LATTICE BOLTZMAN METHOD

FUNDAMENTAL OF LBM

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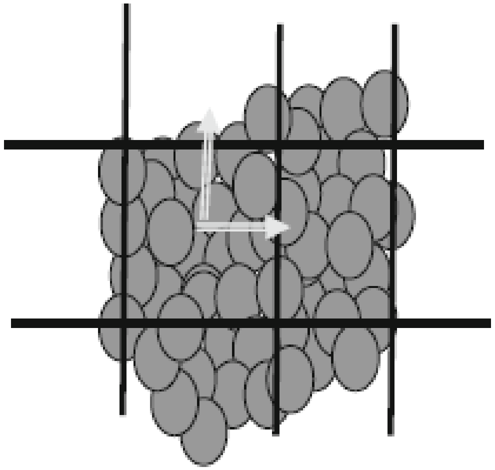
INTRODUCTION

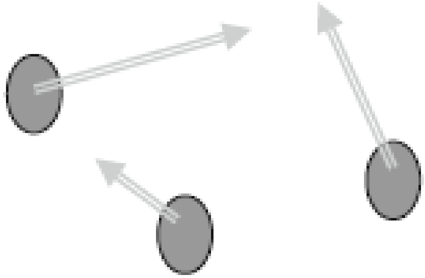
- LBM is a class of computational fluid dynamics (CFD) methods for fluid simulation
- LBM is a relatively new simulation technique for complex fluid systems
- LBM is to bridge the gap between micro-scale and macro-scale by not considering each particle behavior alone

INTRODUCTION



Continuum
(macroscopic-scale),FD,FV,FE

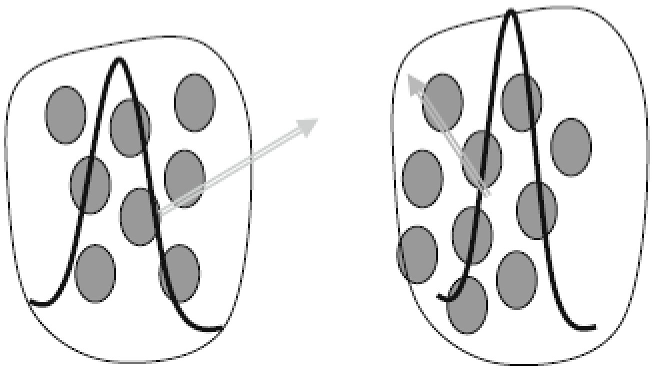
NS equation



Molecular Dynamics
(microscopic-scale)

Hamilton’s Equation

INTRODUCTION



Lattice Boltzmann Method
(Mesoscopic-scale)

Boltzmann Equation

* **Mesoscopic** means intermediate between the microscopic and the macroscopic

ADVANTAGES OF LBM

- Easy to treat multi-phase and multi-component flow
- It can be naturally adapted to parallel process computing
- No need to solve Laplace equation at each time-step
- It can handle a problem in micro & macro-scale with reliable accuracy

DISADVANTAGES OF LBM

- Needs more computer memory compared with NS solver

LIMITATION OF LBM

- High-Mach no. flows in aerodynamics
- Consistent thermo-hydrodynamic scheme
- For m.phase/m.component models, the interface thickness is usually large and the density ratio across the interface is small when compared with real fluids

Boltzmann Transport Equation

If no collisions take place between the molecules

$$f(r + c\mathrm{d}t, c + F\mathrm{d}t, t + \mathrm{d}t)\mathrm{d}r\mathrm{d}c - f(r, c, t)\mathrm{d}r\mathrm{d}c = 0$$

If collisions take place between the molecules

$$f(r + c\mathrm{d}t, c + F\mathrm{d}t, t + \mathrm{d}t)\mathrm{d}r\mathrm{d}c - f(r, c, t)\mathrm{d}r\mathrm{d}c = \Omega(f)\mathrm{d}r\mathrm{d}c\mathrm{d}t$$

Where:

c= velocity
r= position
t= time

$$c = \frac{dr}{dt} \qquad dc = Fdt$$

Therefore,

dr=cdt
dc= Fdt

Boltzmann Transport Equation

$$f(r + c\mathrm{d}t, c + F\mathrm{d}t, t + \mathrm{d}t)\mathrm{d}r\mathrm{d}c - f(r, c, t)\mathrm{d}r\mathrm{d}c = \Omega(f)\mathrm{d}r\mathrm{d}c\mathrm{d}t$$

Collision operator

Dividing the above equation by *dtdrdc* and as a limit dt→ 0, yields

$$\frac{df}{dt} = \Omega(f)$$

The above equation state that the **total rate of change of the distribution function is equal to the rate of the collision**

Boltzmann Transport Equation

The total rate of change can be expanded as:

$$\mathrm{d}f = \frac{\partial f}{\partial r}\mathrm{d}r + \frac{\partial f}{\partial c}\mathrm{d}c + \frac{\partial f}{\partial t}\mathrm{d}t$$

Dividing by dt, yields

$$\frac{\mathrm{df}}{\mathrm{d}t} = \frac{\partial f}{\partial r}\frac{\mathrm{dr}}{\mathrm{d}t} + \frac{\partial f}{\partial c}\frac{\mathrm{dc}}{\mathrm{d}t} + \frac{\partial f}{\partial t}$$

where,

$$\frac{dr}{dt} = c \qquad \frac{dc}{dt} = a$$

Therefore:

$$\frac{\mathrm{df}}{\mathrm{d}t} = \frac{\partial f}{\partial r} c + \frac{\partial f}{\partial c} a + \frac{\partial f}{\partial t}$$

Previously, $\dfrac{df}{dt} = \Omega(f)$

So, $$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial r} \cdot c + \frac{F}{m} \cdot \frac{\partial f}{\partial c} = \Omega$$

For system without external force, the Boltzmann equation can be written as:

$$\frac{\partial f}{\partial t} + c \cdot \nabla f = \Omega$$

The BGKW Approximation

- It is difficult to solve Boltzmann equation because the collision term is very complicated.
- Therefore BGKW approximation introduce a simplified model for collision operator,$\boldsymbol{\Omega}$.

$$\Omega = \omega(f^{\mathrm{eq}} - f) = \frac{1}{\tau}(f^{\mathrm{eq}} - f)$$

Where $\boldsymbol{\omega} = 1/\tau$

$\boldsymbol{\omega}$ is collision frequency
τ is relaxation factor
f^{eq} is Maxwell Boltzmann distribution function

Previously,

$$\frac{\partial f}{\partial t} + c \cdot \nabla f = \Omega$$

After introducing BGKW approximation, the Boltzmann equation (without external force) can be approximated as:

$$\underbrace{\frac{\partial f}{\partial t} + c \cdot \nabla f}_{\text{Streaming process}} = \underbrace{\frac{1}{\tau}\left(f^{\mathrm{eq}} - f\right)}_{\text{Collision process}}$$

Streaming process

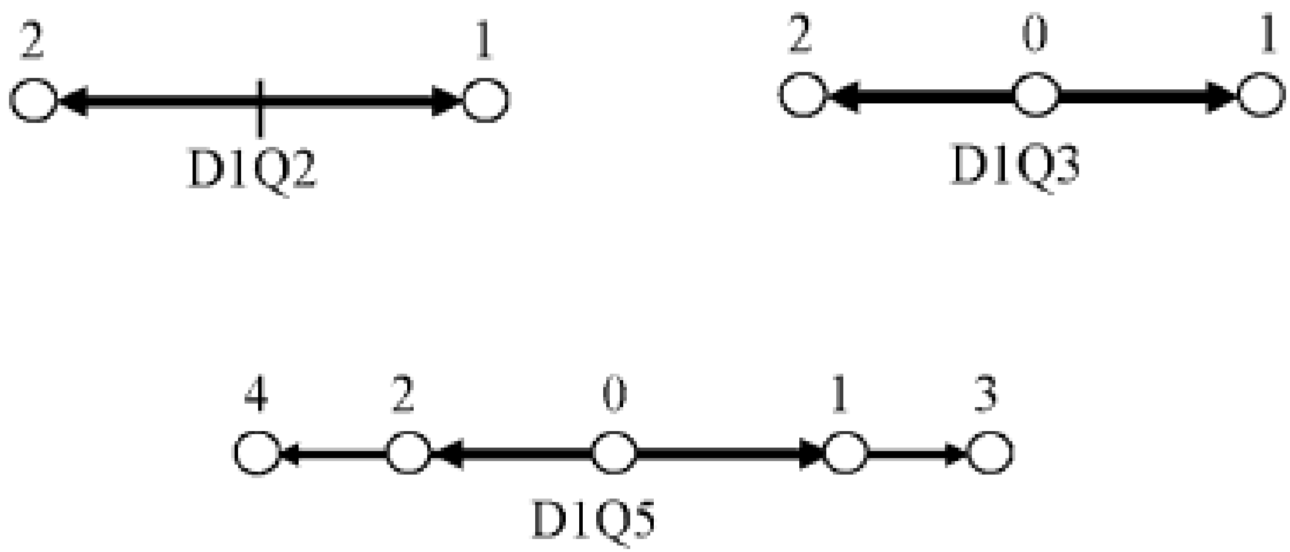
Collision process

Lattice Arrangement

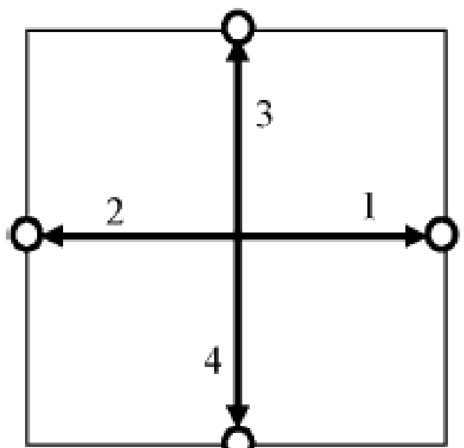
- Use terminology D*n*Q*m*
  - *n* represent dimension
  - *m* refers to the speed model

- One-Dimensional (1D)
  - D1Q2,D1Q3,D1Q5
- Two-Dimensional (2D)
  - D2Q4,D2Q5,D2Q9
- Three-Dimensional (3D)
  - D3Q15,D3Q19

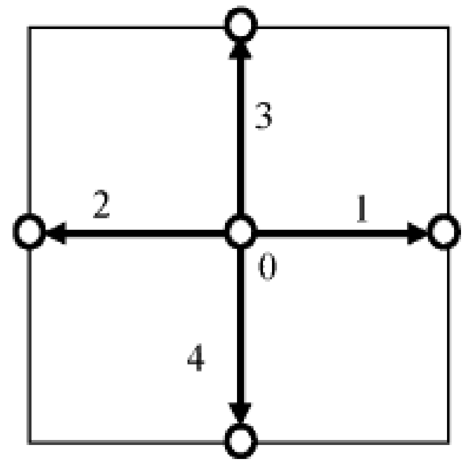
One-Dimensional (1D)



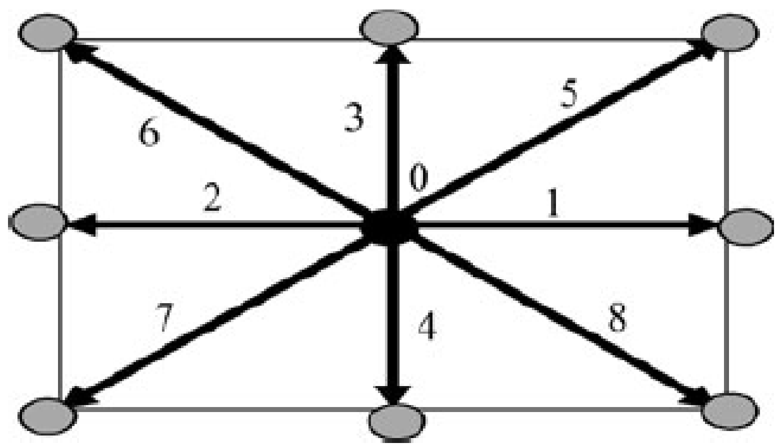
Two-Dimensional (2D)



D2Q4

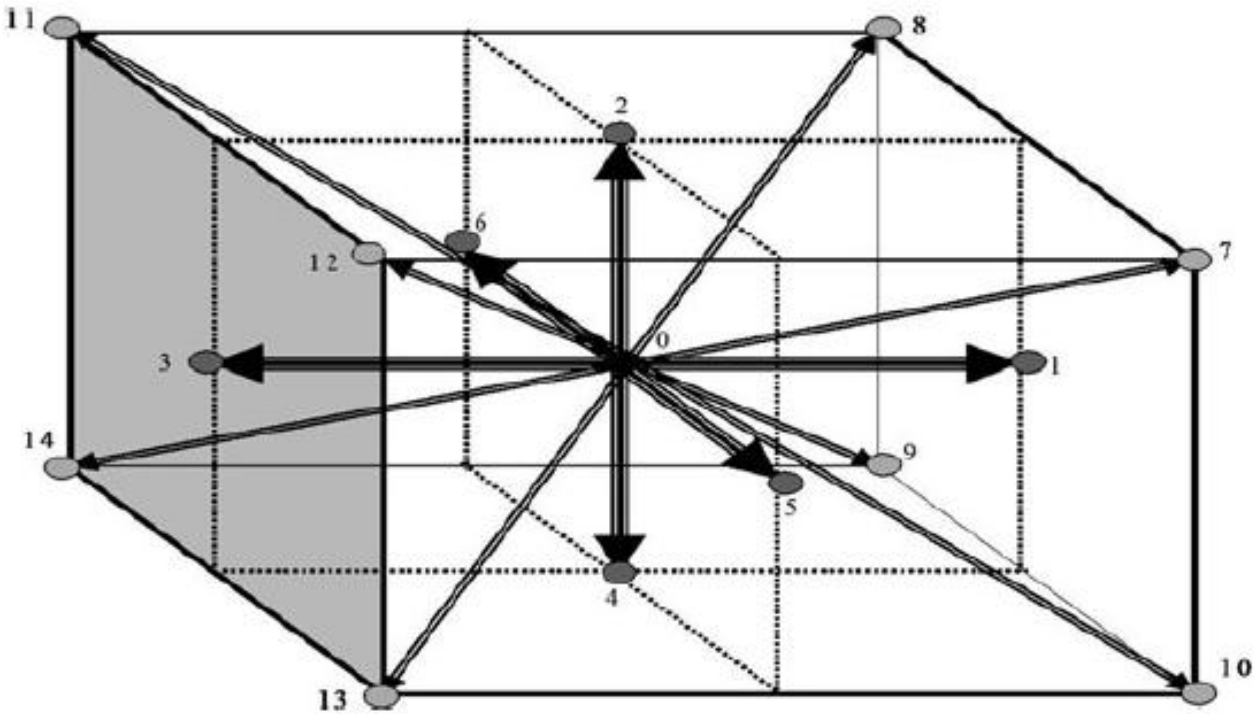


D2Q5



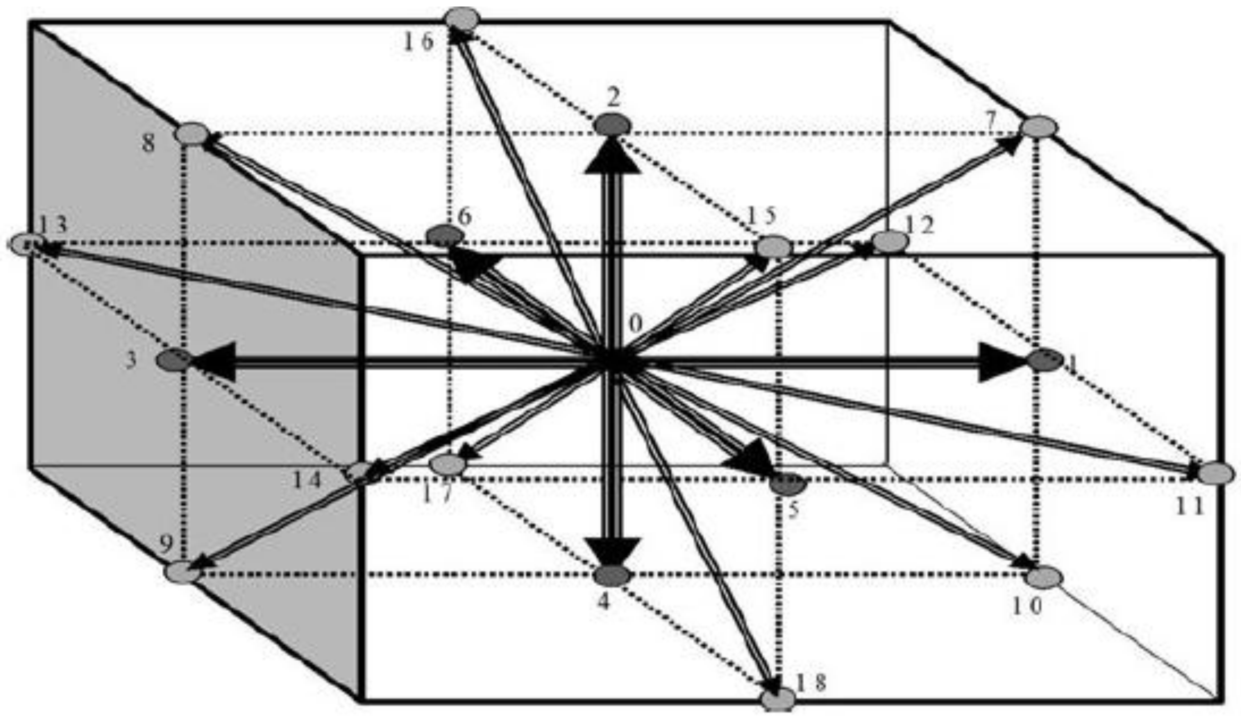
D2Q9

Three-Dimensional (3D)



D3Q15.

Three-Dimensional (3D)



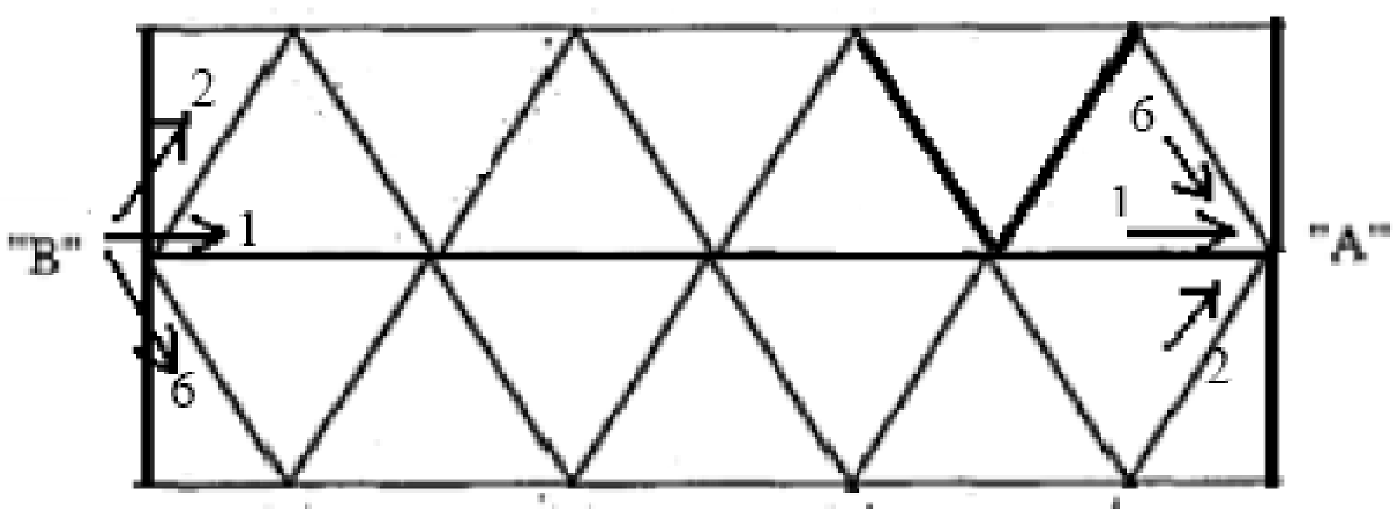
D3Q19.

Boundary Conditions

There are several types of BC used in LBM:

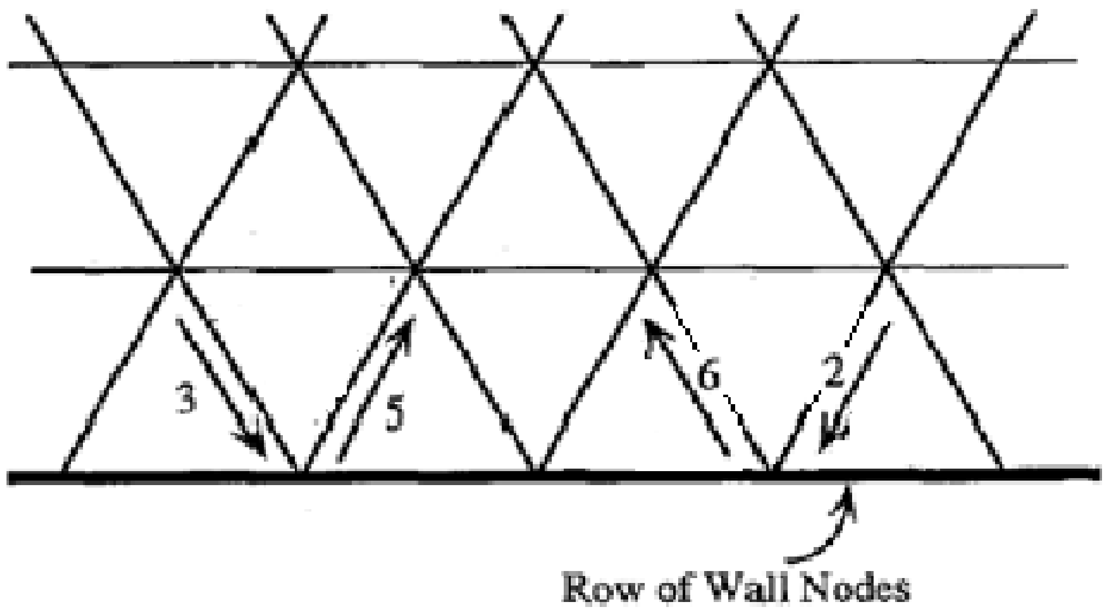
- Periodic boundary condition
- Free slip boundary condition
- Bounce-back boundary condition
- Von neumann (flux) boundary condition
- Drichlet boundary condition

Periodic boundary condition



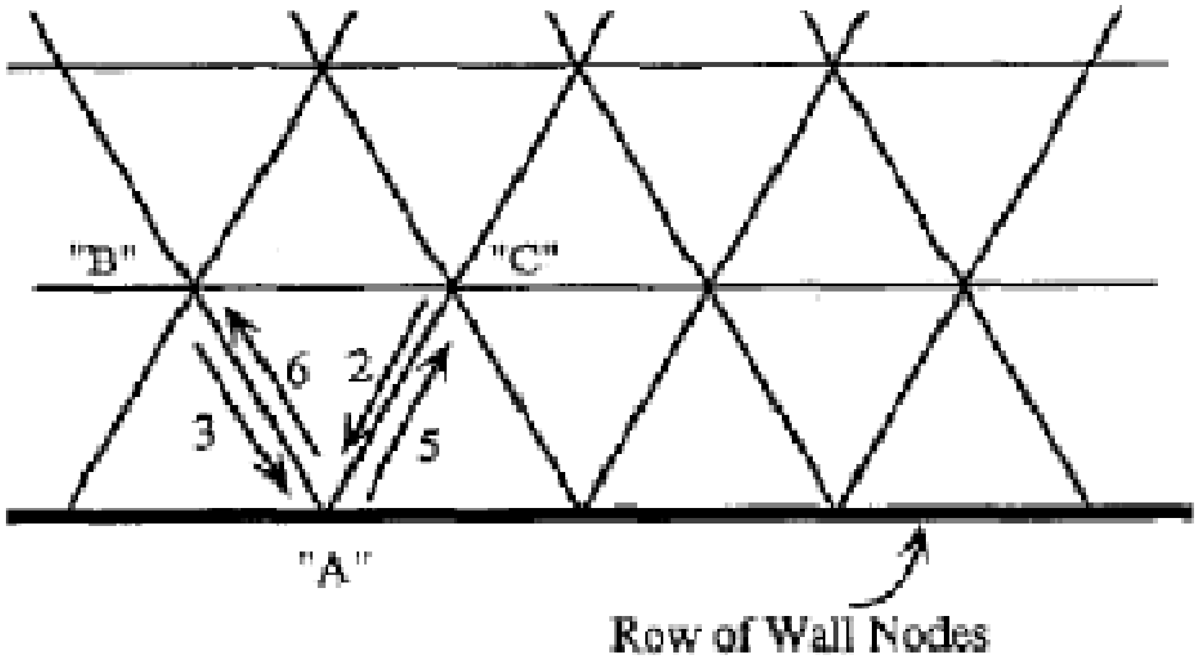
Periodic boundary condition

Free slip boundary condition



Free slip boundary condition

Bounce-back boundary condition



Bounceback boundary condition

THANK YOU
Q & A

"People who succeed in life are those who see objects clearly and lead him without derogating"~Cecil B. DeMille

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