Mathematical Models for Productivity Rate of Automated Lines with Reliability Attributes of Mechanisms

Ryspek Usubamatov[1, a] and Tan Chan Sin[1, b]

[1]School of Manufacturing Engineering, Campus Tetap Pauh, University Malaysia Perlis, 02600 Arau, Perlis, Malaysia

[a]ryspek@unimap.edu.my, [b]tcs5077@hotmail.com

Keywords: Productivity, Reliability, Automated lines

Abstract. Automated lines with complex structures consist of stations and mechanisms with different levels of reliability. Most publications that present the mathematical models for productivity of automated lines are based on simplifications to derive the approximate equations of productivity. Simplification is based on the premise that all stations of the automated lines have one level of reliability, and balancing of technological process on stations is conducted evenly, etc. However, manufacturers need correct and clear mathematical models to enable the calculation of the productivity of the automated lines with high accuracy. This paper presents the analytical approach to the productivity rate of the automated lines with stations and mechanisms, with different failure rates, and processing times. The proposed mathematical models allow for the output of automated lines to be modelled with results that are close to actual productivity.

Introduction

Mathematical models for the productivity rate of the industrial machines with complex designs are primary attributes, to evaluate the efficiency of manufacturing system. Methodology for calculation of the system productivity is presented by several key publications that consider all aspects of manufacturing processes [1-4].

Efficiency of expensive production systems like automated lines with complex design depends on reliability of main mechanisms and units. Reliability problem in industry is not new and its attributes of engineering are well presented and made it possible to describe analytically manufacturing problems [5-7]. The theory of reliability provides standard attributes and modes of calculation that can be used for the mathematical modeling of reliability of manufacturing systems. However, known attributes of reliability describe properties of an industrial machine separately from its productivity rate and other indices. In literature, there are publications which describe the reliability of manufacturing systems using a probabilistic approach that allow for the calculation of the average magnitudes of searching parameters [6-7]. Several publications are dedicated to studying the productivity and reliability attributes of automated lines with different designs [8]. Analysis of the equations for productivity of multi-station automated lines of different structures shows that for simplification, the attributes of reliability of stations are accepted by average magnitude. Such simplification formulates equations for productivity rate of the automated lines which are unable to give accurate results in calculation [3].

Mathematical models of industrial machines' productivity rate are derived according to the level of consideration of the manufacturing processes. This article presents the equations of productivity that include the technological and technical aspects of the manufacturing systems and do not consider aspects of maintenance of machines at prescribed planned overhaul repair time.

The reliability theory represents the indices by the following attributes: the machine failure rate $\lambda = 1/m_w$; mean time to work m_w; mean time to repair m_r; and availability A. These attributes of reliability are used to develop analytical equations for the productivity rate of machines with complex designs like automated lines of different structures. This paper presents mathematical models of productivity rate of automated lines, which stations failure rates are different and results of productivity calculations will have values close to the actual output.

Analytical approach

The actual reliability of the industrial machine is high and probability of failure is low. However, reliability of automated lines is decreasing with increasing the number of stations that have hard linking. Any random failures of a station lead to stops of whole automated line with other workable stations. These random dawn times of automated lines is calculated by the rules of probability theory. Naturally, reliability level of manufacturing machines, units and systems are different. However, some reliability attributes like the repair time is short and accepted in average normative repair time. These regulations are used in mathematical models for productivity of the manufacturing systems with different failure rates of stations.

The mathematical models for productivity of automated lines are represented by different equations [9]. The typical designs of the automated lines are presented by three types of structure: a automated line of serial, parallel, and serial-parallel structures. All these designs can be presented by linear, circular or rotary arrangements.

- Productivity of the serial automated line with mechanisms of different reliability. Generally, machines with complex designs like automated lines consist different stations (drilling heads, milling head, boring head, power head, control station, etc.) and common units and devices such as a control system, a transport mechanism, etc., that serve the whole automated line (Fig. 1). This is typical serial automated line with hard linked stations and units. Failure of any mechanism leads to a stop of the whole line.

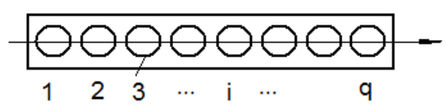

Figure 1. Scheme of an automated line with q serial stations

The equation of productivity of serial automated line with q stations represented by the following equation [3, 9].

$$Q = \frac{1}{T} \times \frac{1}{1 + \sum_{i=1}^{g} t_{ei} / T} = \frac{1}{\frac{t_{mo}}{q} + t_a} \times \frac{1}{1 + \sum_{i=1}^{g} t_{ei} / \left(\frac{t_{mo}}{q} + t_a \right)} \qquad (1)$$

where t_{mo} is total machining time of a product, t_a is auxiliary time to prepare machining process, $T = (t_{mo}/q) + t_a$ is the cycle time, q is number of serial stations, $\sum_{i=1}^{g} t_{e.i}$ is time losses due to n mechanisms and units of an automated line referred to one product, $Q = 1/T$ is cyclic productivity, A is availability of an automated line.

The time losses for each station and mechanisms are different. Hence, the equation for the time losses of stations and common mechanisms and units can be represented by the following equation:

$$\sum_{i=1}^{n} t_{ei} = \frac{\theta_{s.1}}{z} + \frac{\theta_{s.2}}{z} + \ldots + \frac{\theta_{s.i}}{z} \ldots + \frac{\theta_{s.q}}{z} + \frac{\theta_c}{z} + \frac{\theta_{tr}}{z} = \sum_{i=1}^{q} t_{s.i} + t_c + t_{tr} \qquad (2)$$

where $\theta_{s.i}$ is idle time of i station, θ_c is idle time of common mechanisms and units, θ_{tr} is idle time of a transport mechanism, the ratio of $\sum_{i=1}^{q} t_{s.i} = \left(\sum_{i=1}^{q} \theta_{s.i}\right)/z$ is time losses referring to one product due to reliability of q stations, the ratio of $t_c = \theta_c/z$ is time losses referring to one product due to reliability of common control system, the ratio of $t_{tr} = \theta_{tr}/z$ is time losses referring to one product due to reliability of common transport mechanism; other parameters are as specified above.

Substituting Eq. (2) into Eq. (1) and transforming it, formulates the following equation for the productivity rate of a machine:

$$Q = \frac{1}{\frac{t_{mo}}{q} + t_a} \times \frac{1}{1 + \left(\sum_{i=1}^{q} t_{s.i} + t_c + t_{tr}\right) / \left(\frac{t_{mo}}{q} + t_a\right)} \tag{3}$$

where all parameters are as specified above.

An automated line after balancing has a bottleneck station, whose machining time is longest. The accepted average machining time $t_{m.o}/q = t_{av}$ of one section presented above should be replaced by the machining time $t_{m.b.}$ of the bottleneck station. Hence, the machining time $t_{m.b.}$ of the bottleneck station can be represented by the equation $t_{m.b} = f_m t_{m.o}/q$, where f_m is the correction factor that expresses the difference in the machining time between the bottleneck station and the average machining time of the station. In these connections, Eq. (3) is corrected and presented as in the following equation:

$$Q = \frac{1}{\frac{t_{mo}}{q} f_m + t_a} \times \frac{1}{1 + \left(\sum_{i=1}^{q} t_{s.i} + t_c + t_{tr}\right) / \left(\frac{t_{mo}}{q} f_m + t_a\right)} = Q_T \times A \tag{4}$$

Eq. (4) has two components. The equation $Q_T = 1/[(t_{mo}/q)f_m + t_a]$ is the cyclic productivity of an automated line and A its availability. The equation of availability of serial automated line can be represented by symbols of idle time and work times

$$A = \frac{1}{1 + \left(\sum_{i=1}^{q} t_{s.i} + t_c + t_{tr}\right) \Big/ T} = \frac{1}{1 + \frac{\sum_{i=1}^{g} \theta_i}{zT}} = \frac{1}{1 + \frac{\sum_{i=1}^{g} \theta_i}{(\theta_w / T)T}} = \frac{1}{1 + \frac{\sum_{i=1}^{g} \theta_i}{\theta_w}} \tag{5}$$

where $\theta_i = m_r n_i$ is idle time of i mechanism, m_r is mean repair time, n_i is number of failures of i mechanism, $\theta_w = m_w b$ is work time of the automated line, m_w is mean time of work between two failures, b is total number of failures. The total number of failures of the automated line is the sum of failures of single mechanisms and units, i.e.,

$b = n_{s.1} + n_{s.2} + n_{s.3} + \ldots + n_{s.i} + \ldots + n_{s.q} + n_c + n_{tr} = \sum_{i=1}^{g} n_i$, where $n_{s.1}$, $n_{s.2}$, $n_{s.3}$, and $n_{s.i}$ is number of failures of station 1, 2, 3 and i, n_c, and n_{tr} is number of failures of control and transport systems accordingly. Substituting defined parameters into Eq. (5) and transforming, gives the following equation of availability

$$A = \frac{1}{1+\dfrac{m_r n_{s.1} + m_r n_{s.2} + \ldots + m_r n_{s.i} + \ldots + m_r n_{s.n} + m_r n_c + m_r n_{tr}}{m_w b}} = \frac{1}{1+\dfrac{m_r (n_{s.1} + n_{s.2} + \ldots + n_{s.i} + \ldots + n_{s.q} + n_c + n_{tr})}{m_w b}} = \tag{6}$$

$$\frac{1}{1+m_r\lambda\dfrac{(n_{s.1} + n_{s.2} + \ldots + n_{s.i} + \ldots + n_{s.q} + n_c + n_{tr})}{b}} = \frac{1}{1+m_r\lambda(f_{s.1} + f_{s.2} + \ldots f_{s.i} \ldots + f_{s.q} + f_c + f_{tr})}$$

where $\lambda = 1/m_{\mathrm{w}}$ is average failure rate of the automated line, $f_{i.i} = n_{i.i}/b$ is coefficient of correction, which show the ratio of number failures of i mechanism to total number of failures of an automated line.

Eq. (6) consists components $\lambda f_{i\cdot i} = \lambda_i$ which is failure rate of i mechanism (station, control unit, etc.). Thus, the mathematical model for productivity of a serial automated line with station, mechanisms and units of different failure rate is represented by the following equation.

$$Q = \frac{1}{\dfrac{t_{mo}}{q} f_m + t_a} \times \frac{1}{1 + m_r\left(\sum_{i=1}^{q} \lambda_{s.i} + \lambda_c + \lambda_{tr}\right)} \tag{7}$$

where all parameters are as specified above.

Equation (7) of productivity of serial automated line is represented via symbols of reliability of each station and mechanism and the machining and auxiliary times of a bottleneck station. The analytical result and failure rate λ_{i} of some mechanism that are represented as product of the average failure rate λ and correction factor $f_{i,}$ ($\lambda_i = \lambda f_i$) will be used for the other equations for the productivity of the automated lines of different designs.

- Productivity of automated lines with parallel, serial - parallel actions have different reliability of mechanisms. Manufacturers produce automated lines with a different number of parallel p and serial q stations. An automated line with parallel structure has identical parallel stations with equal reliability. Increasing the number of parallel stations in the line lead to increasing the productivity rate, but reliability attributes of lines with complex design impose restrictions in machine designs. Optimization of the structure for the automated line with parallel stations is not a simple process that should be considered by economical approaches.

The schematic diagram of such a line is represented in Fig. 2.

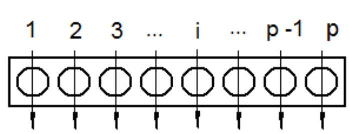

Fig. 2 Scheme of an automated line of parallel structure with p parallel stations

The equation of the productivity rate for the automated line with parallel stations is represented by the following [3, 9].

$$Q = \frac{p}{t_m + t_a} \times \frac{1}{1 + m_r[p_s(\lambda_{s.} + \lambda_f) + \lambda_c]} \tag{8}$$

where p is number of products machined, p_s is the number of parallel station arrangements, $p = p_s$, λ_f is failure rate of the feeder, λ_c is failure rate of the control system that serve whole line, other parameters are as specified above.

Eq. (8) enables the calculation of the maximum productivity rate by using mathematical limit of equation with the variable p, when other parameters are constant.

$$\lim_{p\to\infty} Q = \lim_{p\to\infty}\left(\frac{p}{t_m + t_a} \times \frac{1}{1 + m_r[p_s(\lambda_{s.} + \lambda_f) + \lambda_c]}\right), \quad Q_{\max} = \frac{1}{m_r(\lambda_{s.} + \lambda_f)(t_m + t_a)} \tag{9}$$

The maximum productivity of the automated line with parallel stations depends on the machining, auxiliary times and failure rate of the station and the meant time to repair the automated line.

Results and discussion

The derived equations for the productivity rate of the typical automated lines with complex designs and different reliability of mechanisms enable the prediction of the real output of production systems. The equations enable the calculations of the productivity rate of the automated line as a function of the number of serial and parallel stations and reliability attributes of the automated lines' mechanisms and units. Simplified approach in calculations based on the average technical parameters of the automated line can give serious deviations in results.

Conclusion

The productivity rate of automated lines is a very important economical index that should be predicted and evaluated by analytical methods as much as possible with accurate results. This paper formulated mathematical models of the productivity rate for typical designs of automated lines of serial and parallel structures which have different reliability levels of the mechanisms and stations. The equations of the productivity rate of automated lines are represented as functions of the technological and technical parameters as well as attributes of reliability of typical mechanisms and units. Represented equations of the productivity rate enable designers to analyse the output of automated lines, with results which are close to real data. Simplified equation of the productivity rate based on the average technical parameters of the failure rates can give considerable deviation from the real result. Engineers and designers of automated lines can use the derived mathematical models of productivity rates in the project stage of automated lines design.

References

1. G. Chryssolouris, Manufacturing systems: theory and practice, 2nd edn, Springer, New York, (2006).
2. M.P. Groover, Fundamentals of modern manufacturing: materials, processes and systems, 4th edn, Wiley, New York, (2010).
3. L. Volchkevich, Automation of production processes, Mashinostroenie, Moscow, (2005).
4. S.B. Gershwin, Manufacturing systems engineering. Prentice Hall, New Jersey, (1994).
5. P.D.T. O'Connor, Practical reliability engineering. 4th edn, Wiley, New York, (2002).
6. A. Birolini, Reliability engineering: theory and practice. 5th edn, Springer, New York, (2007).
7. T. Freiheit, Y, Koren, S.J. Hu, Productivity of parallel production lines with unreliable machines and material handling. IEEE Trans 1(1), (2004): 98 – 103.
8. R. Usubamatov, K.A. Ismail, J.M. Shah, Mathematical models for productivity and availability of automated lines, International Journal of Advanced Manufacturing Technology, Vol. 66, Issue 1, (2013), pp. 59-69.