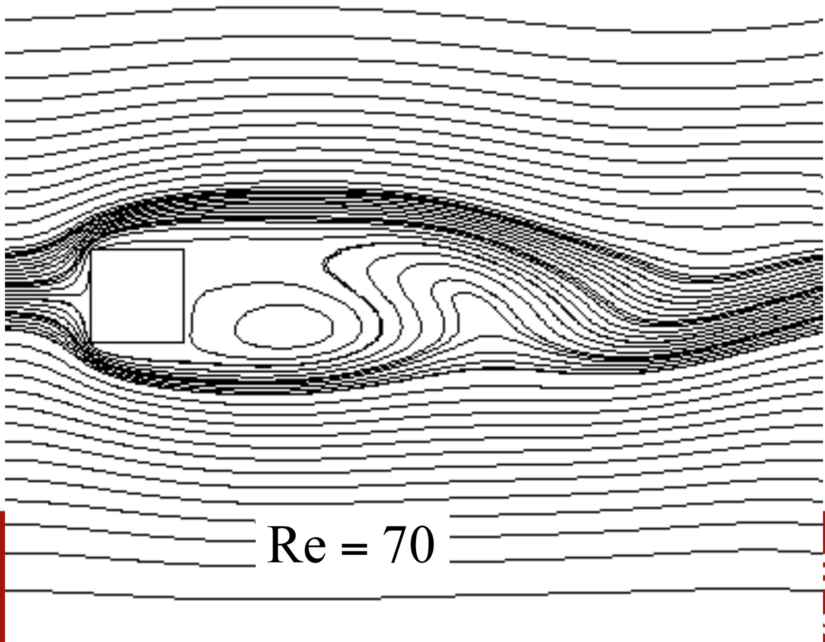
Present simulaion

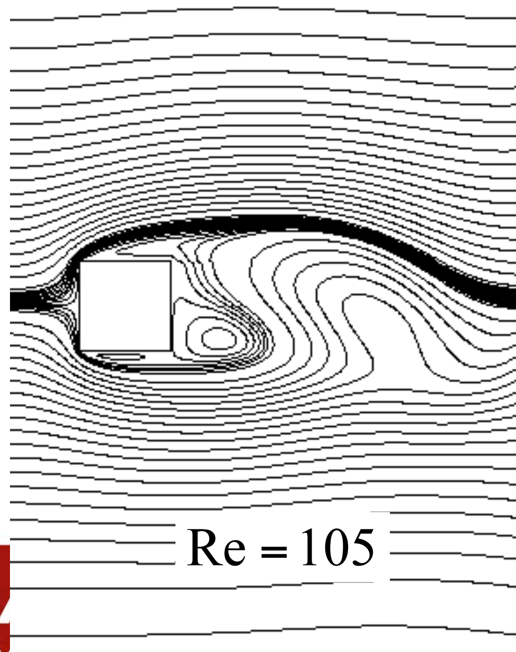
$\mathrm{Re} = 103$

Flow pattern for various Reynolds numbers





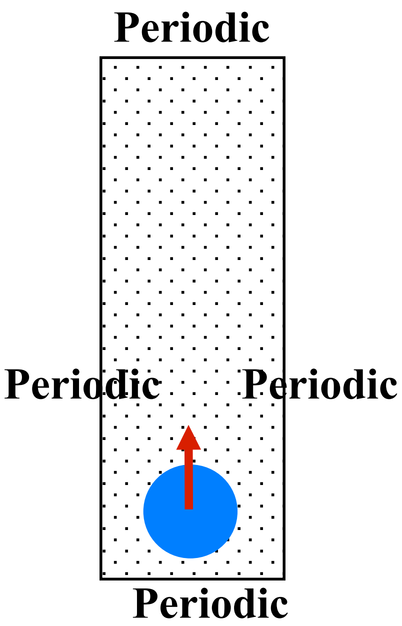




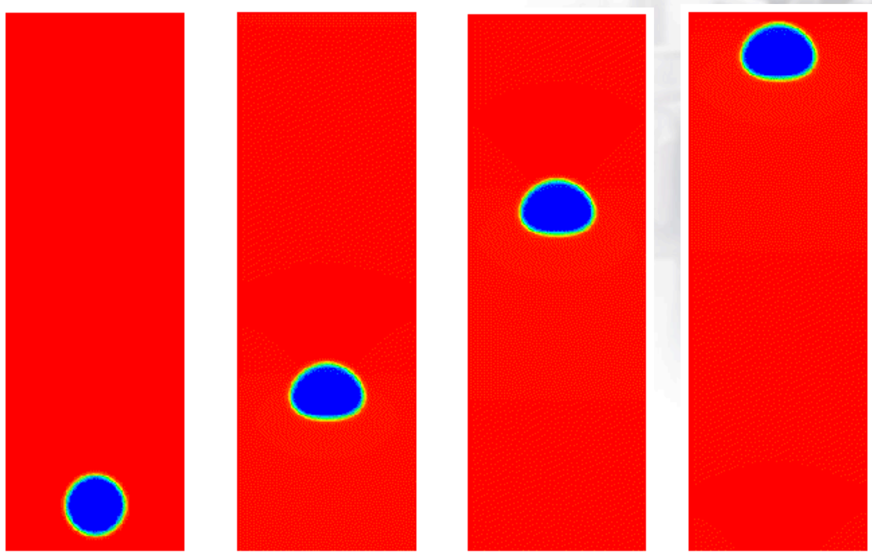
Multiphase flow

Bubble rises

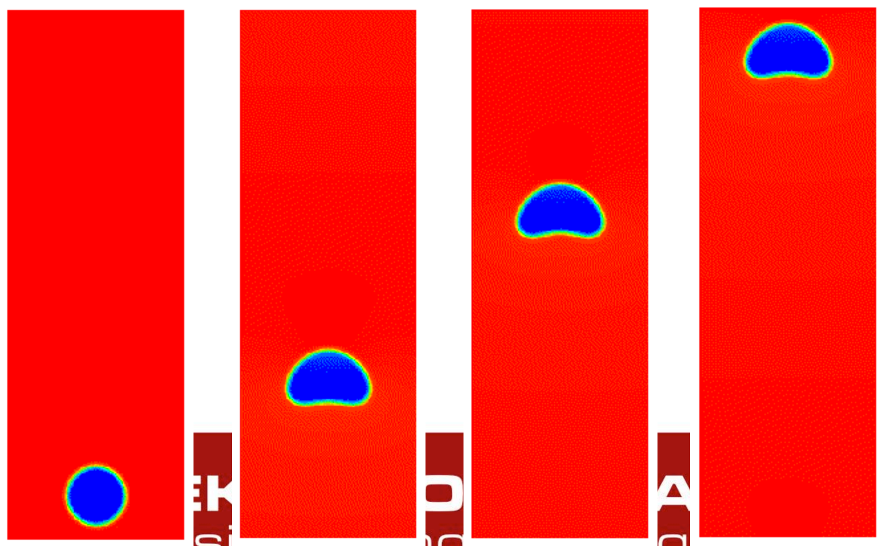
Model for simulation



Morton Number , $Mo = 10$



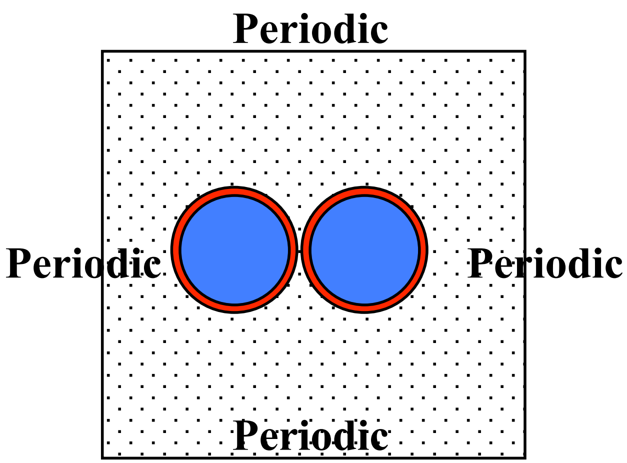
Morton Number , $Mo = 20$

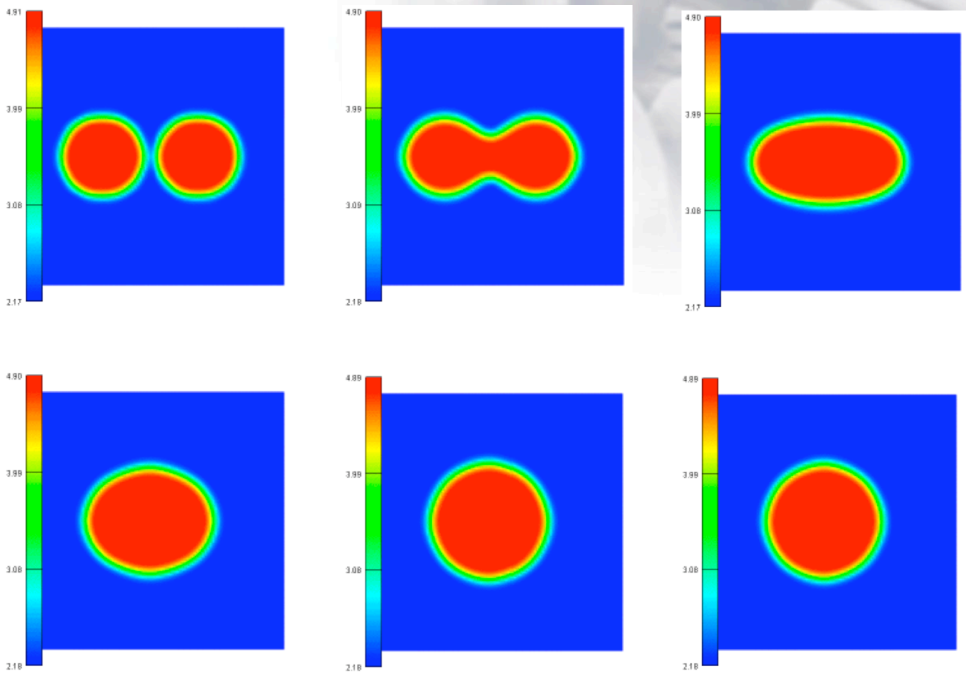


Multiphase flow

Bubbles Coalesce

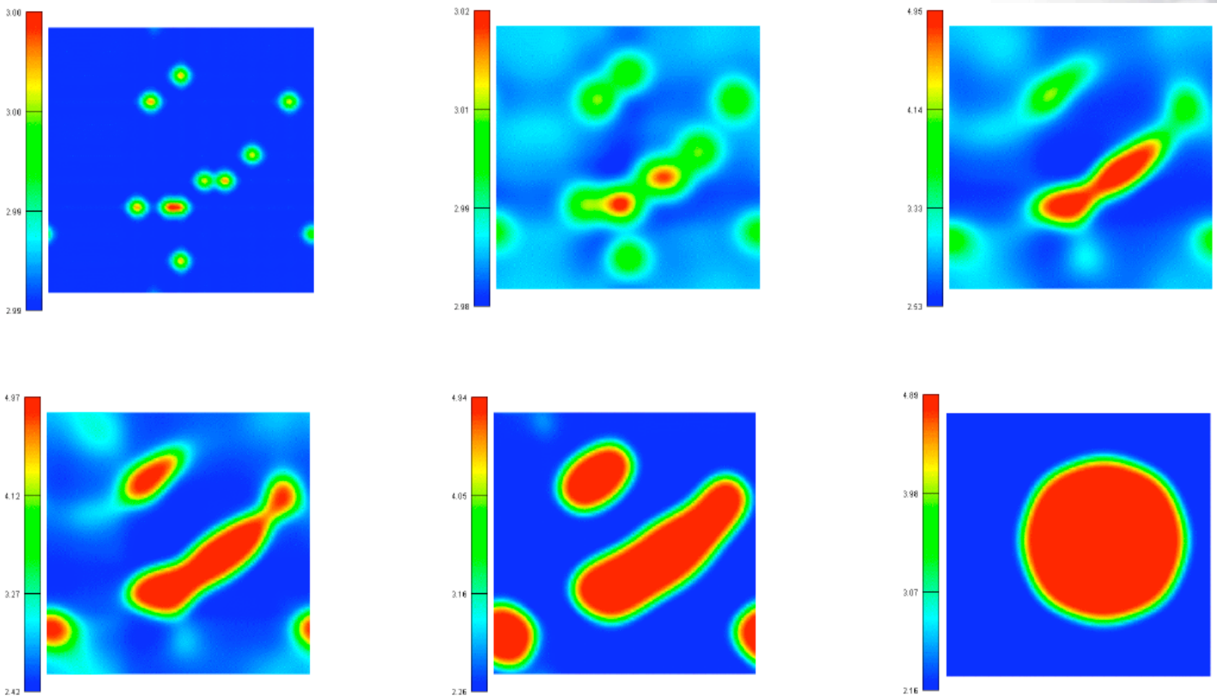
Model for simulation





Multiphase flow

Phase separation



Thermal BGK Boltzmann equation

Energy distribution function to calculate temperature

$$\frac{\partial g_i}{\partial t} + \mathbf{c}_i \frac{\partial g_i}{\partial \mathbf{x}} = -\frac{g_i - g_i^{eq}}{\tau_g}$$

Equilibrium energy distribution function

$$g_i^{eq} = \rho T \omega_i \left[1 + 3(\mathbf{c}_i \cdot \mathbf{u})\right]$$

Macroscopic parameters

Density

$$\rho = \sum_i f_i d\mathbf{c} = \sum_i f_i^{eq} d\mathbf{c}$$

Velocity

$$\mathbf{u} = \sum_i \mathbf{c} f_i d\mathbf{c} = \sum_i \mathbf{c} f_i^{eq} d\mathbf{c}$$

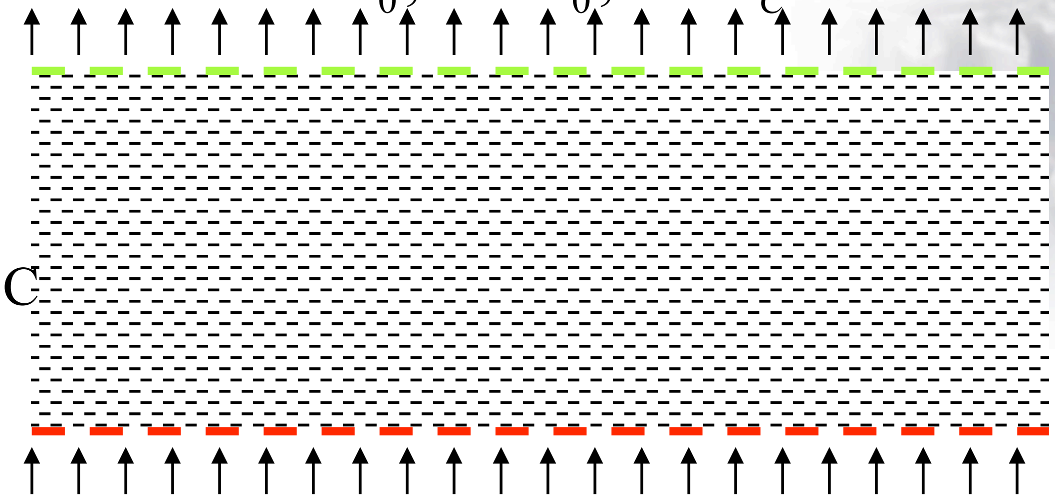
Temperature

$$T = \sum_i g_i d\mathbf{c} = \sum_i g_i^{eq} d\mathbf{c}$$

Numerical Test: Thermal Porous Couette Flow

$U = U_0, V = V_0, T = T_C$

Periodic B.C



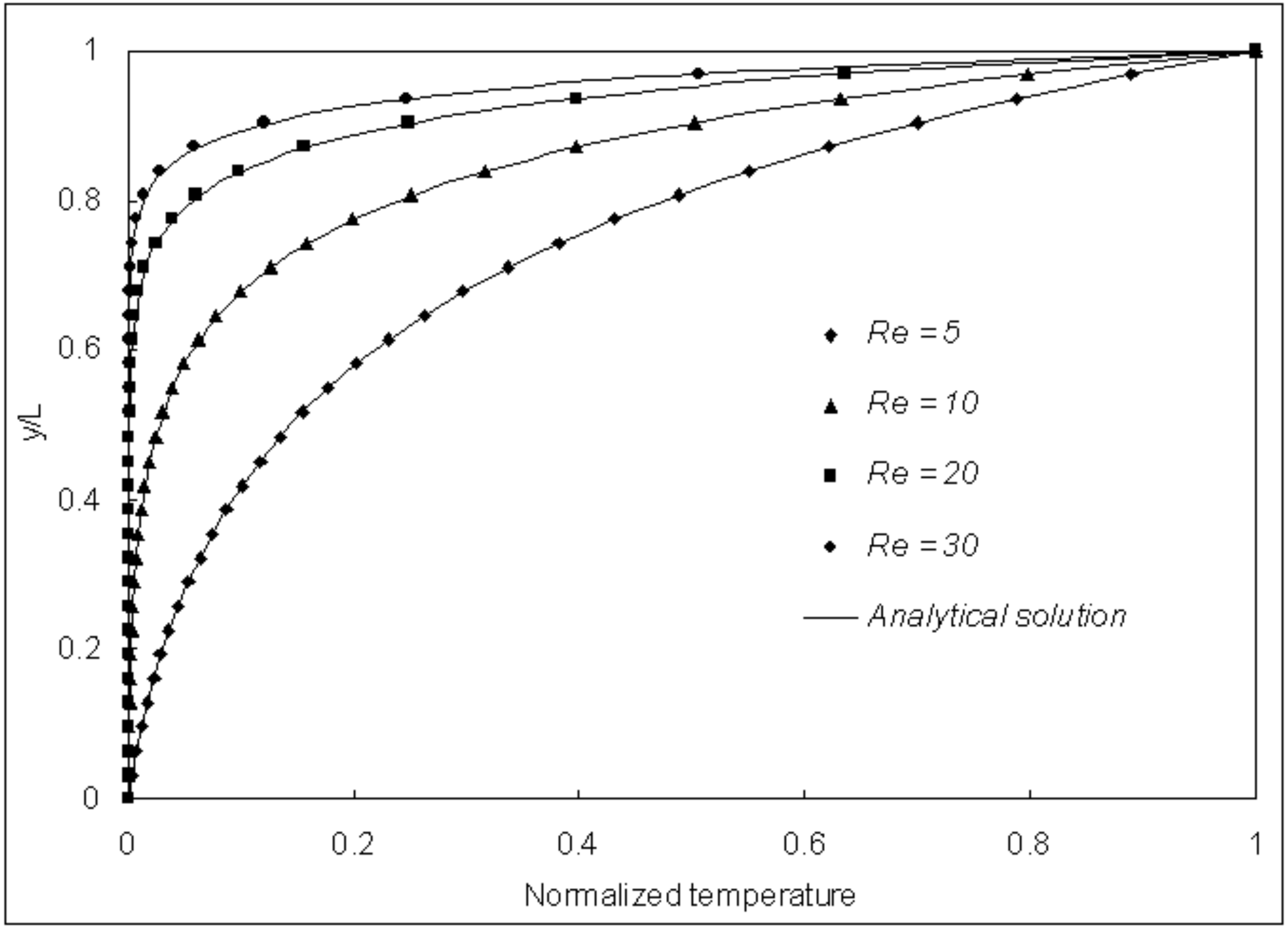
Periodic B.C

$U = 0, V = V_0, T = T_H$

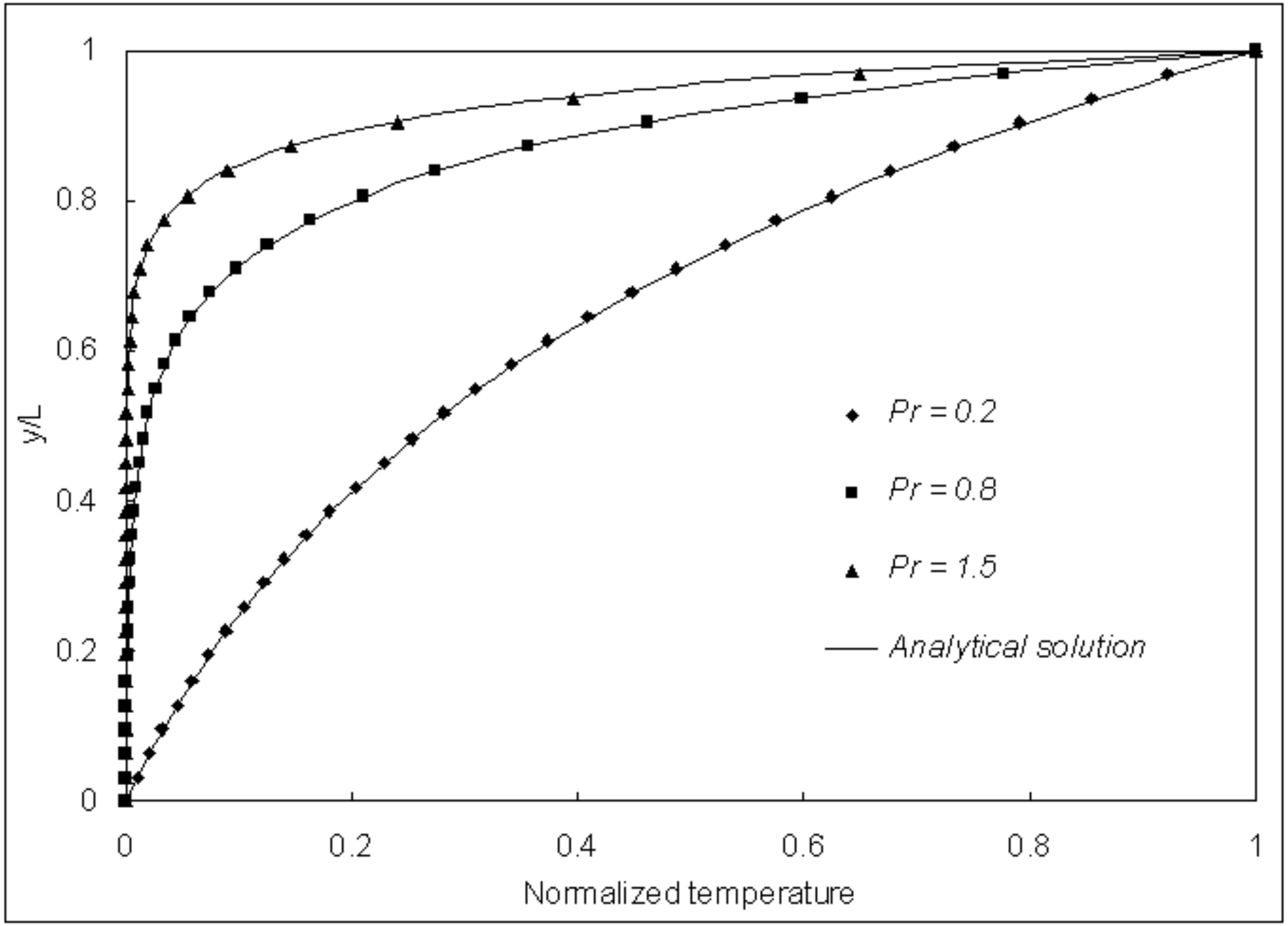
Figure: Simulation model for Couette flow

Analytical solution

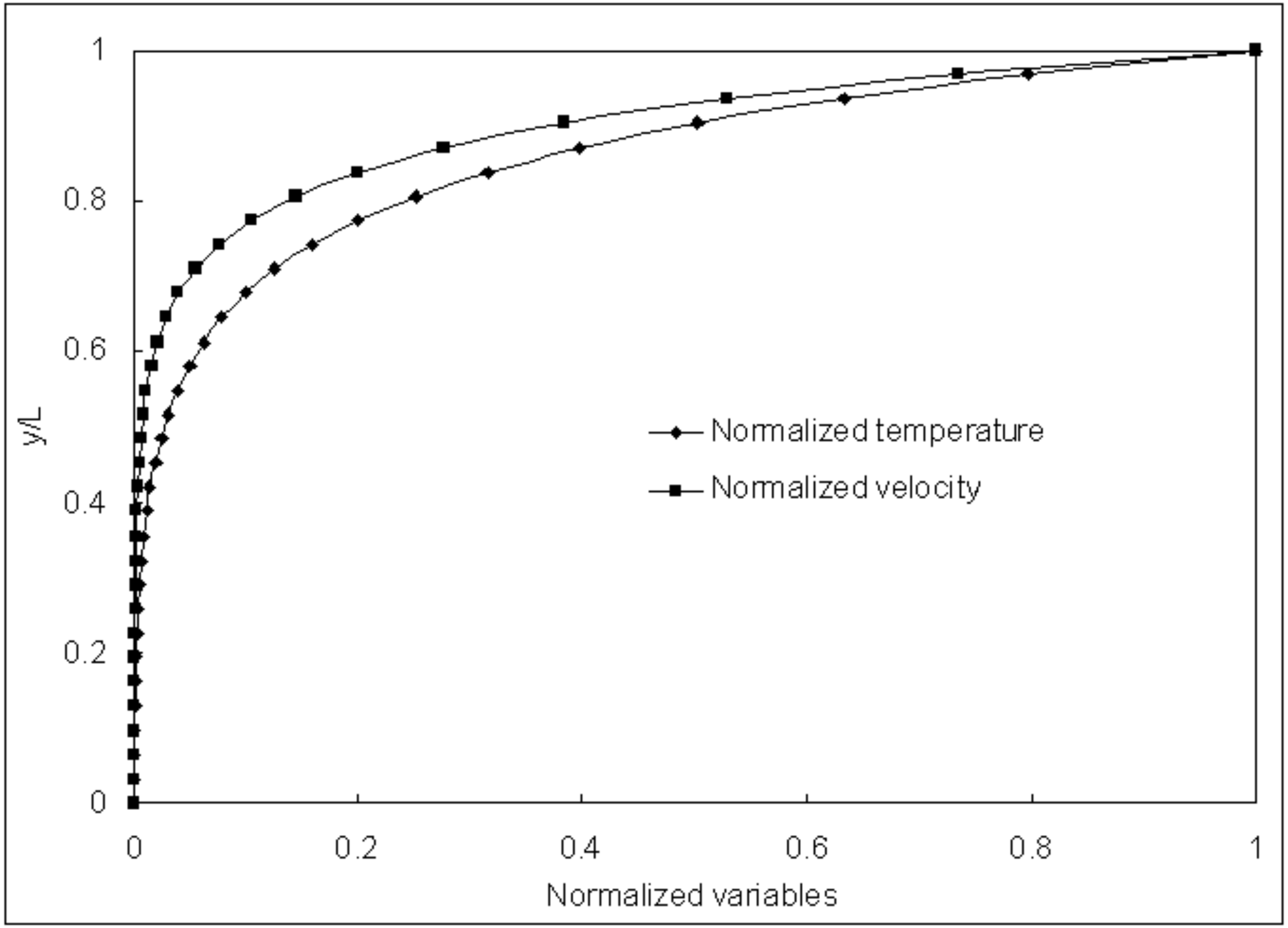
$$u = U_0 \left(\frac{e^{(\mathrm{Re}\, y/\mathrm{L})} - 1}{e^{\mathrm{Re}} - 1} \right), \quad T = T_C + \Delta T \left(\frac{e^{(\mathrm{Pr}\,\mathrm{Re}\, y/\mathrm{L})} - 1}{e^{\mathrm{Pr}\,\mathrm{Re}} - 1} \right)$$



Graph: Temperature profile at $Pr = 0.71$ and $Ra = 100$



Graph: Temperature profile at $Re = 10$ and $Ra = 100$

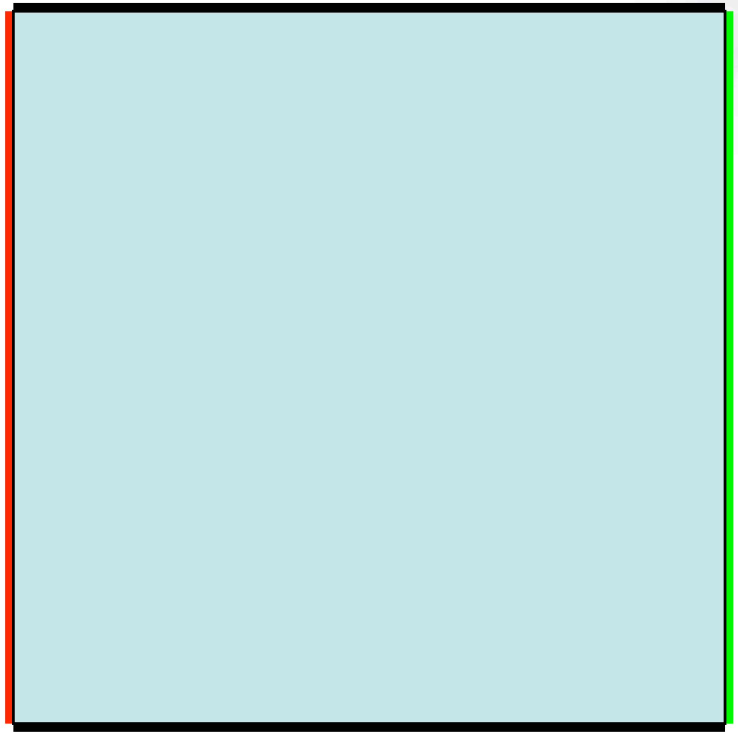


Graph: Temperature profile at $Re = 10, Pr = 0.71$ and $Ra = 60000$

Numerical Test: Natural Convection in a Square Cavity

$$\frac{\partial T}{\partial y} = 0$$

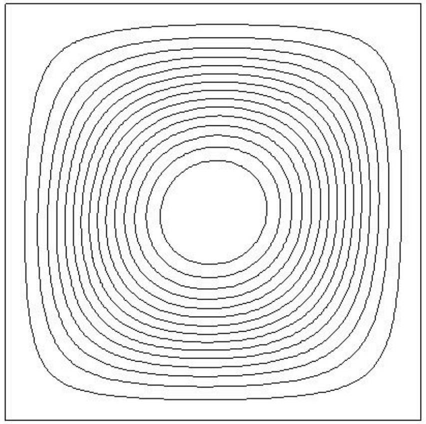
$T = T_H$



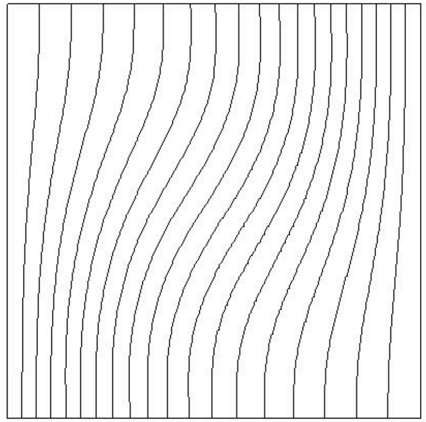
$T = T_C$

$$\frac{\partial T}{\partial y} = 0$$

$Ra = 10^3$



Streamline



Isotherms

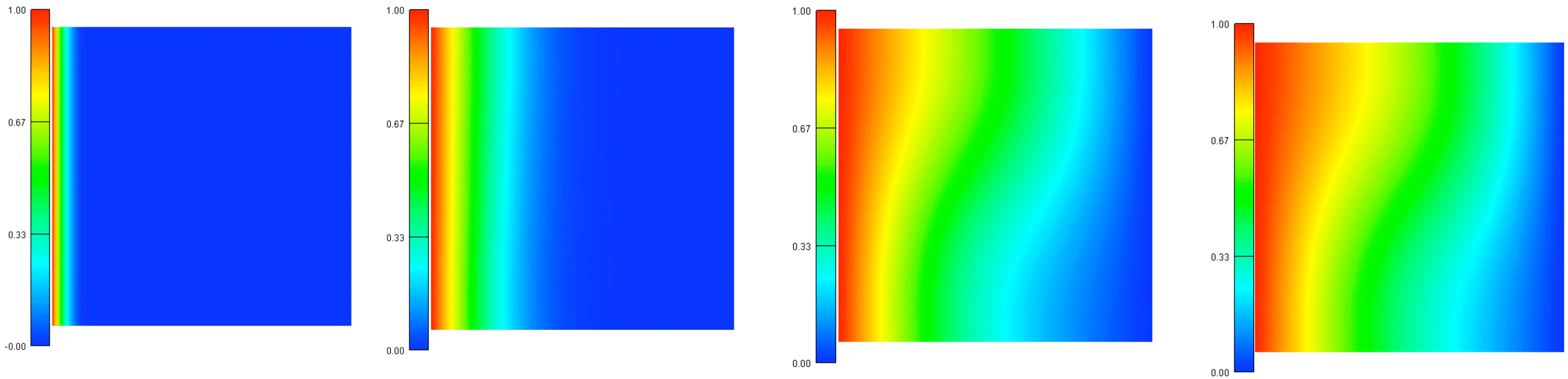
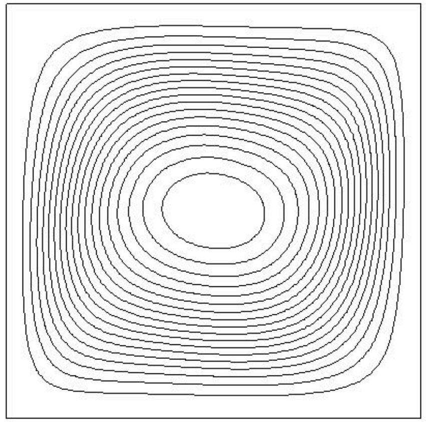
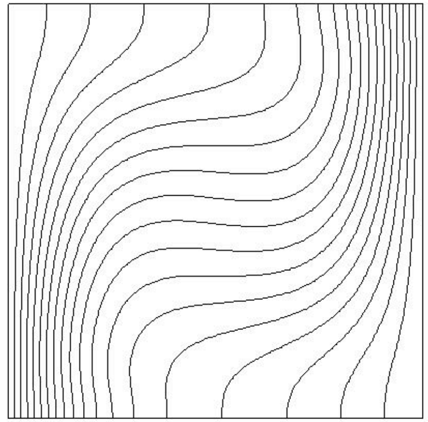


Figure: Animation of temperature

$$Ra = 10^4$$



Streamline



Isotherms

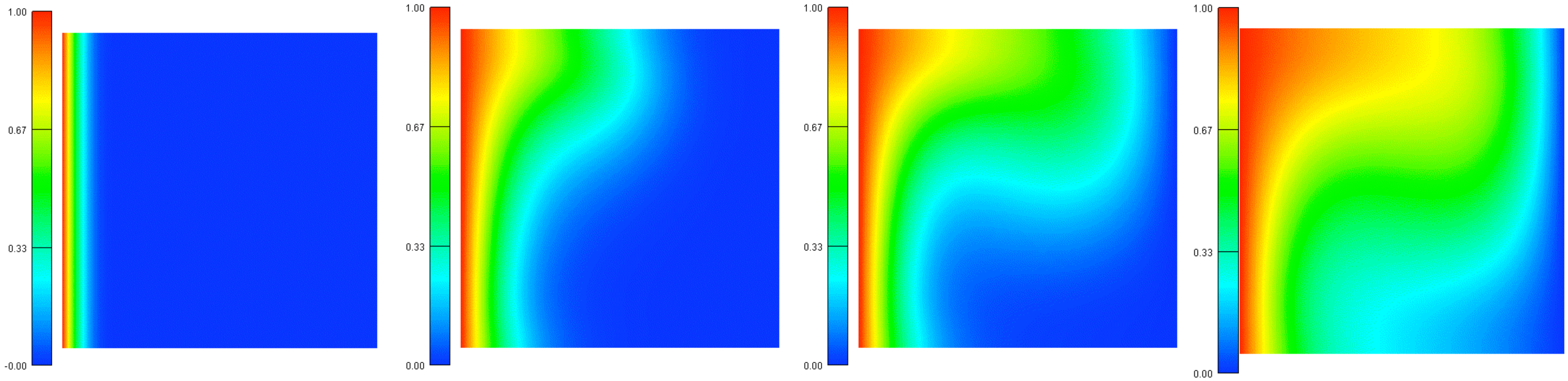
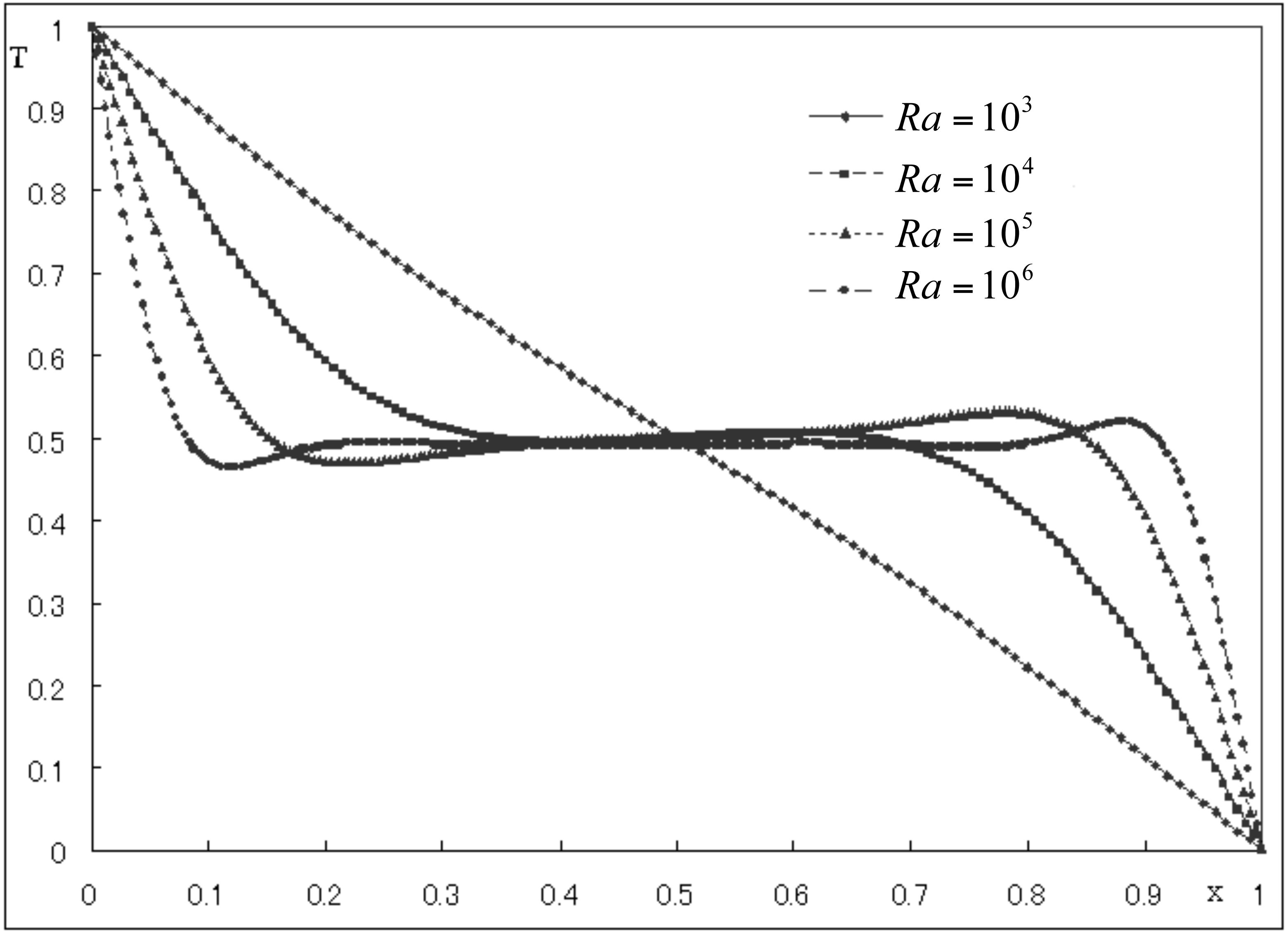
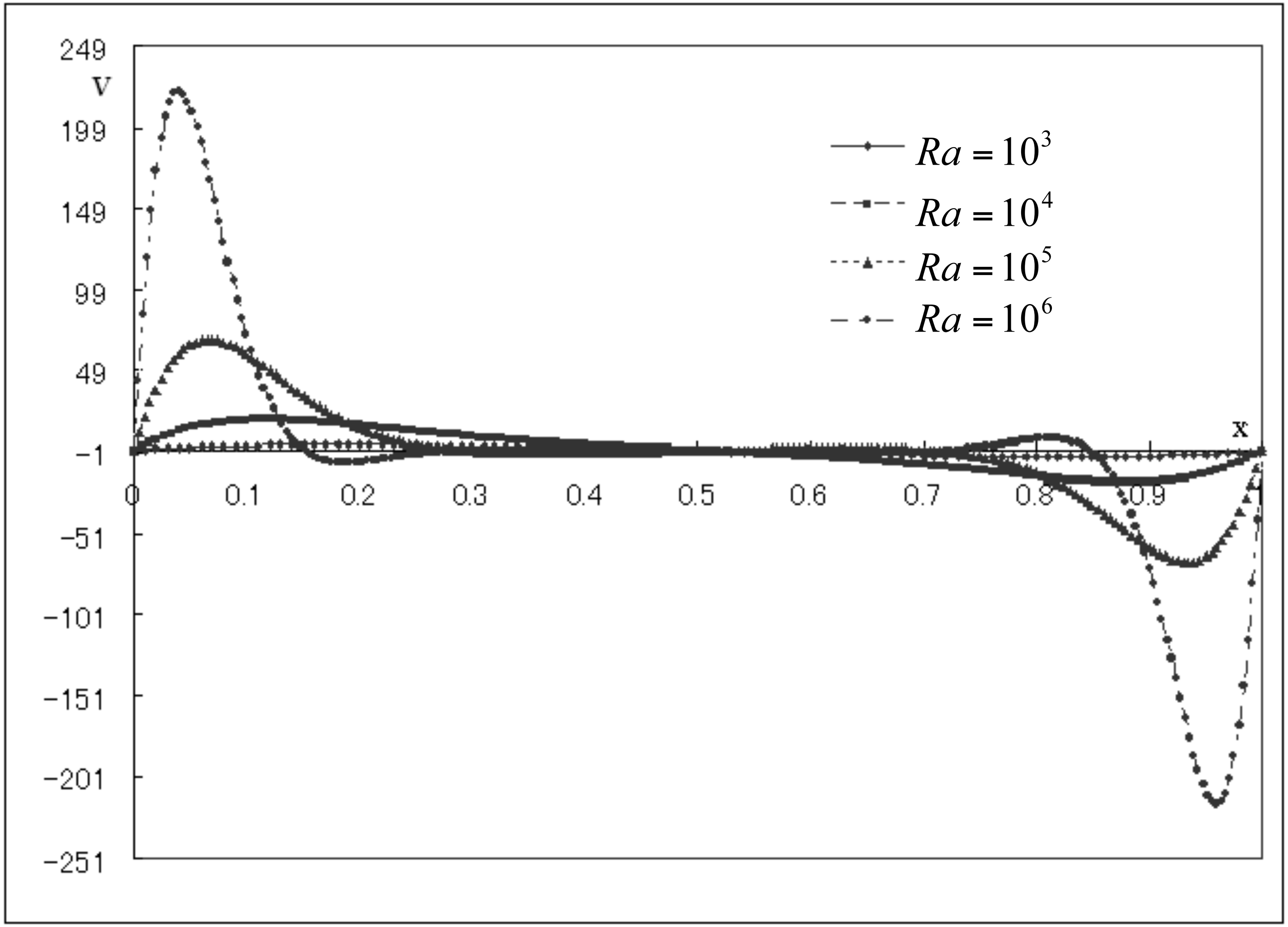


Figure: Animation of temperature

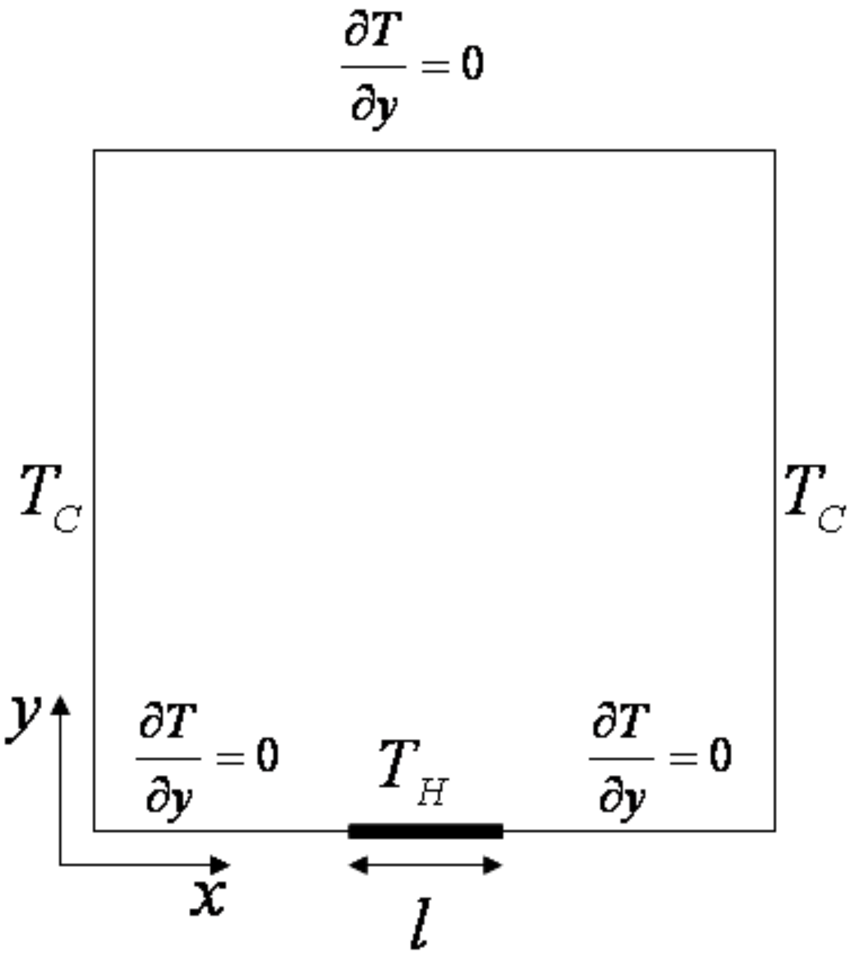


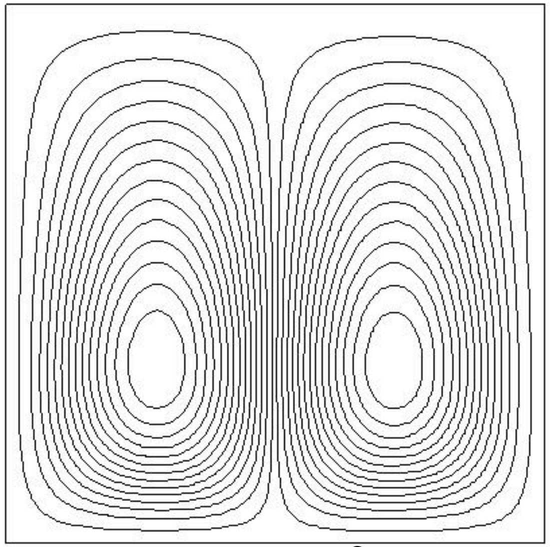
Graph: Temperature profile at the mid-height of the cavity



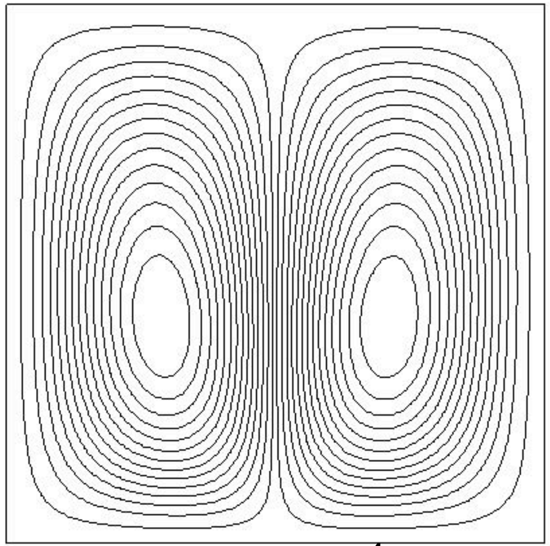
Graph: Vertical velocity profile at the mid-height of the cavity

Numerical Test: localized heating

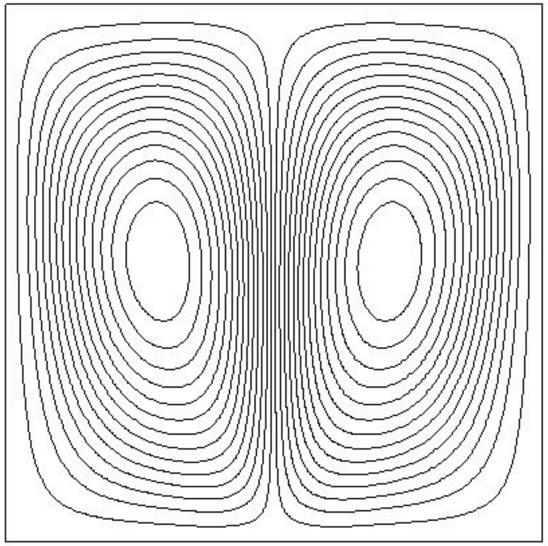




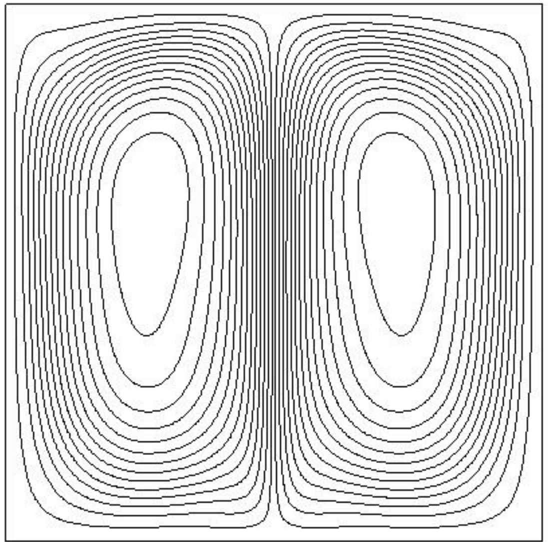
$Ra = 10^3$



$Ra = 10^4$

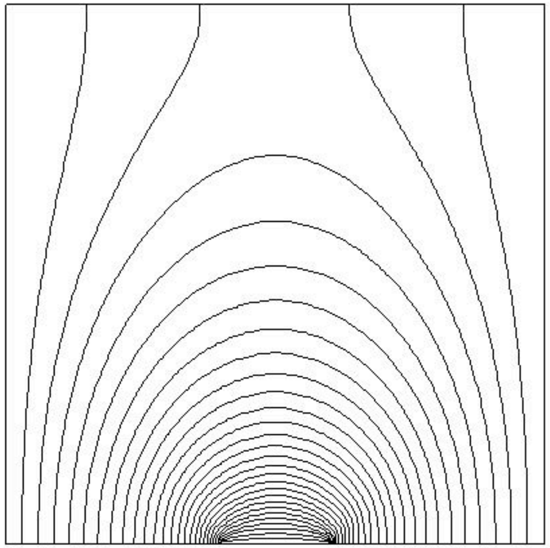


$Ra = 10^5$

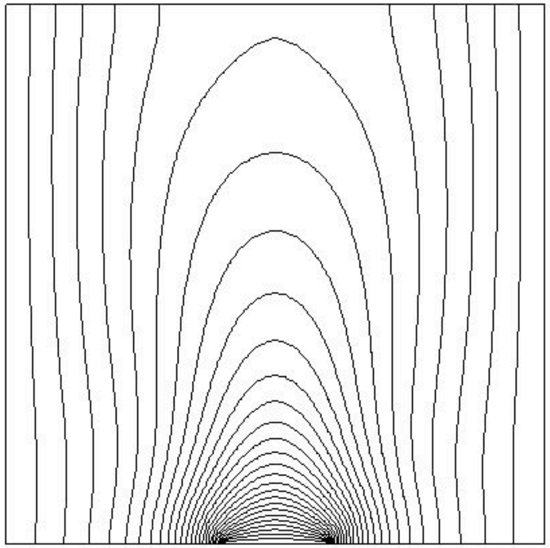


$Ra = 10^6$

Streamlines



$Ra = 10^3$



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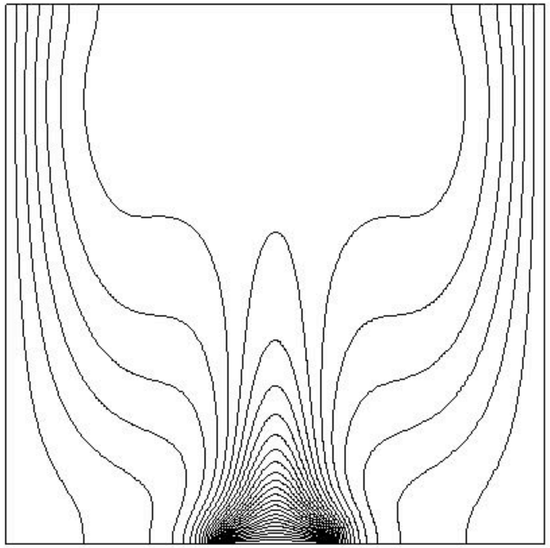
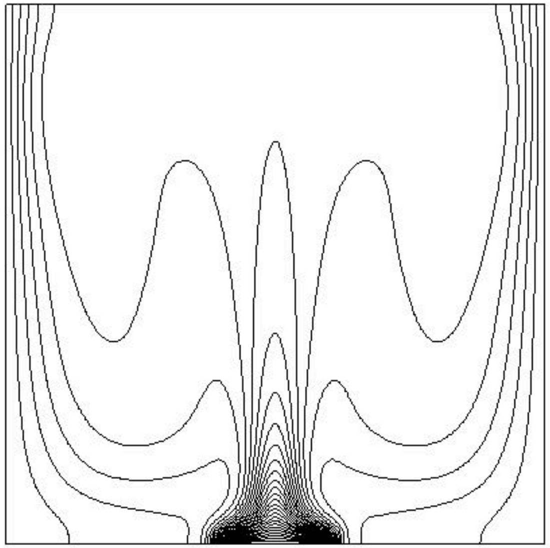
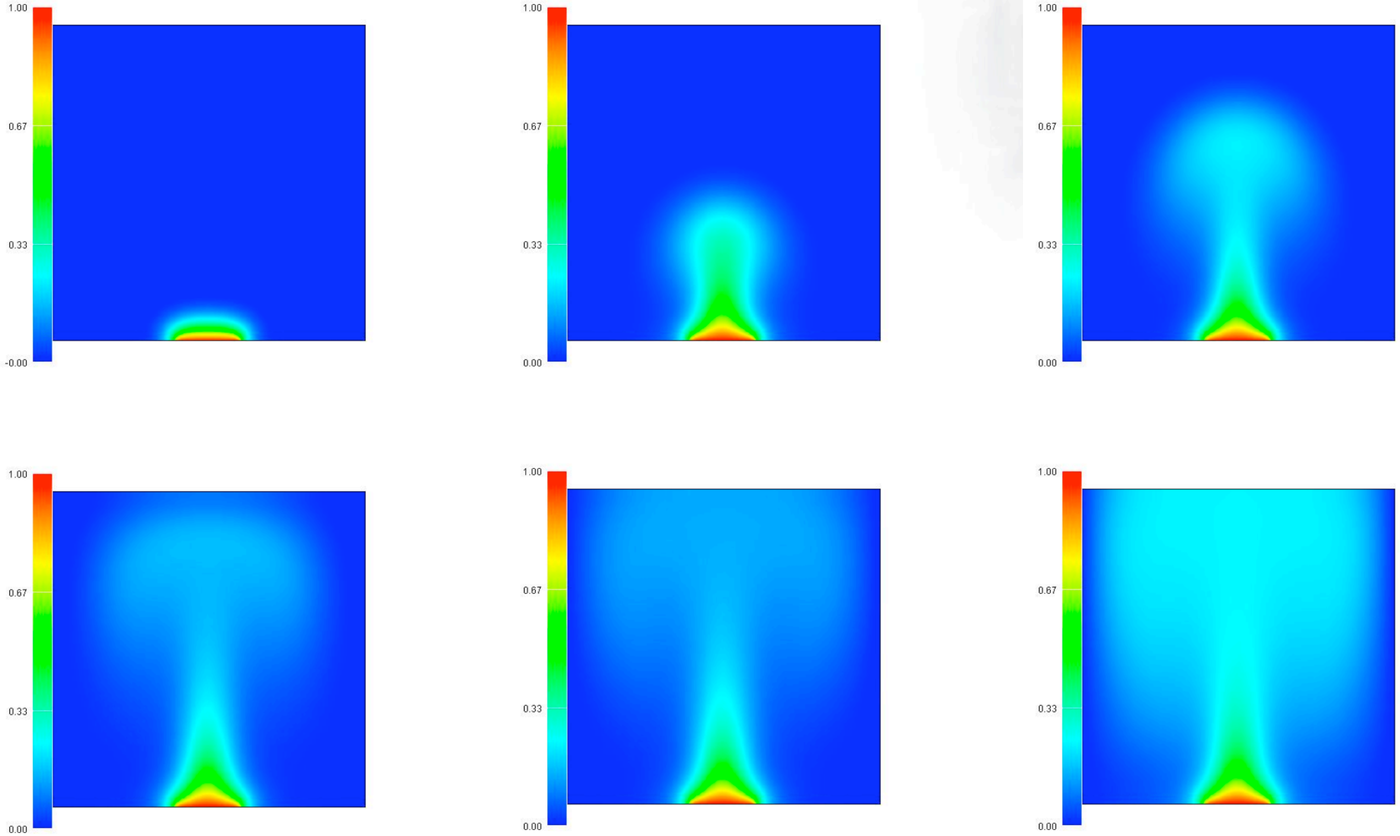


$Ra = 10^5$



$Ra = 10^6$

Isotherms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0.00
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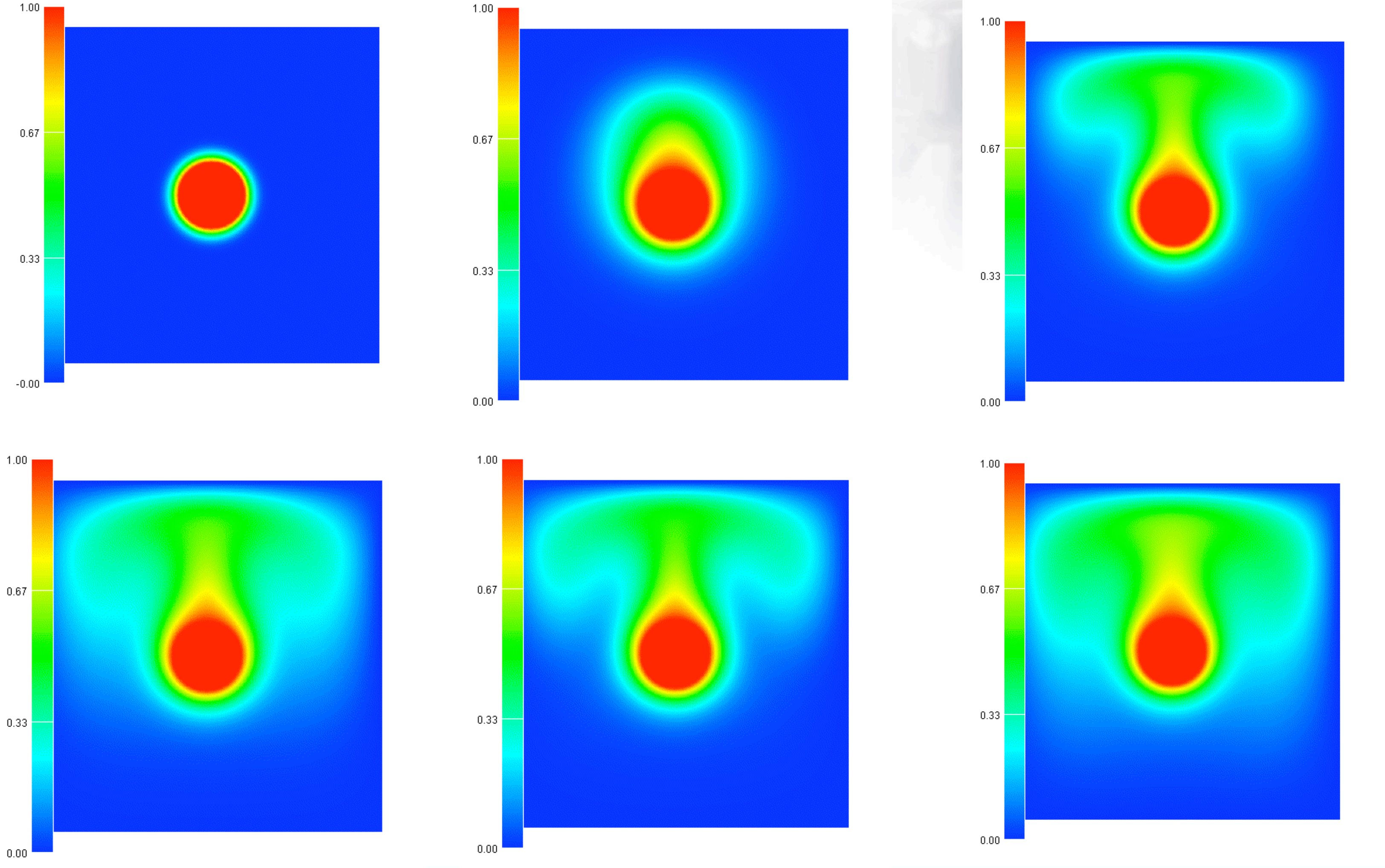
0.00

Numerical Test: Heated cylinder in enclosure



Conclusion

- Only one simple equation required to simulate isothermal fluid flow (two equations for thermal fluid flow)
- Full momentum and energy equation can be derived from Boltzmann equations.
- Short and simple computer program source code.
- Program source code can be easily modified, etc

Future research

- Flow in nano-and micro-tube.
- Fluid structure interaction (flow behind fish motion).
- Droplet motion on inclined plate.