Numerical Prediction of Contaminant Removal from Cavity in Horizontal Channel by Constrained Interpolated Profile Method

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Abstract. In this paper, Constrained Interpolated Profile Method (CIP) was used to simulate contaminants removal from square cavity in channel flow. Predictions were conducted for the range of aspect ratios from 0.25 to 4.0. The inlet parabolic flow with various Reynolds number from 50 to 1000 was used for the whole presentation with the same properties of contaminants and fluid. The obtained results indicated that the percentage of removal increased at high aspect ratio of cavity and higher Reynolds number of flow but it shows more significant changes as increasing aspect ratio rather than increasing Reynolds number. High removal rate was found at the beginning of the removal process.

Introduction

Modelling of transport and dispersion in two-phase particle-laden flows include sediment grains in rivers, cloud particles in the atmosphere, fluidized beds and fluid-solid cyclone separators. Analyzing the hydrodynamic removal of contaminants from equipment is another related field of particle-laden flow study. Superior cleaning process is a matter of interest in the chemical and food industries. Studies of mass transfer in a channel have been discussed in many papers [1-3], and they examined the flow under steady-state conditions, some of them looked at transient flow.

In the early days, experimental approach was the only option for the researchers as it can produce accurate results and based on the real condition. Different authors have measured fluid-solid interaction in a variety of ways such as stereo imaging method [1], magnetic resonance imaging [2] and particle image velocimetry technique [3]. However, precise equipments is needed to get reliable results but they are very expensive need to be supported by research fund. As an alternative approach, many researchers considered computational scheme in their investigations. Kosinski et al. [4,5] provided extensive numerical results on the subject. The behaviour of one particle in a lid-driven cavity flow to thousands of particles in an expansion horizontal pipe has been studied in their research works. On the other hand, another numerical work by Mickaily and Middleman [6], they developed a numerical model to predict the thinning rate of a viscous film of liquid and compared with the experimental observation for smooth and roughened surface.

In recent study, Nor Azwadi et al. [7] investigate the effect of heat to the removal rate on contaminated cavity in channel by using Constrained Interpolated Profile Method (CIP). It shown that the present of heat will improve removal process from cavity. Experimental work on removal of contaminant in cavity was done by Fang et al. [8] and shown at shallow cavity gives better results. Fang et al [9] also has extend their previous work by studying the effect of mixed convection flow numerically.
It appears from the aforementioned investigations that numerous investigations have been conducted on the hydrodynamics contaminant removal. However, to the best of authors’ knowledge, there are no literature reporting on the transient behaviour of solid particle in cavity at wide range of cavity ratio and Reynolds number in isothermal condition. To close this gap, focused has been given on the effect of cavity aspect ratio on transient hydrodynamic removal of a contaminant from a cavity.

**Mathematical Formulation**

![Figure 1. Model of cavity channel flow with contaminant.](image)

Figure 1 shows a model of pipeline with a square cavity full with. The governing equation for the flow field is represented by a vorticity transport equation expressed as:

\[
\frac{\partial \Omega}{\partial t} + U \frac{\partial \Omega}{\partial X} + V \frac{\partial \Omega}{\partial Y} = \frac{1}{Re} \left( \frac{\partial^2 \Omega}{\partial X^2} + \frac{\partial^2 \Omega}{\partial Y^2} \right)
\]  

(1)

In the Cubic Interpolated Profile method (CIP), Eq. (1) and its spatial derivatives were split into advection and non-advection phases [10,11]. In this method, the advection phase of the spatial quantities in the grid interval was approximated with constrained polynomial using the value of its spatial derivative at neighbouring grid points as follow:

\[
F_{i,j}(X,Y) = \left[ (a_1 X + a_2 Y + a_3) \bar{X} + a_4 \bar{Y} + \Omega_X \right] \bar{X} + \left[ (a_5 \bar{Y} + a_6 \bar{X} + a_7) \bar{Y} + \Omega_Y \right] \bar{Y} + \Omega
\]

(8)

\[
F_{X,i,j}(X,Y) = \left[ (3a_1 \bar{X} + 2a_2 \bar{Y} + a_3) \bar{X} + (a_4 + a_5 \bar{Y}) \bar{Y} + \Omega_X \right]
\]

(9)

\[
F_{Y,i,j}(X,Y) = \left[ (2a_2 \bar{Y} + a_3) \bar{X} + (3a_5 \bar{Y} + 2a_6 \bar{X} + 2a_7) \bar{Y} + \Omega_Y \right]
\]

(10)

In the two-dimensional case, the advected profile was approximated as follows:

\[
\Omega^x_{i,j} = F_{i,j}(X + \eta Y + \xi)
\]

(11)

\[
\Omega^x_{X,i,j} = F_{X,i,j}(X + \eta Y + \xi)
\]

(12)

\[
\Omega^x_{Y,i,j} = F_{Y,i,j}(X + \eta Y + \xi)
\]

(13)
where $\eta = -U\Delta T$ and $\xi = -V\Delta T$. Then, calculated spatial quantities were later been used to solve the non-advection phase of Eqs. (5) to (7) together with vorticity formulation.

To predict the dynamics of solid particles in shear-driven cavity flow, the equation of motion for solid particle can be expressed as in Eq. 14 and drag force can be expressed as:

$$m_p \frac{dv_p}{dt} = f_p$$  \hspace{1cm} (14)

$$f_p = C_D A_p \rho \frac{|u - v_p| (u - v_p)}{2}$$  \hspace{1cm} (15)

where $m_p$, $v_p$, $f_p$, $A_p$ are the mass of particle, velocity, drag force acting on particle due to the surrounding fluid, projected area of solid particle and $C_D$ is the drag coefficient which given as:

$$C_D = \frac{24}{Re_p}$$  \hspace{1cm} (16)

The particle’s Reynolds number in the above equation was defined as follow:

$$Re_p = \frac{d_p |u - v_p|}{\nu}$$  \hspace{1cm} (17)

where $d_p$ is the diameter of solid particle. In the computational technique, the particles’ velocity $v_p^{n+1}$ can be determined since the initial value of $v_p^0$ was known, thus the new position of solid particle can be determined as follow:

$$x_p^{n+1} = v_p^{n+1} \Delta t + x_p^n$$  \hspace{1cm} (19)

**Results and Discussion**

Transient hydrodynamic removal of solid particles from a cavity was presented. It covered a range of Reynolds number up to 1000 and the aspect ratio of 0.25 to 4.0. Properties of the contaminated flow were similar for the fluid flowing in the channel so that the energy exchange between the two phases could be neglected. The simulation consists of 1,600 particles distributed randomly in the cavity. Since the point-force Lagrangian approach was utilized, very small particle size was selected with respect to the grid size. The rate of contaminant removal at Reynolds number of 50 ($Re = 50$) and aspect ratio of 1.0 and 4.0 (AR = 1 and AR = 4) were compared with the experimental data by Fang et al [8]. As can be seen in Figure 2, our finding is comparable with Fang et al. [8], which shows that the removal rate and percentage of removal at steady state was higher for AR = 4 compared to lower aspect ratio.

The rate of contaminant removal was increased significantly during the unsteady start-up and reached the maximum value at the early phase of particle motion. The percentage of particles removed from the cavity for various aspect ratios and Reynolds number of 50 is depicted in Figure 3. The figure clearly demonstrates the effect of deeper penetration of the flow inside the cavity, caused by the increased in aspect ratio and leads to improved cavity clearance. At low aspect ratio or deep cavity, less removal was observed due to more than one circulation present inside the cavity. In addition, particles with less centrifugal force were trapped in the cavity, which prevented
them from escaping from the weak vortex formed in the deep cavity. Consistent with the findings by Fang et al [8], the present study recorded minimum removal about 1.5% and maximum about 69% when aspect ratios were 0.25 and 4.0 respectively, as shown in Figure 4.

Figure 2: Percentage of particle removal against time for Reynolds number 50 for present study and experimental [8] for AR = 1 and AR = 4.

Figure 3: (a) Removal rate for various aspect ratio at Re = 50 and (b) Comparison of present study with Fang et al [8].

Figure 4: removal rate for various aspect ratio at various Reynolds number.
The percentage of particles removed from the cavity for various aspect ratios and Reynolds number at steady state is illustrated in Figure 4. As expected, when the aspect ratio increased, the percentage of removal increased for all Reynolds number. It is also shown similar relation that higher Reynolds number will gives higher removal. However, an interesting finding from the figure was that the lowest Reynolds number of 50 gave the highest percentage of removal at aspect ratio of 4.0 except for Re = 1000. A possible explanation was that at high Reynolds number (Re=100 to Re=400), a big and unstructured vortex was formed inside the cavity. This vortex trapped more particles and randomly placed inside the cavity permanently. Different with Re=1000, it produce bigger circulation in the cavity and push more of particles than at Re=50.

Conclusion

Particle removal from rectangular cavity in channel flow has been studied using Constrained Interpolated Profile Method (CIP) to solve advection part of Navier-Stokes equations. The effect of cavity aspect ratio on the hydrodynamic removal of particles contained was illustrated for different Reynolds number. The results from our study were captured at the transient state of solution for both fluid and particles. The rate of removal increased during the unsteady start-up period and reached the maximum value at the early phase of particle motion. Variation of aspect ratio showed significant changes in the percentage of removed particles compared to Reynolds number.

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References